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Logic

(contents)

- 1) The Mathematical analysis of Logic
By Geo. Boole: Cambridge: 1847
- 2) Recent publications on Logical Science
an article in the Edinburgh Review
by Sir Wm Hamilton
- 3) First notions of Logic
By Augustus De Morgan: London: 1840
- 4) Statement in answer to an apertion
made by Sir Wm Hamilton Bart:
By A De Morgan: London: 1847
- 5) A letter to Augustus De Morgan Esq
.... in his claims to an independent
rediscovery of a new principle in
the theory of Syllogism
By Sir W. Hamilton:
London & Edinburgh: 1847
subjoined:— the previous correspondence
& a P.S in answer to the above No 4
- 6) Mill's Logic
an article in the Dublin Review
(no 68)

H. P. M. Key
from his Father
2 May 1871

THE MATHEMATICAL ANALYSIS OF LOGIC,

BEING AN ESSAY TOWARDS A CALCULUS
OF DEDUCTIVE REASONING.

BY GEORGE BOOLE.

Ἐπικοινωνοῦσι δὲ πᾶσαι αἱ ἐπιστήμαι ἀλλήλαις κατὰ τὰ κοινά. Κοινὰ δὲ
λέγω, οἷς χρῶνται ὡς ἐκ τούτων ἀποδεικνύντες· ἀλλ' οὐ περὶ ὧν δεικνύουσιν,
οὔτε ὁ δεικνύουσι.

ARISTOTLE, *Anal. Post.*, lib. 1. cap. xi.

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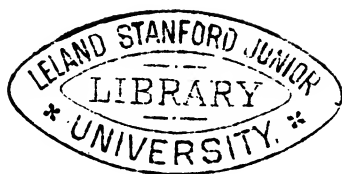
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ERRATA.

- Page 5, note, for vi. read iv.
" 8, line 31, for first read fact.
" 17, " 12, for abstraction read election.
" 53, " 12, for numbers read members.
" 66, " 12 and 32, for z read w.
" 80, " 27, for 3 read 2.



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STANFORD

PREFACE.

IN presenting this Work to public notice, I deem it not irrelevant to observe, that speculations similar to those which it records have, at different periods, occupied my thoughts. In the spring of the present year my attention was directed to the question then moved between Sir W. Hamilton and Professor De Morgan; and I was induced by the interest which it inspired, to resume the almost-forgotten thread of former inquiries. It appeared to me that, although Logic might be viewed with reference to the idea of quantity,* it had also another and a deeper system of relations. If it was lawful to regard it from *without*, as connecting itself through the medium of Number with the intuitions of Space and Time, it was lawful also to regard it from *within*, as based upon facts of another order which have their abode in the constitution of the Mind. The results of this view, and of the inquiries which it suggested, are embodied in the following Treatise.

It is not generally permitted to an Author to prescribe the mode in which his production shall be judged; but there are two conditions which I may venture to require of those who shall undertake to estimate the merits of this performance. The first is, that no preconceived notion of the impossibility of its objects shall be permitted to interfere with that candour and impartiality which the investigation of Truth demands; the second is, that their judgment of the system as a whole shall not be founded either upon the examination of only

* See p. 42.

PREFACE.

a part of it, or upon the measure of its conformity with any received system, considered as a standard of reference from which appeal is denied. It is in the general theorems which occupy the latter chapters of this work,—results to which there is no existing counterpart,—that the claims of the method, as a Calculus of Deductive Reasoning, are most fully set forth.

What may be the final estimate of the value of the system, I have neither the wish nor the right to anticipate. The estimation of a theory is not simply determined by its truth. It also depends upon the importance of its subject, and the extent of its applications; beyond which something must still be left to the arbitrariness of human Opinion. If the utility of the application of Mathematical forms to the science of Logic were solely a question of Notation, I should be content to rest the defence of this attempt upon a principle which has been stated by an able living writer: “Whenever the nature of the subject permits the reasoning process to be without danger carried on mechanically, the language should be constructed on as mechanical principles as possible; while in the contrary case it should be so constructed, that there shall be the greatest possible obstacle to a mere mechanical use of it.”* In one respect, the science of Logic differs from all others; the perfection of its method is chiefly valuable as an evidence of the speculative truth of its principles. To supersede the employment of common reason, or to subject it to the rigour of technical forms, would be the last desire of one who knows the value of that intellectual toil and warfare which imparts to the mind an athletic vigour, and teaches it to contend with difficulties and to rely upon itself in emergencies.

* Mill's *System of Logic, Ratiocinative and Inductive*, Vol. II. p. 292.

LINCOLN, Oct. 29, 1847.

MATHEMATICAL ANALYSIS OF LOGIC.

INTRODUCTION.

THEY who are acquainted with the present state of the theory of Symbolical Algebra, are aware, that the validity of the processes of analysis does not depend upon the interpretation of the symbols which are employed, but solely upon the laws of their combination. Every system of interpretation which does not affect the truth of the relations supposed, is equally admissible, and it is thus that the same process may, under one scheme of interpretation, represent the solution of a question on the properties of numbers, under another, that of a geometrical problem, and under a third, that of a problem of dynamics or optics. This principle is indeed of fundamental importance; and it may with safety be affirmed, that the recent advances of pure analysis have been much assisted by the influence which it has exerted in directing the current of investigation.

But the full recognition of the consequences of this important doctrine has been, in some measure, retarded by accidental circumstances. It has happened in every known form of analysis, that the elements to be determined have been conceived as measurable by comparison with some fixed standard. The predominant idea has been that of magnitude, or more strictly, of numerical ratio. The expression of magnitude, or

of operations upon magnitude, has been the express object for which the symbols of Analysis have been invented, and for which their laws have been investigated. Thus the abstractions of the modern Analysis, not less than the ostensive diagrams of the ancient Geometry, have encouraged the notion, that Mathematics are essentially, as well as actually, the Science of Magnitude.

The consideration of that view which has already been stated, as embodying the true principle of the Algebra of Symbols, would, however, lead us to infer that this conclusion is by no means necessary. If every existing interpretation is shewn to involve the idea of magnitude, it is only by induction that we can assert that no other interpretation is possible. And it may be doubted whether our experience is sufficient to render such an induction legitimate. The history of pure Analysis is, it may be said, too recent to permit us to set limits to the extent of its applications. Should we grant to the inference a high degree of probability, we might still, and with reason, maintain the sufficiency of the definition to which the principle already stated would lead us. We might justly assign it as the definitive character of a true Calculus, that it is a method resting upon the employment of Symbols, whose laws of combination are known and general, and whose results admit of a consistent interpretation. That to the existing forms of Analysis a quantitative interpretation is assigned, is the result of the circumstances by which those forms were determined, and is not to be construed into a universal condition of Analysis. It is upon the foundation of this general principle, that I purpose to establish the Calculus of Logic, and that I claim for it a place among the acknowledged forms of Mathematical Analysis, regardless that in its object and in its instruments it must at present stand alone.

That which renders Logic possible, is the existence in our minds of general notions,—our ability to conceive of a class, and to designate its individual members by a common name.

The theory of Logic is thus intimately connected with that of Language. A successful attempt to express logical propositions by symbols, the laws of whose combinations should be founded upon the laws of the mental processes which they represent, would, so far, be a step toward a philosophical language. But this is a view which we need not here follow into detail.* Assuming the notion of a class, we are able, from any conceivable collection of objects, to separate by a mental act, those which belong to the given class, and to contemplate them apart from the rest. Such, or a similar act of election, we may conceive to be repeated. The group of individuals left under consideration may be still further limited, by mentally selecting those among them which belong to some other recognised class, as well as to the one before contemplated. And this process may be repeated with other elements of distinction, until we arrive at an individual possessing all the distinctive characters which we have taken into account, and a member, at the same time, of every class which we have enumerated. It is in fact a method similar to this which we employ whenever, in common language, we accumulate descriptive epithets for the sake of more precise definition.

Now the several mental operations which in the above case we have supposed to be performed, are subject to peculiar laws. It is possible to assign relations among them, whether as respects the repetition of a given operation or the succession of different ones, or some other particular, which are never violated. It is, for example, true that the result of two successive acts is

* This view is well expressed in one of Blanco White's Letters :—"Logic is for the most part a collection of technical rules founded on classification. The Syllogism is nothing but a result of the classification of things, which the mind naturally and necessarily forms, in forming a language. All abstract terms are classifications ; or rather the labels of the classes which the mind has settled." —*Memoirs of the Rev. Joseph Blanco White*, vol. II. p. 163. See also, for a very lucid introduction, Dr. Latham's *First Outlines of Logic applied to Language*, Becker's *German Grammar*, &c. Extreme Nominalists make Logic entirely dependent upon language. For the opposite view, see Cudworth's *Eternal and Immutable Morality*, Book VI. Chap. III.

unaffected by the order in which they are performed ; and there are at least two other laws which will be pointed out in the proper place. These will perhaps to some appear so obvious as to be ranked among necessary truths, and so little important as to be undeserving of special notice. And probably they are noticed for the first time in this Essay. Yet it may with confidence be asserted, that if they were other than they are, the entire mechanism of reasoning, nay the very laws and constitution of the human intellect, would be vitally changed. A Logic might indeed exist, but it would no longer be the Logic we possess.

Such are the elementary laws upon the existence of which, and upon their capability of exact symbolical expression, the method of the following Essay is founded ; and it is presumed that the object which it seeks to attain will be thought to have been very fully accomplished. Every logical proposition, whether categorical or hypothetical, will be found to be capable of exact and rigorous expression, and not only will the laws of conversion and of syllogism be thence deducible, but the resolution of the most complex systems of propositions, the separation of any proposed element, and the expression of its value in terms of the remaining elements, with every subsidiary relation involved. Every process will represent deduction, every mathematical consequence will express a logical inference. The generality of the method will even permit us to express arbitrary operations of the intellect, and thus lead to the demonstration of general theorems in logic analogous, in no slight degree, to the general theorems of ordinary mathematics. No inconsiderable part of the pleasure which we derive from the application of analysis to the interpretation of external nature, arises from the conceptions which it enables us to form of the universality of the dominion of law. The general formulæ to which we are conducted seem to give to that element a visible presence, and the multitude of particular cases to which they apply, demonstrate the extent of its sway. Even the symmetry

of their analytical expression may in no fanciful sense be deemed indicative of its harmony and its consistency. Now I do not presume to say to what extent the same sources of pleasure are opened in the following Essay. The measure of that extent may be left to the estimate of those who shall think the subject worthy of their study. But I may venture to assert that such occasions of intellectual gratification are not here wanting. The laws we have to examine are the laws of one of the most important of our mental faculties. The mathematics we have to construct are the mathematics of the human intellect. Nor are the form and character of the method, apart from all regard to its interpretation, undeserving of notice. There is even a remarkable exemplification, in its general theorems, of that species of excellence which consists in freedom from exception. And this is observed where, in the corresponding cases of the received mathematics, such a character is by no means apparent. The few who think that there is that in analysis which renders it deserving of attention for its own sake, may find it worth while to study it under a form in which every equation can be solved and every solution interpreted. Nor will it lessen the interest of this study to reflect that every peculiarity which they will notice in the form of the Calculus represents a corresponding feature in the constitution of their own minds.

It would be premature to speak of the value which this method may possess as an instrument of scientific investigation. I speak here with reference to the theory of reasoning, and to the principle of a true classification of the forms and cases of Logic considered as a Science.* The aim of these investigations was in the first instance confined to the expression of the received logic, and to the forms of the Aristotelian arrangement,

* "Strictly a Science"; also "an Art."—*Whately's Elements of Logic*. Indeed ought we not to regard all Art as applied Science; unless we are willing, with "the multitude," to consider Art as "guessing and aiming well"?—*Plato, Philebus*.

but it soon became apparent that restrictions were thus introduced, which were purely arbitrary and had no foundation in the nature of things. These were noted as they occurred, and will be discussed in the proper place. When it became necessary to consider the subject of hypothetical propositions (in which comparatively less has been done), and still more, when an interpretation was demanded for the general theorems of the Calculus, it was found to be imperative to dismiss all regard for precedent and authority, and to interrogate the method itself for an expression of the just limits of its application. Still, however, there was no special effort to arrive at novel results. But among those which at the time of their discovery appeared to be such, it may be proper to notice the following.

A logical proposition is, according to the method of this Essay, expressible by an equation the form of which determines the rules of conversion and of transformation, to which the given proposition is subject. Thus the law of what logicians term simple conversion, is determined by the fact, that the corresponding equations are symmetrical, that they are unaffected by a mutual change of place, in those symbols which correspond to the convertible classes. The received laws of conversion were thus determined, and afterwards another system, which is thought to be more elementary, and more general. See Chapter, *On the Conversion of Propositions*.

The premises of a syllogism being expressed by equations, the elimination of a common symbol between them leads to a third equation which expresses the conclusion, this conclusion being always the most general possible, whether Aristotelian or not. Among the cases in which no inference was possible, it was found, that there were two distinct forms of the final equation. It was a considerable time before the explanation of this ~~first~~ was discovered, but it was at length seen to depend upon the presence or absence of a true medium of comparison between the premises. The distinction which is thought to be new is illustrated in the Chapter, *On Syllogisms*.

The nonexclusive character of the disjunctive conclusion of a hypothetical syllogism, is very clearly pointed out in the examples of this species of argument.

The class of logical problems illustrated in the chapter, *On the Solution of Elective Equations*, is conceived to be new: and it is believed that the method of that chapter affords the means of a perfect analysis of any conceivable system of propositions, an end toward which the rules for the conversion of a single categorical proposition are but the first step.

However, upon the originality of these or any of these views, I am conscious that I possess too slight an acquaintance with the literature of logical science, and especially with its older literature, to permit me to speak with confidence.

It may not be inappropriate, before concluding these observations, to offer a few remarks upon the general question of the use of symbolical language in the mathematics. Objections have lately been very strongly urged against this practice, on the ground, that by obviating the necessity of thought, and substituting a reference to general formulæ in the room of personal effort, it tends to weaken the reasoning faculties.

Now the question of the use of symbols may be considered in two distinct points of view. First, it may be considered with reference to the progress of scientific discovery, and secondly, with reference to its bearing upon the discipline of the intellect.

And with respect to the first view, it may be observed that as it is one fruit of an accomplished labour, that it sets us at liberty to engage in more arduous toils, so it is a necessary result of an advanced state of science, that we are permitted, and even called upon, to proceed to higher problems, than those which we before contemplated. The practical inference is obvious. If through the advancing power of scientific methods, we find that the pursuits on which we were once engaged, afford no longer a sufficiently ample field for intellectual effort, the remedy is, to proceed to higher inquiries, and, in new tracks, to seek for difficulties yet unsubdued. And such is,

indeed, the actual law of scientific progress. We must be content, either to abandon the hope of further conquest, or to employ such aids of symbolical language, as are proper to the stage of progress, at which we have arrived. Nor need we fear to commit ourselves to such a course. We have not yet arrived so near to the boundaries of possible knowledge, as to suggest the apprehension, that scope will fail for the exercise of the inventive faculties.

In discussing the second, and scarcely less momentous question of the influence of the use of symbols upon the discipline of the intellect, an important distinction ought to be made. It is of most material consequence, whether those symbols are used with a full understanding of their meaning, with a perfect comprehension of that which renders their use lawful, and an ability to expand the abbreviated forms of reasoning which they induce, into their full syllogistic development; or whether they are mere unsuggestive characters, the use of which is suffered to rest upon authority.

The answer which must be given to the question proposed, will differ according as the one or the other of these suppositions is admitted. In the former case an intellectual discipline of a high order is provided, an exercise not only of reason, but of the faculty of generalization. In the latter case there is no mental discipline whatever. It were perhaps the best security against the danger of an unreasoning reliance upon symbols, on the one hand, and a neglect of their just claims on the other, that each subject of applied mathematics should be treated in the spirit of the methods which were known at the time when the application was made, but in the best form which those methods have assumed. The order of attainment in the individual mind would thus bear some relation to the actual order of scientific discovery, and the more abstract methods of the higher analysis would be offered to such minds only, as were prepared to receive them.

The relation in which this Essay stands at once to Logic and

to Mathematics, may further justify some notice of the question which has lately been revived, as to the relative value of the two studies in a liberal education. One of the chief objections which have been urged against the study of Mathematics in general, is but another form of that which has been already considered with respect to the use of symbols in particular. And it need not here be further dwelt upon, than to notice, that if it avails anything, it applies with an equal force against the study of Logic. The canonical forms of the Aristotelian syllogism are really symbolical; only the symbols are less perfect of their kind than those of mathematics. If they are employed to test the validity of an argument, they as truly supersede the exercise of reason, as does a reference to a formula of analysis. Whether men do, in the present day, make this use of the Aristotelian canons, except as a special illustration of the rules of Logic, may be doubted; yet it cannot be questioned that when the authority of Aristotle was dominant in the schools of Europe, such applications were habitually made. And our argument only requires the admission, that the case is possible.

But the question before us has been argued upon higher grounds. Regarding Logic as a branch of Philosophy, and defining Philosophy as the "science of a real existence," and "the research of causes," and assigning as its *main* business the investigation of the "why, ($\tau\omicron\delta\ \delta\iota\omicron\tau\iota$)," while Mathematics display only the "that, ($\tau\omicron\delta\ \delta\tau\iota$)," Sir W. Hamilton has contended, not simply, that the superiority rests with the study of Logic, but that the study of Mathematics is at once dangerous and useless.* The pursuits of the mathematician "have not only not trained him to that acute scent, to that delicate, almost instinctive, tact which, in the twilight of probability, the search and discrimination of its finer facts demand; they have gone to cloud his vision, to indurate his touch, to all but the blazing light, the iron chain of demonstration, and left him out of the narrow confines of his science, to a passive *credulity* in any premises, or to

* *Edinburgh Review*, vol. LXII. p. 409, and *Letter to A. De Morgan, Esq.*

an absolute *incredulity* in all." In support of these ~~and of other charges, both argument and copious authority~~ are adduced.* I shall not attempt a complete discussion of the topics which are suggested by these remarks. My object is not controversy, and the observations which follow are offered ~~not in the spirit~~ of antagonism, but in the hope of contributing to the formation of just views upon an important subject. Of Sir W. Hamilton it is impossible to speak otherwise than with that respect which is due to genius and learning.

Philosophy is then described as the *science of a real existence and the research of causes*. And that no doubt may rest upon the meaning of the word *cause*, it is further said, that philosophy "mainly investigates the *why*." These definitions are common among the ancient writers. Thus Seneca, one of Sir W. Hamilton's authorities, *Epistle LXXXVIII.*, "The philosopher seeks and knows the *causes* of natural things, of which the mathematician searches out and computes the numbers and the measures." It may be remarked, in passing, that in whatever degree the belief has prevailed, that the business of philosophy is immediately with *causes*; in the same degree has every science whose object is the investigation of *laws*, been lightly esteemed. Thus the Epistle to which we have referred, bestows, by contrast with Philosophy, a separate condemnation on Music and Grammar, on Mathematics and Astronomy, although it is that of Mathematics only that Sir W. Hamilton has quoted.

Now we might take our stand upon the conviction of many thoughtful and reflective minds, that in the extent of the meaning above stated, Philosophy is impossible. The business of true Science, they conclude, is with laws and phenomena. The nature of Being, the mode of the operation of Cause, the *why*,

* The arguments are in general better than the authorities. Many writers quoted in condemnation of mathematics (Aristo, Seneca, Jerome, Augustine, Cornelius Agrippa, &c.) have borne a no less explicit testimony against other sciences, nor least of all, against that of logic. The treatise of the last named writer *De Vanitate Scientiarum*, must surely have been referred to by mistake.—*Vide cap. cii.*

they hold to be beyond the reach of our^{*} intelligence. But we do not require the vantage-ground of this position; nor is it doubted that whether the aim of Philosophy is attainable or not, the desire which impels us to the attempt is an instinct of our higher nature. Let it be granted that the problem which has baffled the efforts of ages, is not a hopeless one; that the "science of a real existence," and "the research of causes," "that kernel" for which "Philosophy is still militant," do not transcend the limits of the human intellect. I am then compelled to assert, that according to this view of the nature of Philosophy, *Logic forms no part of it*. On the principle of a true classification, we ought no longer to associate Logic and Metaphysics, but Logic and Mathematics.

Should any one after what has been said, entertain a doubt upon this point, I must refer him to the evidence which will be afforded in the following Essay. He will there see Logic resting like Geometry upon axiomatic truths, and its theorems constructed upon that general doctrine of symbols, which constitutes the foundation of the recognised Analysis. In the *Logic* of Aristotle he will be led to view a collection of the formulæ of the science, expressed by another, but, (it is thought) less perfect scheme of symbols. I feel bound to contend for the absolute exactness of this parallel. It is no escape from the conclusion to which it points to assert, that Logic not only constructs a science, but also inquires into the origin and the nature of its own principles,—a distinction which is denied to Mathematics. "It is wholly beyond the domain of mathematicians," it is said, "to inquire into the origin and nature of their principles."—*Review*, page 415. But upon what ground can such a distinction be maintained? What definition of the term Science will be found sufficiently arbitrary to allow such differences?

The application of this conclusion to the question before us is clear and decisive. The mental discipline which is afforded by the study of Logic, *as an exact science*, is, in species, the same as that afforded by the study of Analysis.

Is it then contended that either Logic or Mathematics can supply a perfect discipline to the Intellect? The most careful and unprejudiced examination of this question leads me to doubt whether such a position can be maintained. The exclusive claims of either must, I believe, be abandoned, nor can any others, partaking of a like exclusive character, be admitted in their room. It is an important observation, which has more than once been made, that it is one thing to arrive at correct premises, and another thing to deduce logical conclusions, and that the business of life depends more upon the former than upon the latter. The study of the exact sciences may teach us the one, and it may give us some general preparation of knowledge and of practice for the attainment of the other, but it is to the union of thought with action, in the field of Practical Logic, the arena of Human Life, that we are to look for its fuller and more perfect accomplishment.

I desire here to express my conviction, that with the advance of our knowledge of all true science, an ever-increasing harmony will be found to prevail among its separate branches. The view which leads to the rejection of one, ought, if consistent, to lead to the rejection of others. And indeed many of the authorities which have been quoted against the study of Mathematics, are even more explicit in their condemnation of Logic. "Natural science," says the Chian Aristo, "is above us, Logical science does not concern us." When such conclusions are founded (as they often are) upon a deep conviction of the preeminent value and importance of the study of Morals, we admit the premises, but must demur to the inference. For it has been well said by an ancient writer, that it is the "characteristic of the liberal sciences, not that they conduct us to Virtue, but that they prepare us for Virtue;" and Melancthon's sentiment, "*abeunt studia in mores*," has passed into a proverb. Moreover, there is a common ground upon which all sincere votaries of truth may meet, exchanging with each other the language of Flamsteed's appeal to Newton, "The works of the Eternal Providence will be better understood through your labors and mine."

FIRST PRINCIPLES.

LET us employ the symbol 1, or unity, to represent the Universe, and let us understand it as comprehending every conceivable class of objects whether actually existing or not, it being premised that the same individual may be found in more than one class, inasmuch as it may possess more than one quality in common with other individuals. Let us employ the letters X, Y, Z, to represent the individual members of classes, X applying to every member of one class, as members of that particular class, and Y to every member of another class as members of such class, and so on, according to the received language of treatises on Logic.

Further let us conceive a class of symbols x, y, z , possessed of the following character.

The symbol x operating upon any subject comprehending individuals or classes, shall be supposed to select from that subject all the Xs which it contains. In like manner the symbol y , operating upon any subject, shall be supposed to select from it all individuals of the class Y which are comprised in it, and so on.

When no subject is expressed, we shall suppose 1 (the Universe) to be the subject understood, so that we shall have

$$x = x \quad (1),$$

the meaning of either term being the selection from the Universe of all the Xs which it contains, and the result of the operation

being in common language, the class X , i. e. the class of which each member is an X .

From these premises it will follow, that the product xy will represent, in succession, the selection of the class Y , and the selection from the class Y of such individuals of the class X as are contained in it, the result being the class whose members are both X s and Y s. And in like manner the product xyz will represent a compound operation of which the successive elements are the selection of the class Z , the selection from it of such individuals of the class Y as are contained in it, and the selection from the result thus obtained of all the individuals of the class X which it contains, the final result being the class common to X , Y , and Z .

From the nature of the operation which the symbols x , y , z , are conceived to represent, we shall designate them as elective symbols. An expression in which they are involved will be called an elective function, and an equation of which the members are elective functions, will be termed an elective equation.

It will not be necessary that we should here enter into the analysis of that mental operation which we have represented by the elective symbol. It is not an act of Abstraction according to the common acceptance of that term, because we never lose sight of the concrete, but it may probably be referred to an exercise of the faculties of Comparison and Attention. Our present concern is rather with the laws of combination and of succession, by which its results are governed, and of these it will suffice to notice the following.

1st. The result of an act of election is independent of the grouping or classification of the subject.

Thus it is indifferent whether from a group of objects considered as a whole, we select the class X , or whether we divide the group into two parts, select the X s from them separately, and then connect the results in one aggregate conception.

We may express this law mathematically by the equation

$$x(u + v) = xu + xv,$$

$u + v$ representing the undivided subject, and u and v the component parts of it.

2nd. It is indifferent in what order two successive acts of election are performed.

Whether from the class of animals we select sheep, and from the sheep those which are horned, or whether from the class of animals we select the horned, and from these such as are sheep, the result is unaffected. In either case we arrive at the class *horned sheep*.

The symbolical expression of this law is

$$xy = yx.$$

3rd. The result of a given act of ~~abstraction~~ ^{election} performed twice, or any number of times in succession, is the result of the same act performed once.

If from a group of objects we select the X s, we obtain a class of which all the members are X s. If we repeat the operation on this class no further change will ensue: in selecting the X s we take the whole. Thus we have

$$xx = x,$$

or
$$x^2 = x;$$

and supposing the same operation to be n times performed, we have

$$x^n = x,$$

which is the mathematical expression of the law above stated.*

The laws we have established under the symbolical forms

$$x(u + v) = xu + xv \dots\dots\dots (1),$$

$$xy = yx \dots\dots\dots (2),$$

$$x^n = x \dots\dots\dots (3),$$

* The office of the elective symbol x , is to select individuals comprehended in the class X . Let the class X be supposed to embrace the universe; then, whatever the class Y may be, we have

$$xy = y.$$

The office which x performs is now equivalent to the symbol $+$, in one at least of its interpretations, and the index law (3) gives

$$+^n = +,$$

which is the known property of that symbol.

are sufficient for the basis of a Calculus. From the first of these, it appears that elective symbols are *distributive*, from the second that they are *commutative*; properties which they possess in common with symbols of *quantity*, and in virtue of which, all the processes of common algebra are applicable to the present system. The one and sufficient axiom involved in this application is that equivalent operations performed upon equivalent subjects produce equivalent results.*

The third law (3) we shall denominate the index law. It is peculiar to elective symbols, and will be found of great importance in enabling us to reduce our results to forms meet for interpretation.

From the circumstance that the processes of algebra may be applied to the present system, it is not to be inferred that the interpretation of an elective equation will be unaffected by such processes. The expression of a truth cannot be negated by

* It is generally asserted by writers on Logic, that all reasoning ultimately depends on an application of the dictum of Aristotle, *de omni et nullo*. "What-ever is predicated universally of any class of things, may be predicated in like manner of any thing comprehended in that class." But it is agreed that this dictum is not immediately applicable in all cases, and that in a majority of instances, a certain previous process of reduction is necessary. What are the elements involved in that process of reduction? Clearly they are as much a part of general reasoning as the dictum itself.

Another mode of considering the subject resolves all reasoning into an application of one or other of the following canons, viz.

1. If two terms agree with one and the same third, they agree with each other.

2. If one term agrees, and another disagrees, with one and the same third, these two disagree with each other.

But the application of these canons depends on mental acts equivalent to those which are involved in the before-named process of reduction. We have to select individuals from classes, to convert propositions, &c., before we can avail ourselves of their guidance. Any account of the process of reasoning is insufficient, which does not represent, as well the laws of the operation which the mind performs in that process, as the primary truths which it recognises and applies.

It is presumed that the laws in question are adequately represented by the fundamental equations of the present Calculus. The proof of this will be found in its capability of expressing propositions, and of exhibiting in the results of its processes, every result that may be arrived at by ordinary reasoning.

a legitimate operation, but it may be limited. The equation $y = z$ implies that the classes Y and Z are equivalent, member for member. Multiply it by a factor x , and we have

$$xy = xz,$$

which expresses that the individuals which are common to the classes X and Y are also common to X and Z, and *vice versa*. This is a perfectly legitimate inference, but the fact which it declares is a less general one than was asserted in the original proposition.

OF EXPRESSION AND INTERPRETATION.

A Proposition is a sentence which either affirms or denies, as, All men are mortal, No creature is independent.

A Proposition has necessarily two terms, as *men, mortal*; the former of which, or the one spoken of, is called the subject; the latter, or that which is affirmed or denied of the subject, the predicate. These are connected together by the copula *is*, or *is not*, or by some other modification of the substantive verb.

The substantive verb is the only verb recognised in Logic; all others are resolvable by means of the verb *to be* and a participle or adjective, *e. g.* "The Romans conquered"; the word *conquered* is both copula and predicate, being equivalent to "*were (copula) victorious*" (predicate).

A Proposition must either be affirmative or negative, and must be also either universal or particular. Thus we reckon in all, four kinds of pure categorical Propositions.

1st. Universal-affirmative, usually represented by A,

Ex. All Xs are Ys.

2nd. Universal-negative, usually represented by E,

Ex. No Xs are Ys.

3rd. Particular-affirmative, usually represented by I,

Ex. Some Xs are Ys.

4th. Particular-negative, usually represented by O,*

Ex. Some Xs are not Ys.

1. To express the class, not-X, that is, the class including all individuals that are not Xs.

The class X and the class not-X together make the Universe. But the Universe is 1, and the class X is determined by the symbol x , therefore the class not-X will be determined by the symbol $1 - x$.

* The above is taken, with little variation, from the Treatises of Aldrich and Whately.

Hence the office of the symbol $1 - x$ attached to a given subject will be, to select from it all the not-Xs which it contains.

And in like manner, as the product xy expresses the entire class whose members are both Xs and Ys, the symbol $y(1 - x)$ will represent the class whose members are Ys but not Xs, and the symbol $(1 - x)(1 - y)$ the entire class whose members are neither Xs nor Ys.

2. To express the Proposition, All Xs are Ys.

As all the Xs which exist are found in the class Y, it is obvious that to select out of the Universe all Ys, and from these to select all Xs, is the same as to select at once from the Universe all Xs.

Hence

$$xy = x,$$

or

$$x(1 - y) = 0, \quad (4).$$

3. To express the Proposition, No Xs are Ys.

To assert that no Xs are Ys, is the same as to assert that there are no terms common to the classes X and Y. Now all individuals common to those classes are represented by xy . Hence the Proposition that No Xs are Ys, is represented by the equation

$$xy = 0, \quad (5).$$

4. To express the Proposition, Some Xs are Ys.

If some Xs are Ys, there are some terms common to the classes X and Y. Let those terms constitute a separate class V, to which there shall correspond a separate elective symbol v , then

$$v = xy, \quad (6).$$

And as v includes all terms common to the classes X and Y, we can indifferently interpret it, as Some Xs, or Some Ys.

5. To express the Proposition, Some Xs are not Ys.

In the last equation write $1 - y$ for y , and we have

$$v = x(1 - y), \quad (7),$$

the interpretation of v being indifferently Some Xs or Some not-Ys.

The above equations involve the complete theory of categorical Propositions, and so far as respects the employment of analysis for the deduction of logical inferences, nothing more can be desired. But it may be satisfactory to notice some particular forms deducible from the third and fourth equations, and susceptible of similar application.

If we multiply the equation (6) by x , we have

$$vx = x^2y = xy \text{ by (3).}$$

Comparing with (6), we find

$$v = vx,$$

or

$$v(1 - x) = 0, \quad (8).$$

And multiplying (6) by y , and reducing in a similar manner, we have

$$v = vy,$$

or

$$v(1 - y) = 0, \quad (9).$$

Comparing (8) and (9),

$$vx = vy = v, \quad (10).$$

And further comparing (8) and (9) with (4), we have as the equivalent of this system of equations the Propositions

$$\left. \begin{array}{l} \text{All Vs are Xs} \\ \text{All Vs are Ys} \end{array} \right\}.$$

The system (10) might be used to replace (6), or the single equation

$$vx = vy, \quad (11),$$

might be used, assigning to vx the interpretation, Some Xs, and to vy the interpretation, Some Ys. But it will be observed that

this system does not express quite so much as the single equation (6), from which it is derived. Both, indeed, express the Proposition, Some Xs are Ys, but the system (10) does not imply that the class V includes *all* the terms that are common to X and Y.

In like manner, from the equation (7) which expresses the Proposition Some Xs are not Ys, we may deduce the system

$$vx = v(1 - y) = v, \quad (12),$$

in which the interpretation of $v(1 - y)$ is Some not-Ys. Since in this case $vy = 0$, we must of course be careful not to interpret vy as Some Ys.

If we multiply the first equation of the system (12), viz.

$$vx = v(1 - y),$$

by y , we have

$$vxy = vy(1 - y);$$

$$\therefore vxy = 0, \quad (13),$$

which is a form that will occasionally present itself. It is not necessary to revert to the primitive equation in order to interpret this, for the condition that vx represents Some Xs, shews us by virtue of (5), that its import will be

Some Xs are not Ys,

the subject comprising *all* the Xs that are found in the class V.

Universally in these cases, difference of form implies a difference of interpretation with respect to the auxiliary symbol v , and each form is interpretable by itself.

Further, these differences do not introduce into the Calculus a needless perplexity. It will hereafter be seen that they give a precision and a definiteness to its conclusions, which could not otherwise be secured.

Finally, we may remark that all the equations by which particular truths are expressed, are deducible from any one general equation, expressing any one general Proposition, from which those particular Propositions are necessary deductions.

This has been partially shewn already, but it is much more fully exemplified in the following scheme.

The general equation $x = y$,

implies that the classes **X** and **Y** are equivalent, member for member; that every individual belonging to the one, belongs to the other also. Multiply the equation by x , and we have

$$x^2 = xy;$$

$$\therefore x = xy,$$

which implies, by (4), that all **Xs** are **Ys**. Multiply the same equation by y , and we have in like manner

$$y = xy;$$

the import of which is, that all **Ys** are **Xs**. Take either of these equations, the latter for instance, and writing it under the form

$$(1 - x)y = 0,$$

we may regard it as an equation in which y , an unknown quantity, is sought to be expressed in terms of x . Now it will be shewn when we come to treat of the Solution of Elective Equations (and the result may here be verified by substitution) that the most general solution of this equation is

$$y = vx,$$

which implies that All **Ys** are **Xs**, and that Some **Xs** are **Ys**. Multiply by x , and we have

$$xy = vx,$$

which indifferently implies that some **Ys** are **Xs** and some **Xs** are **Ys**, being the particular form at which we before arrived.

For convenience of reference the above and some other results have been classified in the annexed Table, the first column of which contains propositions, the second equations, and the third the conditions of final interpretation. It is to be observed, that the auxiliary equations which are given in this column are not independent: they are implied either in the equations of the second column, or in the condition for

the interpretation of v . But it has been thought better to write them separately, for greater ease and convenience. And it is further to be borne in mind, that although three different forms are given for the expression of each of the *particular* propositions, everything is really included in the first form.

TABLE.

The class X	x	
The class not-X	$1 - x$	
All Xs are Ys } All Ys are Xs }	$x = y$	
All Xs are Ys	$x (1 - y) = 0$	
No Xs are Ys	$xy = 0$	
All Ys are Xs } Some Xs are Ys }	$y = vx$	$vx = \text{some Xs}$ $v (1 - x) = 0.$
No Ys are Xs } Some not-Xs are Ys }	$y = v (1 - x)$	$v (1 - x) = \text{some not-Xs}$ $vx = 0.$
Some Xs are Ys	$\left\{ \begin{array}{l} v = xy \\ \text{or } vx = vy \\ \text{or } vx (1 - y) = 0 \end{array} \right.$	$\left\{ \begin{array}{l} v = \text{some Xs or some Ys} \\ vx = \text{some Xs, } vy = \text{some Ys} \\ v (1 - x) = 0, v (1 - y) = 0. \end{array} \right.$
Some Xs are not Ys	$\left\{ \begin{array}{l} v = x (1 - y) \\ \text{or } vx = v (1 - y) \\ \text{or } vxy = 0 \end{array} \right.$	$\left\{ \begin{array}{l} v = \text{some Xs, or some not-Ys} \\ vx = \text{some Xs, } v (1 - y) = \text{some not-Ys} \\ v (1 - x) = 0, vy = 0. \end{array} \right.$

OF THE CONVERSION OF PROPOSITIONS.

A Proposition is said to be converted when its terms are transposed ; when nothing more is done, this is called simple conversion ; *e.g.*

No virtuous man is a tyrant, *is converted into*

No tyrant is a virtuous man.

Logicians also recognise conversion *per accidens*, or by limitation, *e.g.*

All birds are animals, *is converted into*

Some animals are birds.

And conversion by *contraposition* or *negation*, as

Every poet is a man of genius, *converted into*

He who is not a man of genius is not a poet.

In one of these three ways every Proposition may be illatively converted, viz. E and I simply, A and O by negation, A and E by limitation.

The primary canonical forms already determined for the expression of Propositions, are

All Xs are Ys, $x(1 - y) = 0$, A.

No Xs are Ys, $xy = 0$, E.

Some Xs are Ys, $v = xy$, I.

Some Xs are not Ys, $v = x(1 - y)$ O.

On examining these, we perceive that E and I are symmetrical with respect to x and y , so that x being changed into y , and y into x , the equations remain unchanged. Hence E and I may be interpreted into

No Ys are Xs,

Some Ys are Xs,

respectively. Thus we have the known rule of the Logicians, that particular affirmative and universal negative Propositions admit of simple conversion.

The equations A and O may be written in the forms

$$(1 - y) \{1 - (1 - x)\} = 0,$$

$$v = (1 - y) \{1 - (1 - x)\}.$$

Now these are precisely the forms which we should have obtained if we had in those equations changed x into $1 - y$, and y into $1 - x$, which would have represented the changing in the original Propositions of the Xs into not-Ys, and the Ys into not-Xs, the resulting Propositions being

All not-Ys are not-Xs,

Some not-Ys are not not-Xs (a).

Or we may, by simply inverting the order of the factors in the second member of O, and writing it in the form

$$v = (1 - y) x,$$

interpret it by I into

Some not-Ys are Xs,

which is really another form of (a). Hence follows the rule, that universal affirmative and particular negative Propositions admit of negative conversion, or, as it is also termed, conversion by contraposition.

The equations A and E, written in the forms

$$(1 - y) x = 0,$$

$$yx = 0,$$

give on solution the respective forms

$$x = vy,$$

$$x = v(1 - y),$$

the correctness of which may be shewn by substituting these values of x in the equations to which they belong, and observing that those equations are satisfied quite independently of the nature of the symbol v . The first solution may be interpreted into

Some Ys are Xs,

and the second into

Some not-Ys are Xs.

From which it appears that universal-affirmative, and universal-negative Propositions are convertible by limitation, or, as it has been termed, *per accidens*.

The above are the laws of Conversion recognized by Abp. Whately. Writers differ however as to the admissibility of negative conversion. The question depends on whether we will consent to use such terms as not-X, not-Y. Agreeing with those who think that such terms ought to be admitted, even although they change the *kind* of the Proposition, I am constrained to observe that the present classification of them is faulty and defective. Thus the conversion of No Xs are Ys, into All Ys are not-Xs, though perfectly legitimate, is not recognised in the above scheme. It may therefore be proper to examine the subject somewhat more fully.

Should we endeavour, from the system of equations we have obtained, to deduce the laws not only of the conversion, but also of the general transformation of propositions, we should be led to recognise the following distinct elements, each connected with a distinct mathematical process.

1st. The negation of a term, *i.e.* the changing of X into not-X, or not-X into X.

2nd. The translation of a Proposition from one *kind* to another, as if we should change

All Xs are Ys into Some Xs are Ys A into I,
which would be lawful; or

All Xs are Ys into No Xs are Y. A into E,
which would be unlawful.

3rd. The simple conversion of a Proposition.

The conditions in obedience to which these processes may lawfully be performed, may be deduced from the equations by which Propositions are expressed.

We have

$$\text{All Xs are Ys} \dots \dots x(1 - y) = 0. \quad \text{A,}$$

$$\text{No Xs are Ys} \dots \dots xy = 0. \quad \text{E.}$$

Write E in the form

$$x \{1 - (1 - y)\} = 0,$$

and it is interpretable by A into

All Xs are not-Ys,

so that we may change

No Xs are Ys into All Xs are not-Ys.

In like manner A interpreted by E gives

No Xs are not-Ys,

so that we may change

All Xs are Ys into No Xs are not-Ys.

From these cases we have the following Rule: A universal-affirmative Proposition is convertible into a universal-negative, and, *vice versâ*, by negation of the predicate.

Again, we have

Some Xs are Ys. $v = xy$,

Some Xs are not Ys $v = x(1 - y)$.

These equations only differ from those last considered by the presence of the term v . The same reasoning therefore applies, and we have the Rule—

A particular-affirmative proposition is convertible into a particular-negative, and *vice versâ*, by negation of the predicate.

Assuming the universal Propositions

All Xs are Ys. $x(1 - y) = 0$,

No Xs are Ys $xy = 0$.

Multiplying by v , we find

$$vx(1 - y) = 0,$$

$$vxy = 0,$$

which are interpretable into

Some Xs are Ys. I,

Some Xs are not Ys. . . . O.

Hence a universal-affirmative is convertible into a particular-affirmative, and a universal-negative into a particular-negative without negation of subject or predicate.

Combining the above with the already proved rule of simple conversion, we arrive at the following system of independent laws of transformation.

1st. An affirmative Proposition may be changed into its corresponding negative (A into E, or I into O), and *vice versa*, by negation of the predicate.

2nd. A universal Proposition may be changed into its corresponding particular Proposition, (A into I, or E into O).

3rd. In a particular-affirmative, or universal-negative Proposition, the terms may be mutually converted.

Wherein negation of a term is the changing of X into not-X, and *vice versa*, and is not to be understood as affecting the *kind* of the Proposition.

Every lawful transformation is reducible to the above rules. Thus we have

All Xs are Ys,

No Xs are not-Ys by 1st rule,

No not-Ys are Xs by 3rd rule,

All not-Ys are not-Xs by 1st rule,

which is an example of *negative conversion*. Again,

No Xs are Ys,

No Ys are Xs 3rd rule,

All Ys are not-Xs 1st rule,

which is the case already deduced.

OF SYLLOGISMS.

A Syllogism consists of three Propositions, the last of which, called the conclusion, is a logical consequence of the two former, called the premises; e. g.

Premises, $\begin{cases} \text{All Ys are Xs.} \\ \text{All Zs are Ys.} \end{cases}$
 Conclusion, All Zs are Xs.

Every syllogism has three and only three terms, whereof that which is the subject of the conclusion is called the *minor* term, the predicate of the conclusion, the *major* term, and the remaining term common to both premises, the middle term. Thus, in the above formula, Z is the minor term, X the major term, Y the middle term.

The figure of a syllogism consists in the situation of the middle term with respect to the terms of the conclusion. The varieties of figure are exhibited in the annexed scheme.

1st Fig.	2nd Fig.	3rd Fig.	4th Fig.
YX	XY	YX	XY
ZY	ZY	YZ	YZ
ZX	ZX	ZX	ZX

When we designate the three propositions of a syllogism by their usual symbols (*A, E, I, O*), and in their actual order, we are said to determine the mood of the syllogism. Thus the syllogism given above, by way of illustration, belongs to the mood *AAA* in the first figure.

The moods of all syllogisms commonly received as valid, are represented by the vowels in the following mnemonic verses.

Fig. 1.—*bArbArA, cElArEnt, dArII, fErIO* que prioris.

Fig. 2.—*cEsArE, cAmEstrEs, fEstIno, bArOkO*, secundæ.

Fig. 3.—*Tertia dArAptI, dIsAmIs, dAtIsI, fElAptOn,*
bOkArdO, fErIsO, habet: quarta insuper addit.

Fig. 4.—*brAmAntIp, cAmEnEs, dImArIs, fEsapO, frEsIsOn.*

THE equation by which we express any Proposition concerning the classes X and Y, is an equation between the symbols *x* and *y*, and the equation by which we express any

Proposition concerning the classes Y and Z, is an equation between the symbols y and z . If from two such equations we eliminate y , the result, if it do not vanish, will be an equation between x and z , and will be interpretable into a Proposition concerning the classes X and Z. And it will then constitute the third member, or Conclusion, of a Syllogism, of which the two given Propositions are the premises.

The result of the elimination of y from the equations

$$\begin{aligned} ay + b &= 0, \\ a'y + b' &= 0, \end{aligned} \quad (14),$$

is the equation $ab' - a'b = 0, \quad (15).$

Now the equations of Propositions being of the first order with reference to each of the variables involved, all the cases of elimination which we shall have to consider, will be reducible to the above case, the constants a, b, a', b' , being replaced by functions of x, z , and the auxiliary symbol v .

As to the choice of equations for the expression of our premises, the only restriction is, that the equations must not *both* be of the form $ay = 0$, for in such cases elimination would be impossible. When both equations are of this form, it is necessary to solve one of them, and it is indifferent which we choose for this purpose. If that which we select is of the form $xy = 0$, its solution is

$$y = v(1 - x), \quad (16),$$

if of the form $(1 - x)y = 0$, the solution will be

$$y = vx, \quad (17),$$

and these are the only cases which can arise. The reason of this exception will appear in the sequel.

For the sake of uniformity we shall, in the expression of particular propositions, confine ourselves to the forms

$$\begin{aligned} vx &= vy, & \text{Some Xs are Ys,} \\ vx &= v(1 - y), & \text{Some Xs are not Ys,} \end{aligned}$$

These have a closer analogy with (16) and (17), than the other forms which might be used.

Between the forms about to be developed, and the Aristotelian canons, some points of difference will occasionally be observed, of which it may be proper to forewarn the reader.

To the right understanding of these it is proper to remark, that the essential structure of a Syllogism is, in some measure, arbitrary. Supposing the order of the premises to be fixed, and the distinction of the major and the minor term to be thereby determined, it is purely a matter of choice which of the two shall have precedence in the Conclusion. Logicians have settled this question in favour of the minor term, but it is clear, that this is a convention. Had it been agreed that the major term should have the first place in the conclusion, a logical scheme might have been constructed, less convenient in some cases than the existing one, but superior in others. What it lost in *barbara*, it would gain in *bramantip*. Convenience is *perhaps* in favour of the adopted arrangement,* but it is to be remembered that it is *merely* an arrangement.

Now the method we shall exhibit, not having reference to one scheme of arrangement more than to another, will always give the more general conclusion, regard being paid only to its abstract lawfulness, considered as a result of pure reasoning. And therefore we shall sometimes have presented to us the spectacle of conclusions, which a logician would pronounce informal, but never of such as a reasoning being would account false.

The Aristotelian canons, however, beside restricting the *order* of the terms of a conclusion, limit their nature also;—and this limitation is of more consequence than the former. We may, by a change of figure, replace the particular conclusion

* The contrary view was maintained by Hobbes. The question is very fairly discussed in Hallam's *Introduction to the Literature of Europe*, vol. III. p. 309. In the rhetorical use of Syllogism, the advantage appears to rest with the rejected form.

of *bramantip*, by the general conclusion of *barbara*; but we cannot thus reduce to rule such inferences, as

Some not-Xs are not Ys.

Yet there are cases in which such inferences may lawfully be drawn, and in unrestricted argument they are of frequent occurrence. Now if an inference of this, or of any other kind, is lawful in itself, it will be exhibited in the results of our method.

We may by restricting the canon of interpretation confine our expressed results within the limits of the scholastic logic; but this would only be to restrict ourselves to the use of a part of the conclusions to which our analysis entitles us.

The classification we shall adopt will be purely mathematical, and we shall afterwards consider the logical arrangement to which it corresponds. It will be sufficient, for reference, to name the premises and the Figure in which they are found.

CLASS 1st.—Forms in which *v* does not enter.

Those which admit of an inference are AA, EA, Fig. 1; AE, EA, Fig. 2; AA, AE, Fig. 4.

Ex. AA, Fig. 1, and, by mutation of premises (change of order), AA, Fig. 4.

All Ys are Xs,	$y(1-x) = 0,$	or $(1-x)y = 0,$
All Zs are Ys,	$z(1-y) = 0,$	or $zy - z = 0.$

Eliminating *y* by (15) we have

$$z(1-x) = 0,$$

$$\therefore \text{All Zs are Xs.}$$

A convenient mode of effecting the elimination, is to write the equation of the premises, so that *y* shall appear only as a factor of one member in the first equation, and only as a factor of the opposite member in the second equation, and then to multiply the equations, omitting the *y*. This method we shall adopt.

Ex. AE, Fig. 2, and, by mutation of premises, EA, Fig. 2.

$$\begin{array}{lll} \text{All Xs are Ys,} & x(1-y)=0, & \text{or } x=xy \\ \text{No Zs are Ys,} & zy=0, & \underline{zy=0} \\ & & \underline{zx=0} \end{array}$$

\therefore No Zs are Xs.

The only case in which there is no inference is AA, Fig. 2,

$$\begin{array}{lll} \text{All Xs are Ys,} & x(1-y)=0, & x=xy \\ \text{All Zs are Ys,} & z(1-y)=0, & \underline{zy=z} \\ & & \underline{zx=zx} \end{array}$$

$\therefore 0=0.$

CLASS 2nd.—When v is introduced by the solution of an equation.

The lawful cases directly or indirectly* determinable by the Aristotelian Rules are AE, Fig. 1; AA, AE, EA, Fig. 3; EA, Fig. 4.

The lawful cases not so determinable, are EE, Fig. 1; EE, Fig. 2; EE, Fig. 3; EE, Fig. 4.

Ex. AE, Fig. 1, and, by mutation of premises, EA, Fig. 4.

$$\begin{array}{lll} \text{All Ys are Xs,} & y(1-x)=0, & y=vx \quad (a) \\ \text{No Zs are Ys,} & zy=0, & \underline{0=zy} \\ & & \underline{0=vzx} \end{array}$$

\therefore Some Xs are not Zs.

The reason why we cannot interpret $vzx=0$ into Some Zs are not-Xs, is that by the very terms of the first equation (a) the interpretation of vx is fixed, as Some Xs; v is regarded as the representative of Some, only with reference to the class X.

* We say *directly* or *indirectly*, mutation or conversion of premises being in some instances required. Thus, AE (fig. 1) is resolvable by Fesapo (fig. 4), or by Ferio (fig. 1). Aristotle and his followers rejected the fourth figure as only a modification of the first, but this being a mere question of form, either scheme may be termed Aristotelian.

D

For the reason of our employing a solution of one of the primitive equations, see the remarks on (16) and (17). Had we solved the second equation instead of the first, we should have had

$$\begin{aligned}(1-x)y &= 0, \\ v(1-z) &= y, \quad (a), \\ v(1-z)(1-x) &= 0, \quad (b), \\ \therefore \text{Some not-Zs are Xs.}\end{aligned}$$

Here it is to be observed, that the second equation (a) fixes the meaning of $v(1-z)$, as Some not-Zs. The full meaning of the result (b) is, that all the not-Zs which are found in the class Y are found in the class X, and it is evident that this could not have been expressed in any other way.

Ex. 2. AA, Fig. 3.

$$\begin{array}{lll}\text{All Ys are Xs,} & y(1-x) = 0, & y = vx \\ \text{All Ys are Zs,} & y(1-z) = 0, & 0 = y(1-z) \\ & & \hline & & 0 = vx(1-z) \\ & & \therefore \text{Some Xs are Zs.}\end{array}$$

Had we solved the second equation, we should have had as our result, Some Zs are Xs. The form of the final equation particularizes what Xs or what Zs are referred to, and this remark is general.

The following, EE, Fig. 1, and, by mutation, EE, Fig. 4, is an example of a lawful case not determinable by the Aristotelian Rules.

$$\begin{array}{lll}\text{No Ys are Xs,} & xy = 0, & 0 = xy \\ \text{No Zs are Ys,} & zy = 0, & y = v(1-z) \\ & & \hline & & 0 = v(1-z)x \\ & & \therefore \text{Some not-Zs are not Xs.}\end{array}$$

CLASS 3rd.—When v is met with in one of the equations, but not introduced by solution.

The lawful cases determinable *directly* or *indirectly* by the Aristotelian Rules, are AI, EI, Fig. 1; AO, EI, OA, IE, Fig. 2; AI, AO, EI, EO, IA, IE, OA, OE, Fig. 3; IA, IE, Fig. 4.

Those not so determinable are OE, Fig. 1; EO, Fig. 4.

The cases in which no inference is possible, are AO, EO, IA, IE, OA, Fig. 1; AI, EO, IA, OE, Fig. 2; OA, OE, AI, EI, AO, Fig. 4.

Ex. 1. AI, Fig. 1, and, by mutation, IA, Fig. 4.

$$\begin{array}{ll} \text{All Ys are Xs,} & y(1-x) = 0 \\ \text{Some Zs are Ys,} & vz = vy \\ & \hline & vz(1-x) = 0 \\ & \therefore \text{Some Zs are Xs.} \end{array}$$

Ex. 2. AO, Fig. 2, and, by mutation, OA, Fig. 2.

$$\begin{array}{lll} \text{All Xs are Ys,} & x(1-y) = 0, & x = xy \\ \text{Some Zs are not Ys,} & vz = v(1-y), & \frac{vy = v(1-z)}{vx = vx(1-z)} \\ & & \hline & & vzx = 0 \\ & & \therefore \text{Some Zs are not Xs.} \end{array}$$

The interpretation of vz as Some Zs, is implied, it will be observed, in the equation $vz = v(1-y)$ considered as representing the proposition Some Zs are not Ys.

The cases not determinable by the Aristotelian Rules are OE, Fig. 1, and, by mutation, EO, Fig. 4.

$$\begin{array}{ll} \text{Some Ys are not Xs,} & vy = v(1-x) \\ \text{No Zs are Ys,} & 0 = zy \\ & \hline & 0 = v(1-x)z \\ & \therefore \text{Some not-Xs are not Zs.} \end{array}$$

The equation of the first premiss here permits us to interpret $v(1-x)$, but it does not enable us to interpret vz .

Of cases in which no inference is possible, we take as examples—

AO, Fig. 1, and, by mutation, OA, Fig. 4,

$$\begin{array}{llll}
 \text{All Ys are Xs,} & y(1-x)=0, & & y(1-x)=0 \\
 \text{Some Zs are not Ys,} & vz=v(1-y) & (a) & v(1-z)=vy \\
 & & & \hline
 & & & v(1-z)(1-x)=0 \quad (b) \\
 & & & 0=0
 \end{array}$$

since the auxiliary equation in this case is $v(1-z)=0$.

Practically it is not necessary to perform this reduction, but it is satisfactory to do so. The equation (a), it is seen, defines vz as Some Zs, but it does not define $v(1-z)$, so that we might stop at the result of elimination (b), and content ourselves with saying, that it is not interpretable into a relation between the classes X and Z.

Take as a second example AI, Fig. 2, and, by mutation, IA, Fig. 2,

$$\begin{array}{llll}
 \text{All Xs are Ys,} & x(1-y)=0, & & x=xy \\
 \text{Some Zs are Ys,} & vz=vy, & & vy=vz \\
 & & & \hline
 & & & vx=vxz \\
 & & & v(1-z)x=0 \\
 & & & 0=0,
 \end{array}$$

the auxiliary equation in this case being $v(1-z)=0$.

Indeed in every case in this class, in which no inference is possible, the result of elimination is reducible to the form $0=0$. Examples therefore need not be multiplied.

CLASS 4th.—When v enters into both equations.

No inference is possible in any case, but there exists a distinction among the unlawful cases which is peculiar to this class. The two divisions are,

1st. When the result of elimination is reducible by the auxiliary equations to the form $0=0$. The cases are II, OI,

Fig. 1; II, OO, Fig. 2; II, IO, OI, OO, Fig. 3; II, IO, Fig. 4.

2nd. When the result of elimination is not reducible by the auxiliary equations to the form $0 = 0$.

The cases are IO, OO, Fig. 1; IO, OI, Fig. 2; OI, OO, Fig. 4.

Let us take as an example of the former case, II, Fig. 3.

$$\begin{array}{lll} \text{Some Xs are Ys,} & vx = vy, & vx = vy \\ \text{Some Zs are Ys,} & v'z = v'y, & \frac{v'y = v'z}{vv'x = vv'z} \end{array}$$

Now the auxiliary equations $v(1-x) = 0$, $v'(1-z) = 0$,
give $vx = v$, $v'z = v'$.

Substituting we have

$$\begin{array}{l} vv' = vv', \\ \therefore 0 = 0. \end{array}$$

As an example of the latter case, let us take IO, Fig. 1,

$$\begin{array}{lll} \text{Some Ys are Xs,} & vy = vx, & vy = vx \\ \text{Some Zs are not Ys,} & v'z = v'(1-y), & \frac{v'(1-z) = v'y}{vv'(1-z) = vv'x} \end{array}$$

Now the auxiliary equations being $v(1-x) = 0$, $v'(1-z) = 0$, the above reduces to $vv' = 0$. It is to this form that all similar cases are reducible. Its interpretation is, that the classes v and v' have no common member, as is indeed evident.

The above classification is purely founded on mathematical distinctions. We shall now inquire what is the logical division to which it corresponds.

The lawful cases of the first class comprehend all those in which, from two universal premises, a universal conclusion may be drawn. We see that they include the premises of *barbara* and *celarent* in the first figure, of *cesare* and *camestres* in the second, and of *bramantip* and *camenes* in the fourth.

The premises of *bramantip* are included, because they admit of an universal conclusion, although not in the same figure.

The lawful cases of the second class are those in which a particular conclusion only is deducible from two universal premises.

The lawful cases of the third class are those in which a conclusion is deducible from two premises, one of which is universal and the other particular.

The fourth class has no lawful cases.

Among the cases in which no inference of any kind is possible, we find six in the fourth class distinguishable from the others by the circumstance, that the result of elimination does not assume the form $0 = 0$. The cases are

$$\left\{ \begin{array}{l} \text{Some Ys are Xs,} \\ \text{Some Zs are not Ys,} \end{array} \right\} \left\{ \begin{array}{l} \text{Some Ys are not Xs,} \\ \text{Some Zs are not Ys,} \end{array} \right\} \left\{ \begin{array}{l} \text{Some Xs are Ys,} \\ \text{Some Zs are not Ys,} \end{array} \right\}$$

and the three others which are obtained by mutation of premises.

It might be presumed that some logical peculiarity would be found to answer to the mathematical peculiarity which we have noticed, and in fact there exists a very remarkable one. If we examine each pair of premises in the above scheme, we shall find that there is *virtually* no middle term, i.e. *no medium of comparison*, in any of them. Thus, in the first example, the individuals spoken of in the first premiss are asserted to belong to the class Y, but those spoken of in the second premiss are *virtually* asserted to belong to the class not-Y: nor can we by any lawful transformation or conversion alter this state of things. The comparison will still be made with the class Y in one premiss, and with the class not-Y in the other.

Now in every case beside the above six, there will be found a middle term, either expressed or implied. I select two of the most difficult cases.

In AO, Fig. 1, viz.

All Ys are Xs,
Some Zs are not Ys,

we have, by *negative conversion* of the first premiss,

All not-Xs are not-Ys,
Some Zs are not Ys,

and the middle term is now seen to be not-Y.

Again, in EO, Fig. 1,

No Ys are Xs,
Some Zs are not Ys,

a proved conversion of the first premiss (see *Conversion of Propositions*), gives

All Xs are not-Ys,
Some Zs are not-Ys,

and the middle term, the true medium of comparison, is plainly not-Y, although as the not-Ys in the one premiss *may be* different from those in the other, no conclusion can be drawn.

The mathematical condition in question, therefore,—the irreducibility of the final equation to the form $0 = 0$,—adequately represents the logical condition of there being no middle term, or common medium of comparison, in the given premises.

I am not aware that the distinction occasioned by the presence or absence of a middle term, in the strict sense here understood, has been noticed by logicians before. The distinction, though real and deserving attention, is indeed by no means an obvious one, and it would have been unnoticed in the present instance but for the peculiarity of its mathematical expression.

What appears to be novel in the above case is the proof of the existence of combinations of premises in which there

is absolutely no medium of comparison. When such a medium of comparison, or true middle term, does exist, the condition that its quantification in both premises together shall exceed its quantification as a single whole, has been ably and clearly shewn by Professor De Morgan to be necessary to lawful inference (*Cambridge Memoirs*, Vol. VIII. Part 3). And this is undoubtedly the true principle of the Syllogism, viewed from the standing-point of Arithmetic.

I have said that it would be possible to impose conditions of interpretation which should restrict the results of this calculus to the Aristotelian forms. Those conditions would be,

1st. That we should agree not to interpret the forms $v(1-x)$, $v(1-z)$.

2ndly. That we should agree to reject every interpretation in which the order of the terms should violate the Aristotelian rule.

Or, instead of the second condition, it might be agreed that, the conclusion being determined, the order of the premises should, if necessary, be changed, so as to make the syllogism formal.

From the *general* character of the system it is indeed plain, that it may be made to represent any conceivable scheme of logic, by imposing the conditions proper to the case contemplated.

We have found it, in a certain class of cases, to be necessary to replace the two equations expressive of universal Propositions, by their solutions; and it may be proper to remark, that it would have been allowable in all instances to have done this,* so that every case of the Syllogism, without ex-

* It may be satisfactory to illustrate this statement by an example. In *Barbara*, we should have

$$\begin{array}{ll} \text{All Ys are Xs,} & y = vx \\ \text{All Zs are Ys,} & z = v'y \\ & \hline & z = vv'x \\ \therefore \text{All Zs are Xs.} & \end{array}$$

ception, might have been treated by equations comprised in the general forms

$$\begin{array}{ll}
 y = vx, & \text{or } y - vx = 0 \dots A, \\
 y = v(1 - x), & \text{or } y + vx - v = 0 \dots E, \\
 vy = vx, & \text{or } vy - vx = 0 \dots I, \\
 vy = v(1 - x), & \text{or } vy + vx - v = 0 \dots O.
 \end{array}$$

Or, we may multiply the resulting equation by $1 - x$, which gives

$$z(1 - x) = 0,$$

whence the same conclusion, All Zs are Xs.

Some additional examples of the application of the system of equations in the text to the demonstration of general theorems, may not be inappropriate.

Let y be the term to be eliminated, and let x stand indifferently for either of the other symbols, then each of the equations of the premises of any given syllogism may be put in the form

$$ay + bx = 0, \quad (\alpha)$$

if the premiss is affirmative, and in the form

$$ay + b(1 - x) = 0, \quad (\beta)$$

if it is negative, a and b being either constant, or of the form $\pm v$. To prove this in detail, let us examine each kind of proposition, making y successively subject and predicate.

A, All Ys are Xs,	$y - vx = 0,$	(γ),
All Xs are Ys,	$x - vy = 0,$	(δ),
E, No Ys are Xs,	$xy = 0,$	
No Xs are Ys,	$y - v(1 - x) = 0,$	(ϵ),
I, Some Xs are Ys,		
Some Ys are Xs,	$vx - vy = 0,$	(ζ),
O, Some Ys are not Xs,	$vy - v(1 - x) = 0,$	(η),
Some Xs are not Ys,	$vx = v(1 - y),$	
	$\therefore vy - v(1 - x) = 0,$	(θ).

The affirmative equations (γ), (δ) and (ζ), belong to (α), and the negative equations (ϵ), (η) and (θ), to (β). It is seen that the two last negative equations are alike, but there is a difference of interpretation. In the former

$$v(1 - x) = \text{Some not-Xs,}$$

in the latter,

$$v(1 - x) = 0.$$

The utility of the two general forms of reference, (α) and (β), will appear from the following application.

1st. A conclusion drawn from two affirmative propositions is itself affirmative.

By (α) we have for the given propositions,

$$\begin{array}{l}
 ay + bx = 0, \\
 a'y + b'z = 0,
 \end{array}$$

Perhaps the system we have actually employed is better, as distinguishing the cases in which v only *may* be employed,

and eliminating

$$ab'z - a'bx = 0,$$

which is of the form (α). Hence, if there is a conclusion, it is affirmative.

2nd. *A conclusion drawn from an affirmative and a negative proposition is negative.*

By (α) and (β), we have for the given propositions

$$\begin{aligned} ay + bx &= 0, \\ a'y + b'(1 - z) &= 0, \\ \therefore a'bx - ab'(1 - z) &= 0, \end{aligned}$$

which is of the form (β). Hence the conclusion, if there is one, is negative.

3rd. *A conclusion drawn from two negative premises will involve a negation, (not-X, not-Z) in both subject and predicate, and will therefore be inadmissible in the Aristotelian system, though just in itself.*

For the premises being

$$\begin{aligned} ay + b(1 - x) &= 0, \\ a'y + b'(1 - x) &= 0, \end{aligned}$$

the conclusion will be

$$ab'(1 - x) - a'b(1 - x) = 0,$$

which is only interpretable into a proposition that has a negation in each term.

4th. *Taking into account those syllogisms only, in which the conclusion is the most general, that can be deduced from the premises,—if, in an Aristotelian syllogism, the minor premises be changed in quality (from affirmative to negative or from negative to affirmative), whether it be changed in quantity or not, no conclusion will be deducible in the same figure.*

An Aristotelian proposition does not admit a term of the form not-Z in the subject,—Now on changing the quantity of the minor proposition of a syllogism, we transfer it from the general form

$$ay + bx = 0,$$

to the general form

$$a'y + b'(1 - z) = 0,$$

see (α) and (β), or *vice versa*. And therefore, in the equation of the conclusion, there will be a change from x to $1 - x$, or *vice versa*. But this is equivalent to the change of Z into not-Z, or not-Z into Z. Now the subject of the original conclusion must have involved a Z and not a not-Z, therefore the subject of the new conclusion will involve a not-Z, and the conclusion will not be admissible in the Aristotelian forms, except by conversion, which would render necessary a change of Figure.

Now the conclusions of this calculus are always the most general that can be drawn, and therefore the above demonstration must not be supposed to extend to a syllogism, in which a particular conclusion is deduced, when a universal one is possible. This is the case with *bramantip* only, among the Aristotelian forms, and therefore the transformation of *bramantip* into *camenes*, and *vice versa*, is the case of restriction contemplated in the preliminary statement of the theorem.

from those in which it *must*. But for the demonstration of certain general properties of the Syllogism, the above system is, from its simplicity, and from the mutual analogy of its forms, very convenient. We shall apply it to the following theorem.*

Given the three propositions of a Syllogism, prove that there is but one order in which they can be legitimately arranged, and determine that order.

All the forms above given for the expression of propositions, are particular cases of the general form,

$$a + bx + cy = 0.$$

5th. If for the minor premiss of an Aristotelian syllogism, we substitute its contradictory, no conclusion is deducible in the same figure.

It is here only necessary to examine the case of *bramantip*, all the others being determined by the last proposition.

On changing the minor of *bramantip* to its contradictory, we have AO, Fig. 4, and this admits of no legitimate inference.

Hence the theorem is true without exception. Many other general theorems may in like manner be proved.

* This elegant theorem was communicated by the Rev. Charles Graves, Fellow and Professor of Mathematics in Trinity College, Dublin, to whom the Author desires further to record his grateful acknowledgments for a very judicious examination of the former portion of this work, and for some new applications of the method. The following example of Reduction *ad impossibile* is among the number :

Reducend Mood,	All Xs are Ys,	$1 - y = v' (1 - x)$
<i>Baroko</i>	Some Zs are not Ys	$\underline{vz = v (1 - y)}$
	Some Zs are not Xs	$vz = vv' (1 - x)$
Reduct Mood,	All Xs are Ys	$1 - y = v' (1 - x)$
<i>Barbara</i>	All Zs are Xs	$\underline{z (1 - x) = 0}$
	All Zs are Ys	$z (1 - y) = 0.$

The conclusion of the reduct mood is seen to be the contradictory of the suppressed minor premiss. Whence, &c. It may just be remarked that the mathematical test of contradictory propositions is, that on eliminating one elective symbol between their equations, the other elective symbol vanishes. The *ostensive* reduction of *Baroko* and *Bohardo* involves no difficulty.

Professor Graves suggests the employment of the equation $x = vy$ for the primary expression of the Proposition All Xs are Ys, and remarks, that on multiplying both members by $1 - y$, we obtain $x (1 - y) = 0$, the equation from which we set out in the text, and of which the previous one is a solution.

Assume then for the premises of the given syllogism, the equations

$$a + bx + cy = 0, \quad (18),$$

$$a' + b'z + c'y = 0, \quad (19),$$

then, eliminating y , we shall have for the conclusion

$$ac' - a'c + bc'x - b'cz = 0, \quad (20).$$

Now taking this as one of our premises, and either of the original equations, suppose (18), as the other, if by elimination of a common term x , between them, we can obtain a result equivalent to the remaining premiss (19), it will appear that there are more than one order in which the Propositions may be lawfully written; but if otherwise, one arrangement only is lawful.

Effecting then the elimination, we have

$$bc(a' + b'z + c'y) = 0, \quad (21),$$

which is equivalent to (19) multiplied by a factor bc . Now on examining the value of this factor in the equations A, E, I, O, we find it in each case to be v or $-v$. But it is evident, that if an equation expressing a given Proposition be multiplied by an extraneous factor, derived from another equation, its interpretation will either be limited or rendered impossible. Thus there will either be no result at all, or the result will be a *limitation* of the remaining Proposition.

If, however, one of the original equations were

$$x = y, \quad \text{or} \quad x - y = 0,$$

the factor bc would be -1 , and would *not* limit the interpretation of the other premiss. Hence if the first member of a syllogism should be understood to represent the double proposition All Xs are Ys, and All Ys are Xs, it would be indifferent in what order the remaining Propositions were written.

A more general form of the above investigation would be, to express the premises by the equations

$$a + bx + cy + dxy = 0, \quad (22),$$

$$a' + b'z + c'y + d'zy = 0, \quad (23).$$

After the double elimination of y and x we should find

$$(bc - ad) (a' + b'z + c'y + d'zy) = 0;$$

and it would be seen that the factor $bc - ad$ must in every case either vanish or express a limitation of meaning.

The determination of the order of the Propositions is sufficiently obvious.

OF HYPOTHETICALS.

A hypothetical Proposition is defined to be *two or more categoricals united by a copula* (or conjunction), and the different kinds of hypothetical Propositions are named from their respective conjunctions, viz. conditional (if), disjunctive (either, or), &c.

In conditionals, that categorical Proposition from which the other results is called the *antecedent*, that which results from it the *consequent*.

Of the conditional syllogism there are two, and only two formulæ.

1st. The constructive,

If A is B, then C is D,
But A is B, therefore C is D.

2nd. The Destructive,

If A is B, then C is D,
But C is not D, therefore A is not B.

A dilemma is a complex conditional syllogism, with several antecedents in the major, and a disjunctive minor.

If we examine either of the forms of conditional syllogism above given, we shall see that the validity of the argument does not depend upon any considerations which have reference to the terms A, B, C, D, considered as the representatives of individuals or of classes. We may, in fact, represent the Propositions A is B, C is D, by the arbitrary symbols X and Y respectively, and express our syllogisms in such forms as the following :

If X is true, then Y is true,
But X is true, therefore Y is true.

Thus, what we have to consider is not objects and classes of objects, but the truths of Propositions, namely, of those

elementary Propositions which are embodied in the terms of our hypothetical premises.

To the symbols X, Y, Z , representative of Propositions, we may appropriate the elective symbols x, y, z , in the following sense.

The hypothetical Universe, 1, shall comprehend all conceivable cases and conjunctures of circumstances.

The elective symbol x attached to any subject expressive of such cases shall select those cases in which the Proposition X is true, and similarly for Y and Z .

If we confine ourselves to the contemplation of a given proposition X , and hold in abeyance every other consideration, then two cases only are conceivable, viz. first that the given Proposition is true, and secondly that it is false.* As these cases together make up the Universe of the Proposition, and as the former is determined by the elective symbol x , the latter is determined by the symbol $1 - x$.

But if other considerations are admitted, each of these cases will be resolvable into others, individually less extensive, the

* It was upon the obvious principle that a Proposition is either true or false, that the Stoics, applying it to assertions respecting future events, endeavoured to establish the doctrine of Fate. It has been replied to their argument, that it involves "an abuse of the word *true*, the precise meaning of which is *id quod res est*. An assertion respecting the future is neither true nor false."—*Copleston on Necessity and Predestination*, p. 36. Were the Stoic axiom, however, presented under the form, It is either certain that a given event will take place, or certain that it will not; the above reply would fail to meet the difficulty. The proper answer would be, that no merely verbal definition can settle the question, what is the actual course and constitution of Nature. When we affirm that it is either certain that an event will take place, or certain that it will not take place, we tacitly assume that the order of events is necessary, that the Future is but an evolution of the Present; so that the state of things which is, completely determines that which shall be. But this (at least as respects the conduct of moral agents) is the very question at issue. Exhibited under its proper form, the Stoic reasoning does not involve an abuse of terms, but a *petitio principii*.

It should be added, that enlightened advocates of the doctrine of Necessity in the present day, viewing the end as appointed only in and through the means, justly repudiate those practical ill consequences which are the reproach of Fatalism.

number of which will depend upon the number of foreign considerations admitted. Thus if we associate the Propositions **X** and **Y**, the total number of conceivable cases will be found as exhibited in the following scheme.

Cases.	Elective expressions.
1st X true, Y true	xy
2nd X true, Y false.....	$x(1 - y)$
3rd X false, Y true.....	$(1 - x)y$
4th X false, Y false.....	$(1 - x)(1 - y) \quad (24).$

If we add the elective expressions for the two first of the above cases the sum is x , which is the elective symbol appropriate to the more general case of **X** being true independently of any consideration of **Y**; and if we add the elective expressions in the two last cases together, the result is $1 - x$, which is the elective expression appropriate to the more general case of **X** being false.

Thus the extent of the hypothetical Universe does not at all depend upon the number of circumstances which are taken into account. And it is to be noted that however few or many those circumstances may be, the sum of the elective expressions representing every conceivable case will be unity. Thus let us consider the three Propositions, **X**, It rains, **Y**, It hails, **Z**, It freezes. The possible cases are the following:

Cases.	Elective expressions.
1st It rains, hails, and freezes,	xyz
2nd It rains and hails, but does not freeze	$xy(1 - z)$
3rd It rains and freezes, but does not hail	$xz(1 - y)$
4th It freezes and hails, but does not rain	$yz(1 - x)$
5th It rains, but neither hails nor freezes	$x(1 - y)(1 - z)$
6th It hails, but neither rains nor freezes	$y(1 - x)(1 - z)$
7th It freezes, but neither hails nor rains	$z(1 - x)(1 - y)$
8th It neither rains, hails, nor freezes	$(1 - x)(1 - y)(1 - z)$
<hr/>	
$1 = \text{sum}$	

Expression of Hypothetical Propositions.

To express that a given Proposition X is true.

The symbol $1 - x$ selects those cases in which the Proposition X is false. But if the Proposition is true, there are no such cases in its hypothetical Universe, therefore

$$1 - x = 0,$$

or $x = 1, \quad (25).$

To express that a given Proposition X is false.

The elective symbol x selects all those cases in which the Proposition is true, and therefore if the Proposition is false,

$$x = 0, \quad (26).$$

And in every case, having determined the elective expression appropriate to a given Proposition, we assert the truth of that Proposition by equating the elective expression to unity, and its falsehood by equating the same expression to 0.

To express that two Propositions, X and Y, are simultaneously true.

The elective symbol appropriate to this case is xy , therefore the equation sought is

$$xy = 1, \quad (27).$$

To express that two Propositions, X and Y, are simultaneously false.

The condition will obviously be

$$(1 - x)(1 - y) = 1,$$

or $x + y - xy = 0, \quad (28).$

To express that either the Proposition X is true, or the Proposition Y is true.

To assert that either one or the other of two Propositions is true, is to assert that it is not true, that they are both false. Now the elective expression appropriate to their both being false is $(1 - x)(1 - y)$, therefore the equation required is

$$(1 - x)(1 - y) = 0,$$

or $x + y - xy = 1, \quad (29).$

E

And, by indirect considerations of this kind, may every disjunctive Proposition, however numerous its members, be expressed. But the following general Rule will usually be preferable.

RULE. Consider what are those distinct and mutually exclusive cases of which it is implied in the statement of the given Proposition, that some one of them is true, and equate the sum of their elective expressions to unity. This will give the equation of the given Proposition.

For the sum of the elective expressions for all distinct conceivable cases will be unity. Now all these cases being mutually exclusive, and it being asserted in the given Proposition that some one case out of a given set of them is true, it follows that all which are not included in that set are false, and that their elective expressions are severally equal to 0. Hence the sum of the elective expressions for the remaining cases, viz. those included in the given set, will be unity. Some one of those cases will therefore be true, and as they are mutually exclusive, it is impossible that more than one should be true. Whence the Rule in question.

And in the application of this Rule it is to be observed, that if the cases contemplated in the given disjunctive Proposition are not mutually exclusive, they must be resolved into an equivalent series of cases which are mutually exclusive.

Thus, if we take the Proposition of the preceding example, viz. Either X is true, or Y is true, and assume that the two members of this Proposition are not exclusive, insomuch that in the enumeration of possible cases, we must reckon that of the Propositions X and Y being both true, then the mutually exclusive cases which fill up the Universe of the Proposition, with their elective expressions, are

1st, X true and Y false,	$x(1 - y),$
2nd, Y true and X false,	$y(1 - x),$
3rd, X true and Y true,	$xy,$

and the sum of these elective expressions equated to unity gives

$$x + y - xy = 1. \quad (30),$$

as before. But if we suppose the members of the disjunctive Proposition to be exclusive, then the only cases to be considered are

$$\text{1st, } X \text{ true, } Y \text{ false,} \quad x(1 - y),$$

$$\text{2nd, } Y \text{ true, } X \text{ false,} \quad y(1 - x),$$

and the sum of these elective expressions equated to 0, gives

$$x - 2xy + y = 1, \quad (31).$$

The subjoined examples will further illustrate this method.

To express the Proposition, Either X is not true, or Y is not true, the members being exclusive. *members*

The mutually exclusive cases are

$$\text{1st, } X \text{ not true, } Y \text{ true,} \quad y(1 - x),$$

$$\text{2nd, } Y \text{ not true, } X \text{ true,} \quad x(1 - y),$$

and the sum of these equated to unity gives

$$x - 2xy + y = 1, \quad (32),$$

which is the same as (31), and in fact the Propositions which they represent are equivalent.

To express the Proposition, Either X is not true, or Y is not true, the members not being exclusive.

To the cases contemplated in the last Example, we must add the following, viz.

$$X \text{ not true, } Y \text{ not true,} \quad (1 - x)(1 - y).$$

The sum of the elective expressions gives

$$x(1 - y) + y(1 - x) + (1 - x)(1 - y) = 1,$$

$$\text{or } xy = 0, \quad (33).$$

To express the disjunctive Proposition, Either X is true, or Y is true, or Z is true, the members being exclusive.

Here the mutually exclusive cases are

- 1st, X true, Y false, Z false, $x(1-y)(1-z)$,
 2nd, Y true, Z false, X false, $y(1-z)(1-x)$,
 3rd, Z true, X false, Y false, $z(1-x)(1-y)$,

and the sum of the elective expressions equated to 1, gives, upon reduction,

$$x + y + z - 2(xy + yz + zx) + 3xyz = 1, \quad (34).$$

The expression of the same Proposition, when the members are in no sense exclusive, will be

$$(1-x)(1-y)(1-z) = 0, \quad (35).$$

And it is easy to see that our method will apply to the expression of any similar Proposition, whose members are subject to any specified amount and character of exclusion.

To express the conditional Proposition, If X is true, Y is true.

Here it is implied that all the cases of X being true, are cases of Y being true. The former cases being determined by the elective symbol x , and the latter by y , we have, in virtue of (4),

$$x(1-y) = 0, \quad (36).$$

To express the conditional Proposition, If X be true, Y is not true.

The equation is obviously

$$xy = 0, \quad (37);$$

this is equivalent to (33), and in fact the disjunctive Proposition, Either X is not true, or Y is not true, and the conditional Proposition, If X is true, Y is not true, are equivalent.

To express that If X is not true, Y is not true.

In (36) write $1-x$ for x , and $1-y$ for y , we have

$$(1-x)y = 0.$$

The results which we have obtained admit of verification in many different ways. Let it suffice to take for more particular examination the equation

$$x - 2xy + y = 1, \quad (38),$$

which expresses the conditional Proposition, Either X is true, or Y is true, the members being in this case exclusive.

First, let the Proposition X be true, then $x = 1$, and substituting, we have

$$1 - 2y + y = 1, \quad \therefore -y = 0, \text{ or } y = 0,$$

which implies that Y is not true.

Secondly, let X be not true, then $x = 0$, and the equation gives

$$y = 1, \quad (39),$$

which implies that Y is true. In like manner we may proceed with the assumptions that Y is true, or that Y is false.

Again, in virtue of the property $x^2 = x$, $y^2 = y$, we may write the equation in the form

$$x^2 - 2xy + y^2 = 1,$$

and extracting the square root, we have

$$x - y = \pm 1, \quad (40),$$

and this represents the actual case; for, as when X is true or false, Y is respectively false or true, we have

$$x = 1 \text{ or } 0,$$

$$y = 0 \text{ or } 1,$$

$$\therefore x - y = 1 \text{ or } -1.$$

There will be no difficulty in the analysis of other cases.

Examples of Hypothetical Syllogism.

The treatment of every form of hypothetical Syllogism will consist in forming the equations of the premises, and eliminating the symbol or symbols which are found in more than one of them. The result will express the conclusion.

1st. Disjunctive Syllogism.

Either X is true, or Y is true (exclusive),	$x + y - 2xy = 1$
But X is true,	$x = 1$
Therefore Y is not true,	$\therefore y = 0$
Either X is true, or Y is true (not exclusive),	$x + y - xy = 1$
But X is not true,	$x = 0$
Therefore Y is true,	$\therefore y = 1$

2nd. Constructive Conditional Syllogism.

If X is true, Y is true,	$x(1 - y) = 0$
But X is true,	$x = 1$
Therefore Y is true,	$\therefore 1 - y = 0$ or $y = 1$.

3rd. Destructive Conditional Syllogism.

If X is true, Y is true,	$x(1 - y) = 0$
But Y is not true,	$y = 0$
Therefore X is not true,	$\therefore x = 0$

4th. Simple Constructive Dilemma, the minor premiss exclusive.

If X is true, Y is true,	$x(1 - y) = 0$, (41),
If Z is true, Y is true,	$z(1 - y) = 0$, (42),
But Either X is true, or Z is true,	$x + z - 2xz = 1$, (43).

From the equations (41), (42), (43), we have to eliminate x and z . In whatever way we effect this, the result is

$$y = 1;$$

whence it appears that the Proposition Y is true.

5th. Complex Constructive Dilemma, the minor premiss not exclusive.

If X is true, Y is true,	$x(1 - y) = 0$,
If W is true, Z is true,	$w(1 - z) = 0$,
Either X is true, or W is true,	$x + w - xw = 1$.

From these equations, eliminating x , we have

$$y + z - yz = 1,$$

which expresses the Conclusion, Either Y is true, or Z is true, the members being non-exclusive.

6th. Complex Destructive Dilemma, the minor premiss exclusive.

If X is true, Y is true, $x(1 - y) = 0$

If W is true, Z is true, $w(1 - z) = 0$

Either Y is not true, or Z is not true, $y + z - 2yz = 1$.

From these equations we must eliminate y and z . The result is

$$xw = 0,$$

which expresses the Conclusion, Either X is not true, or Y is not true, the members *not being exclusive*.

7th. Complex Destructive Dilemma, the minor premiss not exclusive.

If X is true, Y is true, $x(1 - y) = 0$

If W is true, Z is true, $w(1 - z) = 0$

Either Y is not true, or Z is not true, $yz = 0$.

On elimination of y and z , we have

$$xw = 0,$$

which indicates the same Conclusion as the previous example.

It appears from these and similar cases, that whether the members of the minor premiss of a Dilemma are exclusive or not, the members of the (disjunctive) Conclusion are never exclusive. This fact has perhaps escaped the notice of logicians.

The above are the principal forms of hypothetical Syllogism which logicians have recognised. It would be easy, however, to extend the list, especially by the blending of the disjunctive and the conditional character in the same Proposition, of which the following is an example.

If X is true, then either Y is true, or Z is true,

$$x(1 - y - z + yz) = 0$$

But Y is not true,

$$y = 0$$

Therefore If X is true, Z is true,

$$\therefore x(1 - z) = 0.$$

That which logicians term a *Causal Proposition* is properly a conditional Syllogism, the major premiss of which is suppressed.

The assertion that the Proposition X is true, *because* the Proposition Y is true, is equivalent to the assertion,

The Proposition Y is true,
Therefore the Proposition X is true;

and these are the minor premiss and conclusion of the conditional Syllogism,

If Y is true, X is true,
But Y is true,
Therefore X is true.

And thus causal Propositions are seen to be included in the applications of our general method.

Note, that there is a family of disjunctive and conditional Propositions, which do not, of right, belong to the class considered in this Chapter. Such are those in which the force of the disjunctive or conditional particle is expended upon the predicate of the Proposition, as if, speaking of the inhabitants of a particular island, we should say, that they are all *either Europeans or Asiatics*; meaning, that it is true of each individual, that he is either a European or an Asiatic. If we appropriate the elective symbol x to the inhabitants, y to Europeans, and z to Asiatics, then the equation of the above Proposition is

$$x = xy + xz, \text{ or } x(1 - y - z) = 0, (a);$$

to which we might add the condition $yz = 0$, since no Europeans are Asiatics. The nature of the symbols x , y , z , indicates that the Proposition belongs to those which we have before designated as *Categorical*. Very different from the above is the Proposition, Either all the inhabitants are Europeans, or they are all Asiatics. Here the disjunctive particle separates Propositions. The case is that contemplated in (31) of the present Chapter; and the symbols by which it is expressed,

although subject to the same laws as those of (a), have a totally different interpretation.*

The distinction is real and important. Every Proposition which language can express may be represented by elective symbols, and the laws of combination of those symbols are in all cases the same; but in one class of instances the symbols have reference to collections of objects, in the other, to the truths of constituent Propositions.

* Some writers, among whom is Dr. Latham (*First Outlines*), regard it as the exclusive office of a conjunction to connect *Propositions*, not *words*. In this view I am not able to agree. The Proposition, Every animal is *either* rational *or* irrational, cannot be resolved into, *Either* every animal is rational, *or* every animal is irrational. The former belongs to pure categoricals, the latter to hypotheticals. In *singular* Propositions, such conversions would seem to be allowable. This animal is *either* rational *or* irrational, is equivalent to, *Either* this animal is rational, *or* it is irrational. This peculiarity of *singular* Propositions would almost justify our ranking them, though truly universals, in a separate class, as Ramus and his followers did.

PROPERTIES OF ELECTIVE FUNCTIONS.

SINCE elective symbols combine according to the laws of quantity, we may, by Maclaurin's theorem, expand a given function $\phi(x)$, in ascending powers of x , known cases of failure excepted. Thus we have

$$\phi(x) = \phi(0) + \phi'(0)x + \frac{\phi''(0)}{1.2}x^2 + \&c., \quad (44).$$

Now $x^2 = x$, $x^3 = x$, &c., whence

$$\phi(x) = \phi(0) + x \left\{ \phi'(0) + \frac{\phi''(0)}{1.2} + \&c. \right\}, \quad (45).$$

Now if in (44) we make $x = 1$, we have

$$\phi(1) = \phi(0) + \phi'(0) + \frac{\phi''(0)}{1.2} + \&c.,$$

whence

$$\phi'(0) + \frac{\phi''(0)}{1.2} + \frac{\phi'''(0)}{1.2.3} + \&c. = \phi(1) - \phi(0).$$

Substitute this value for the coefficient of x in the second member of (45), and we have*

$$\phi(x) = \phi(0) + \{\phi(1) - \phi(0)\}x, \quad (46),$$

* Although this and the following theorems have only been proved for those forms of functions which are expansible by Maclaurin's theorem, they may be regarded as true for all forms whatever; this will appear from the applications. The reason seems to be that, as it is only through the one form of expansion that elective functions become interpretable, no conflicting interpretation is possible.

The development of $\phi(x)$ may also be determined thus. By the known formula for expansion in factorials,

$$\phi(x) = \phi(0) + \Delta\phi(0)x + \frac{\Delta^2\phi(0)}{1.2}x(x-1) + \&c.$$

which we shall also employ under the form

$$\phi(x) = \phi(1)x + \phi(0)(1-x), \quad (47).$$

Every function of x , in which integer powers of that symbol are alone involved, is by this theorem reducible to the first order. The quantities $\phi(0)$, $\phi(1)$, we shall call the moduli of the function $\phi(x)$. They are of great importance in the theory of elective functions, as will appear from the succeeding Propositions.

PROP. 1. Any two functions $\phi(x)$, $\psi(x)$, are equivalent, whose corresponding moduli are equal.

This is a plain consequence of the last Proposition. For since

$$\phi(x) = \phi(0) + \{\phi(1) - \phi(0)\}x,$$

$$\psi(x) = \psi(0) + \{\psi(1) - \psi(0)\}x,$$

it is evident that if $\phi(0) = \psi(0)$, $\phi(1) = \psi(1)$, the two expansions will be equivalent, and therefore the functions which they represent will be equivalent also.

The converse of this Proposition is equally true, viz.

If two functions are equivalent, their corresponding moduli are equal.

Among the most important applications of the above theorem, we may notice the following.

Suppose it required to determine for what forms of the function $\phi(x)$, the following equation is satisfied, viz.

$$\{\phi(x)\}^n = \phi(x).$$

Now x being an elective symbol, $x(x-1) = 0$, so that all the terms after the second, vanish. Also $\Delta\phi(0) = \phi(1) - \phi(0)$, whence

$$\phi\{x = \phi(0)\} + \{\phi(1) - \phi(0)\}x.$$

The mathematician may be interested in the remark, that this is not the only case in which an expansion stops at the second term. The expansions of the compound operative functions $\phi\left(\frac{d}{dx} + x^{-1}\right)$ and $\phi\left\{x + \left(\frac{d}{dx}\right)^{-1}\right\}$ are, respectively,

$$\phi\left(\frac{d}{dx}\right) + \phi'\left(\frac{d}{dx}\right)x^{-1},$$

and

$$\phi(x) + \phi'(x)\left(\frac{d}{dx}\right)^{-1}.$$

See *Cambridge Mathematical Journal*, Vol. IV. p. 219.

Here we at once obtain for the expression of the conditions in question,

$$\{\phi(0)\}^n = \phi(0). \quad \{\phi(1)\}^n = \phi(1), \quad (48).$$

Again, suppose it required to determine the conditions under which the following equation is satisfied, viz.

$$\phi(x) \psi(x) = \chi(x),$$

The general theorem at once gives

$$\phi(0) \psi(0) = \chi(0). \quad \phi(1) \psi(1) = \chi(1), \quad (49).$$

This result may also be proved by substituting for $\phi(x)$, $\psi(x)$, $\chi(x)$, their expanded forms, and equating the coefficients of the resulting equation properly reduced.

All the above theorems may be extended to functions of more than one symbol. For, as different elective symbols combine with each other according to the same laws as symbols of quantity, we can first expand a given function with reference to any particular symbol which it contains, and then expand the result with reference to any other symbol, and so on in succession, the order of the expansions being quite indifferent.

Thus the given function being $\phi(xy)$ we have

$$\phi(xy) = \phi(x0) + \{\phi(x1) - \phi(x0)\} y,$$

and expanding the coefficients with reference to x , and reducing

$$\begin{aligned} \phi(xy) = & \phi(00) + \{\phi(10) - \phi(00)\} x + \{\phi(01) - \phi(00)\} y \\ & + \{\phi(11) - \phi(10) - \phi(01) + \phi(00)\} xy, \end{aligned} \quad (50),$$

to which we may give the elegant symmetrical form

$$\begin{aligned} \phi(xy) = & \phi(00)(1-x)(1-y) + \phi(01)y(1-x) \\ & + \phi(10)x(1-y) + \phi(11)xy, \end{aligned} \quad (51),$$

wherein we shall, in accordance with the language already employed, designate $\phi(00)$, $\phi(01)$, $\phi(10)$, $\phi(11)$, as the moduli of the function $\phi(xy)$.

By inspection of the above general form, it will appear that any functions of two variables are equivalent, whose corresponding moduli are all equal.

Thus the conditions upon which depends the satisfaction of the equation,

$$\{\phi(xy)\}^n = \phi(xy)$$

are seen to be

$$\begin{aligned} \{\phi(00)\}^n &= \phi(00), & \{\phi(01)\}^n &= \phi(01), \\ \{\phi(10)\}^n &= \phi(10), & \{\phi(11)\}^n &= \phi(11), \end{aligned} \quad (52).$$

And the conditions upon which depends the satisfaction of the equation

$$\phi(xy) \psi(xy) = \chi(xy),$$

are

$$\begin{aligned} \phi(00) \psi(00) &= \chi(00), & \phi(01) \psi(01) &= \chi(01), \\ \phi(10) \psi(10) &= \chi(10), & \phi(11) \psi(11) &= \chi(11), \end{aligned} \quad (53).$$

It is very easy to assign by induction from (47) and (51), the general form of an expanded elective function. It is evident that if the number of elective symbols is m , the number of the moduli will be 2^m , and that their separate values will be obtained by interchanging in every possible way the values 1 and 0 in the places of the elective symbols of the given function. The several terms of the expansion of which the moduli serve as coefficients, will then be formed by writing for each 1 that recurs under the functional sign, the elective symbol x , &c., which it represents, and for each 0 the corresponding $1 - x$, &c., and regarding these as factors, the product of which, multiplied by the modulus from which they are obtained, constitutes a term of the expansion.

Thus, if we represent the moduli of any elective function $\phi(xy \dots)$ by $a_1, a_2, \dots a_r$, the function itself, when expanded and arranged with reference to the moduli, will assume the form

$$\phi(xy) = a_1 t_1 + a_2 t_2 \dots + a_r t_r, \quad (54),$$

in which $t_1 t_2 \dots t_r$ are functions of $x, y \dots$, resolved into factors of the forms $x, y, \dots 1 - x, 1 - y, \dots$ &c. These functions satisfy individually the index relations

$$t_1^n = t_1, \quad t_2^n = t_2, \quad \&c. \quad (55),$$

and the further relations,

$$t_1 t_2 = 0 \dots t_1 t_2 = 0, \quad \&c. \quad (56),$$

the product of any two of them vanishing. This will at once be inferred from inspection of the particular forms (47) and (51). Thus in the latter we have for the values of t_1, t_2 , &c., the forms

$$xy, x(1-y), (1-x)y, (1-x)(1-y);$$

and it is evident that these satisfy the index relation, and that their products all vanish. We shall designate $t_1 t_2 \dots$ as the constituent functions of $\phi(xy)$, and we shall define the peculiarity of the vanishing of the binary products, by saying that those functions are *exclusive*. And indeed the classes which they represent are mutually exclusive.

The sum of all the constituents of an expanded function is unity. An elegant proof of this Proposition will be obtained by expanding 1 as a function of any proposed elective symbols. Thus if in (51) we assume $\phi(xy) = 1$, we have $\phi(11) = 1$,

$$\phi(10) = 1, \quad \phi(01) = 1, \quad \phi(00) = 1, \quad \text{and (51) gives}$$

$$1 = xy + x(1-y) + (1-x)y + (1-x)(1-y), \quad (57).$$

It is obvious indeed, that however numerous the symbols involved, all the moduli of unity are unity, whence the sum of the constituents is unity.

We are now prepared to enter upon the question of the general interpretation of elective equations. For this purpose we shall find the following Propositions of the greatest service.

PROP. 2. If the first member of the general equation $\phi(xy\dots) = 0$, be expanded in a series of terms, each of which is of the form at , a being a modulus of the given function, then for every numerical modulus a which does not vanish, we shall have the equation

$$at = 0,$$

and the combined interpretations of these several equations will express the full significance of the original equation.

For, representing the equation under the form

$$a_1 t_1 + a_2 t_2 \dots + a_r t_r = 0, \quad (58).$$

Multiplying by t_1 , we have, by (56),

$$a_1 t_1 = 0, \quad (59),$$

whence if α_1 is a numerical constant which does not vanish,

$$t_1 = 0,$$

and similarly for all the moduli which do not vanish. And inasmuch as from these constituent equations we can form the given equation, their interpretations will together express its entire significance.

Thus if the given equation were

$$x - y = 0, \quad \text{Xs and Ys are identical, (60),}$$

we should have $\phi(11) = 0$, $\phi(10) = 1$, $\phi(01) = -1$, $\phi(00) = 0$, so that the expansion (51) would assume the form

$$x(1 - y) - y(1 - x) = 0,$$

whence, by the above theorem,

$$x(1 - y) = 0, \quad \text{All Xs are Ys,}$$

$$y(1 - x) = 0, \quad \text{All Ys are Xs,}$$

results which are together equivalent to (60).

It may happen that the simultaneous satisfaction of equations thus deduced, may require that one or more of the elective symbols should vanish. This would only imply the nonexistence of a class: it may even happen that it may lead to a final result of the form

$$1 = 0,$$

which would indicate the nonexistence of the logical Universe. Such cases will only arise when we attempt to unite contradictory Propositions in a single equation. The manner in which the difficulty seems to be evaded in the result is characteristic.

It appears from this Proposition, that the differences in the interpretation of elective functions depend solely upon the number and position of the vanishing moduli. No change in the value of a modulus, but one which causes it to vanish, produces any change in the interpretation of the equation in which it is found. If among the infinite number of different values which we are thus permitted to give to the moduli which do not vanish in a proposed equation, any one value should be

preferred, it is unity, for when the moduli of a function are all either 0 or 1, the function itself satisfies the condition

$$\{\phi(xy \dots)\}^n = \phi(xy \dots),$$

and this at once introduces symmetry into our Calculus, and provides us with fixed standards for reference.

PROP. 3. If $w = \phi(xy \dots)$, w, x, y, \dots being elective symbols, and if the second member be completely expanded and arranged in a series of terms of the form at , we shall be permitted to equate separately to 0 every term in which the modulus a does not satisfy the condition

$$a^n = a,$$

and to leave for the value of $\overset{w}{x}$ the sum of the remaining terms.

As the nature of the demonstration of this Proposition is quite unaffected by the number of the terms in the second member, we will for simplicity confine ourselves to the supposition of there being four, and suppose that the moduli of the two first only, satisfy the index law.

We have then

$$w = a_1 t_1 + a_2 t_2 + a_3 t_3 + a_4 t_4, \quad (61),$$

with the relations $a_1^n = a_1, \quad a_2^n = a_2,$

in addition to the two sets of relations connecting t_1, t_2, t_3, t_4 , in accordance with (55) and (56).

Squaring (61), we have

$$w = a_1 t_1 + a_2 t_2 + a_3^2 t_3 + a_4^2 t_4,$$

and subtracting (61) from this,

$$(a_3^2 - a_3) t_3 + (a_4^2 - a_4) t_4 = 0;$$

and it being an hypothesis, that the coefficients of these terms do not vanish, we have, by Prop. 2,

$$t_3 = 0, \quad t_4 = 0, \quad (62),$$

whence (61) becomes

$$\overset{w}{x} = a_1 t_1 + a_2 t_2.$$

The utility of this Proposition will hereafter appear.

PROP. 4. The functions $t_1 t_2 \dots t_r$ being mutually exclusive, we shall always have

$$\psi(a_1 t_1 + a_2 t_2 \dots + a_r t_r) = \psi(a_1) t_1 + \psi(a_2) t_2 \dots + \psi(a_r) t_r, \quad (63),$$

whatever may be the values of $a_1 a_2 \dots a_r$ or the form of ψ .

Let the function $a_1 t_1 + a_2 t_2 \dots + a_r t_r$ be represented by $\phi(xy \dots)$, then the moduli $a_1 a_2 \dots a_r$ will be given by the expressions

$$\phi(11\dots), \phi(10\dots), (\dots) \phi(00\dots).$$

$$\begin{aligned} \text{Also } \psi(a_1 t_1 + a_2 t_2 \dots + a_r t_r) &= \psi\{\phi(xy \dots)\} \\ &= \psi\{\phi(11\dots)\} xy \dots + \psi\{\phi(10\dots)\} x(1-y) \dots \\ &\quad + \psi\{\phi(00\dots)\} (1-x)(1-y) \dots \\ &= \psi(a_1) xy \dots + \psi(a_2) x(1-y) \dots + \psi(a_r) (1-x)(1-y) \dots \\ &= \psi(a_1) t_1 + \psi(a_2) t_2 \dots + \psi(a_r) t_r, \quad (64). \end{aligned}$$

It would not be difficult to extend the list of interesting properties, of which the above are examples. But those which we have noticed are sufficient for our present requirements. The following Proposition may serve as an illustration of their utility.

PROP. 5. Whatever process of reasoning we apply to a single given Proposition, the result will either be the same Proposition or a limitation of it.

Let us represent the equation of the given Proposition under its most general form,

$$a_1 t_1 + a_2 t_2 \dots + a_r t_r = 0, \quad (65),$$

resolvable into as many equations of the form $t = 0$ as there are moduli which do not vanish.

Now the most general transformation of this equation is

$$\psi(a_1 t_1 + a_2 t_2 \dots + a_r t_r) = \psi(0), \quad (66),$$

provided that we attribute to ψ a perfectly arbitrary character, allowing it even to involve new elective symbols, having *any proposed relation* to the original ones.

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The development of (66) gives, by the last Proposition,

$$\psi(a_1)t_1 + \psi(a_2)t_2 \dots + \psi(a_r)t_r = \psi(0).$$

To reduce this to the general form of reference, it is only necessary to observe that since

$$t_1 + t_2 \dots + t_r = 1,$$

we may write for $\psi(0)$,

$$\psi(0)(t_1 + t_2 \dots + t_r),$$

whence, on substitution and transposition,

$$\{\psi(a_1) - \psi(0)\}t_1 + \{\psi(a_2) - \psi(0)\}t_2 \dots + \{\psi(a_r) - \psi(0)\}t_r = 0.$$

From which it appears, that if a be any modulus of the original equation, the corresponding modulus of the transformed equation will be

$$\psi(a) - \psi(0).$$

If $a = 0$, then $\psi(a) - \psi(0) = \psi(0) - \psi(0) = 0$, whence there are no *new terms* in the transformed equation, and therefore there are no *new Propositions* given by equating its constituent members to 0.

Again, since $\psi(a) - \psi(0)$ may vanish without a vanishing, terms may be wanting in the transformed equation which existed in the primitive. Thus some of the constituent truths of the original Proposition may entirely disappear from the interpretation of the final result.

Lastly, if $\psi(a) - \psi(0)$ do not vanish, it must either be a numerical constant, or it must involve new elective symbols. In the former case, the term in which it is found will give

$$t = 0,$$

which is one of the constituents of the original equation: in the latter case we shall have

$$\{\psi(a) - \psi(0)\}t = 0,$$

in which t has a limiting factor. The interpretation of this equation, therefore, is a limitation of the interpretation of (65).

The purport of the last investigation will be more apparent to the mathematician than to the logician. As from any mathematical equation an infinite number of others may be deduced, it seemed to be necessary to shew that when the original equation expresses a logical Proposition, every member of the derived series, even when obtained by expansion under a functional sign, admits of exact and consistent interpretation.

OF THE SOLUTION OF ELECTIVE EQUATIONS.

IN whatever way an elective symbol, considered as unknown, may be involved in a proposed equation, it is possible to assign its complete value in terms of the remaining elective symbols considered as known. It is to be observed of such equations, that from the very nature of elective symbols, they are necessarily linear, and that their solutions have a very close analogy with those of linear differential equations, arbitrary elective symbols in the one, occupying the place of arbitrary constants in the other. The method of solution we shall in the first place illustrate by particular examples, and, afterwards, apply to the investigation of general theorems.

Given $(1 - x)y = 0$, (All Ys are Xs), to determine y in terms of x .

As y is a function of x , we may assume $y = vx + v'(1 - x)$, (such being the expression of an arbitrary function of x), the moduli v and v' remaining to be determined. We have then

$$(1 - x) \{vx + v'(1 - x)\} = 0,$$

or, on actual multiplication,

$$v'(1 - x) = 0:$$

that this may be generally true, without imposing any restriction upon x , we must assume $v' = 0$, and there being no condition to limit v , we have

$$y = vx, \quad (67).$$

This is the complete solution of the equation. The condition that y is an elective symbol requires that v should be an elective

symbol also (since it must satisfy the index law), its interpretation in other respects being arbitrary.

Similarly the solution of the equation, $xy = 0$, is

$$y = v(1 - x), \quad (68).$$

Given $(1 - x)zy = 0$, (All Ys which are Zs are Xs), to determine y .

As y is a function of x and z , we may assume

$$y = v(1 - x)(1 - z) + v'(1 - x)z + v''x(1 - z) + v'''zx.$$

And substituting, we get

$$v'(1 - x)z = 0,$$

whence $v' = 0$. The complete solution is therefore

$$y = v(1 - x)(1 - z) + v''x(1 - z) + v'''zx, \quad (69),$$

v', v'', v''' , being arbitrary elective symbols, and the rigorous interpretation of this result is, that Every Y is *either* a not-X and not-Z, or an X and not-Z, or an X and Z.

It is deserving of note that the above equation may, in consequence of its linear form, be solved by adding the two particular solutions with reference to x and z ; and replacing the arbitrary constants which each involves by an arbitrary function of the other symbol, the result is

$$y = x\phi(z) + (1 - z)\psi(x), \quad (70).$$

To shew that this solution is equivalent to the other, it is only necessary to substitute for the arbitrary functions $\phi(z)$, $\psi(x)$, their equivalents

$$wz + w'(1 - z) \text{ and } w''x + w'''(1 - x),$$

we get $y = wxz + (w' + w'')x(1 - z) + w'''(1 - x)(1 - z)$.

In consequence of the perfectly arbitrary character of w' and w'' , we may replace their sum by a single symbol w' , whence

$$y = wxz + w'x(1 - z) + w'''(1 - x)(1 - z),$$

which agrees with (69).

The solution of the equation $wx(1-y)z=0$, expressed by arbitrary functions, is

$$z = (1-w)\phi(xy) + (1-x)\psi(wy) + y\chi(wx), \quad (71).$$

These instances may serve to shew the analogy which exists between the solutions of elective equations and those of the corresponding order of linear differential equations. Thus the expression of the integral of a partial differential equation, either by arbitrary functions or by a series with arbitrary coefficients, is in strict analogy with the case presented in the two last examples. To pursue this comparison further would minister to curiosity rather than to utility. We shall prefer to contemplate the problem of the solution of elective equations under its most general aspect, which is the object of the succeeding investigations.

To solve the general equation $\phi(xy)=0$, with reference to y .

If we expand the given equation with reference to x and y , we have

$$\begin{aligned} \phi(00)(1-x)(1-y) + \phi(01)(1-x)y + \phi(10)x(1-y) \\ + \phi(11)xy = 0, \quad (72), \end{aligned}$$

the coefficients $\phi(00)$ &c. being numerical constants.

Now the general expression of y , as a function of x , is

$$y = vx + v'(1-x),$$

v and v' being unknown symbols to be determined. Substituting this value in (72), we obtain a result which may be written in the following form,

$$\begin{aligned} [\phi(10) + \{\phi(11) - \phi(10)\}v]x + [\phi(00) + \{\phi(00) - \phi(00)\}v'] \\ (1-x) = 0; \end{aligned}$$

and in order that this equation may be satisfied without any way restricting the generality of x , we must have

$$\begin{aligned} \phi(10) + \{\phi(11) - \phi(10)\}v &= 0, \\ \phi(00) + \{\phi(01) - \phi(00)\}v' &= 0, \end{aligned}$$

from which we deduce

$$v = \frac{\phi(10)}{\phi(10) - \phi(11)}, \quad v' = \frac{\phi(00)}{\phi(01) - \phi(00)},$$

wherefore

$$y = \frac{\phi(10)}{\phi(10) - \phi(11)} x + \frac{\phi(00)}{\phi(00) - \phi(01)} (1 - x), \quad (73).$$

Had we expanded the original equation with respect to y only, we should have had

$$\phi(x0) + \{\phi(x1) - \phi(x0)\} y = 0;$$

but it might have startled those who are unaccustomed to the processes of Symbolical Algebra, had we from this equation deduced

$$y = \frac{\phi(x0)}{\phi(x0) - \phi(x1)},$$

because of the apparently meaningless character of the second member. Such a result would however have been perfectly lawful, and the expansion of the second member would have given us the solution above obtained. I shall in the following example employ this method, and shall only remark that those to whom it may appear doubtful, may verify its conclusions by the previous method.

To solve the general equation $\phi(xyz) = 0$, or in other words to determine the value of z as a function of x and y .

Expanding the given equation with reference to z , we have

$$\phi(xy0) + \{\phi(xy1) - \phi(xy0)\} . z = 0;$$

$$\therefore z = \frac{\phi(xy0)}{\phi(xy0) - \phi(xy1)} \dots (74),$$

and expanding the second member as a function of x and y by aid of the general theorem, we have

$$\begin{aligned} z = & \frac{\phi(110)}{\phi(110) - \phi(111)} xy + \frac{\phi(100)}{\phi(100) - \phi(101)} x(1 - y) \\ & + \frac{\phi(010)}{\phi(010) - \phi(011)} (1 - x)y + \frac{\phi(000)}{\phi(000) - \phi(001)} (1 - x)(1 - y) \\ & \dots\dots\dots (75) \end{aligned}$$

and this is the complete solution required. By the same method we may resolve an equation involving any proposed number of elective symbols.

In the interpretation of any general solution of this nature, the following cases may present themselves.

The values of the moduli $\phi(00)$, $\phi(01)$, &c. being constant, one or more of the coefficients of the solution may assume the form $\frac{0}{0}$ or $\frac{1}{0}$. In the former case, the indefinite symbol $\frac{0}{0}$ must be replaced by an arbitrary elective symbol v . In the latter case, the term, which is multiplied by a factor $\frac{1}{0}$ (or by any numerical constant except 1), must be separately equated to 0, and will indicate the existence of a subsidiary Proposition. This is evident from (62).

Ex. Given $x(1-y)=0$, All Xs are Ys, to determine y as a function of x .

Let $\phi(xy)=x(1-y)$, then $\phi(10)=1$, $\phi(11)=0$, $\phi(01)=0$, $\phi(00)=0$; whence, by (73),

$$\begin{aligned} y &= \frac{1}{1-0} x + \frac{0}{0-0} (1-x) \\ &= x + \frac{0}{0} (1-x) \\ &= x + v (1-x), \quad (76), \end{aligned}$$

v being an arbitrary elective symbol. The interpretation of this result is that the class Y consists of the entire class X with an indefinite remainder of not-Xs. This remainder is indefinite in the highest sense, *i.e.* it may vary from 0 up to the entire class of not-Xs.

Ex. Given $x(1-z)+z=y$, (the class Y consists of the entire class Z, with such not-Zs as are Xs), to find Z.

Here $\phi(xyz)=x(1-z)-y+z$, whence we have the following set of values for the moduli,

$$\begin{aligned} \phi(110) &= 0, & \phi(111) &= 0, & \phi(100) &= 1, & \phi(101) &= 1, \\ \phi(010) &= -1, & \phi(011) &= 0, & \phi(000) &= 0, & \phi(001) &= 1, \end{aligned}$$

and substituting these in the general formula (75), we have

$$z = \frac{0}{0} xy + \frac{1}{0} x(1-y) + (1-x)y, \quad (77),$$

the infinite coefficient of the second term indicates the equation

$$x(1 - y) = 0, \text{ All Xs are Ys ;}$$

and the indeterminate coefficient of the first term being replaced by v , an arbitrary elective symbol, we have

$$z = (1 - x)y + vxy,$$

the interpretation of which is, that the class Z consists of all the Y s which are not X s, and an *indefinite* remainder of Y s which are X s. Of course this indefinite remainder may vanish. The two results we have obtained are logical inferences (not very obvious ones) from the original Propositions, and they give us all the information which it contains respecting the class Z , and its constituent elements.

Ex. Given $x = y(1 - z) + z(1 - y)$. The class X consists of all Y s which are not- Z s, and all Z s which are not- Y s: required the class Z .

We have

$$\phi(xyz) = x - y(1 - z) - z(1 - y),$$

$$\phi(110) = 0, \quad \phi(111) = 1, \quad \phi(100) = 1, \quad \phi(101) = 0,$$

$$\phi(010) = -1, \quad \phi(011) = 0, \quad \phi(000) = 0, \quad \phi(001) = -1;$$

whence, by substituting in (75),

$$z = x(1 - y) + y(1 - x), \quad (78),$$

the interpretation of which is, the class Z consists of all X s which are not Y s, and of all Y s which are not X s; an inference strictly logical.

Ex. Given $y\{1 - z(1 - x)\} = 0$, All Y s are Z s and not- X s.

Proceeding as before to form the moduli, we have, on substitution in the general formulæ,

$$z = \frac{1}{6}xy + \frac{9}{6}x(1 - y) + y(1 - x) + \frac{9}{6}(1 - x)(1 - y),$$

$$\text{or } z = y(1 - x) + vx(1 - y) + v'(1 - x)(1 - y)$$

$$= y(1 - x) + (1 - y)\phi(x), \quad (79),$$

with the relation

$$xy = 0:$$

from these it appears that No Y s are X s, and that the class Z

consists of all Ys which are not Xs, and of an indefinite remainder of not-Ys.

This method, in combination with Lagrange's method of indeterminate multipliers, may be very elegantly applied to the treatment of simultaneous equations. Our limits only permit us to offer a single example, but the subject is well deserving of further investigation.

Given the equations $x(1-z)=0$, $z(1-y)=0$, All Xs are Zs, All Zs are Ys, to determine the complete value of z with any subsidiary relations connecting x and y .

Adding the second equation multiplied by an indeterminate constant λ , to the first, we have

$$x(1-z) + \lambda z(1-y) = 0,$$

whence determining the moduli, and substituting in (75),

$$z = xy + \frac{1}{1-\lambda} x(1-y) + \frac{\lambda}{1-\lambda} (1-x)y, \quad (80),$$

from which we derive

$$z = xy + v(1-x)y,$$

with the subsidiary relation

$$x(1-y) = 0:$$

the former of these expresses that the class Z consists of all Xs that are Ys, with an indefinite remainder of not-Xs that are Ys; the latter, that All Xs are Ys, being in fact the conclusion of the syllogism of which the two given Propositions are the premises.

By assigning an appropriate meaning to our symbols, all the equations we have discussed would admit of interpretation in hypotheticals, but it may suffice to have considered them as examples of categoricals.

That peculiarity of elective symbols, in virtue of which every elective equation is reducible to a system of equations $t_1 = 0$, $t_2 = 0$, &c., so constituted, that all the binary products $t_1 t_2$, $t_1 t_3$, &c., vanish, represents a general doctrine in Logic with reference to the ultimate analysis of Propositions, of which it may be desirable to offer some illustration.

Any of these constituents $t_1, t_2, \&c.$ consists only of factors of the forms $x, y, \dots 1 - w, 1 - z, \&c.$ In categoricals it therefore represents a compound class, *i.e.* a class defined by the presence of certain qualities, and by the absence of certain other qualities.

Each constituent equation $t_i = 0$, &c. expresses a denial of the existence of some class so defined, and the different classes are mutually exclusive.

Thus all categorical Propositions are resolvable into a denial of the existence of certain compound classes, no member of one such class being a member of another.

The Proposition, All Xs are Ys, expressed by the equation $x(1 - y) = 0$, is resolved into a denial of the existence of a class whose members are Xs and not-Ys.

The Proposition Some Xs are Ys, expressed by $v = xy$, is resolvable as follows. On expansion,

$$v - xy = vx(1 - y) + vy(1 - x) + v(1 - x)(1 - y) - xy(1 - v);$$

$$\therefore vx(1 - y) = 0, vy(1 - x) = 0, v(1 - x)(1 - y) = 0, (1 - v)xy = 0.$$

The three first imply that there is no class whose members belong to a certain unknown Some, and are 1st, Xs and not Ys; 2nd, Ys and not Xs; 3rd, not-Xs and not-Ys. The fourth implies that there is no class whose members are Xs and Ys without belonging to this unknown Some.

From the same analysis it appears that *all hypothetical Propositions may be resolved into denials of the coexistence of the truth or falsity of certain assertions.*

Thus the Proposition, If X is true, Y is true, is resolvable by its equation $x(1 - y) = 0$, into a denial that the truth of X and the falsity of Y coexist.

And the Proposition Either X is true, or Y is true, members exclusive, is resolvable into a denial, first, that X and Y are both true; secondly, that X and Y are both false.

But it may be asked, is not something more than a system of negations necessary to the constitution of an affirmative Proposition? is not a positive element required? Undoubtedly

there is need of one; and this positive element is supplied in categoricalals by the assumption (which may be regarded as a prerequisite of reasoning in such cases) that there is a Universe of conceptions, and that each individual it contains either belongs to a proposed class or does not belong to it; in hypotheticalals, by the assumption (equally prerequisite) that there is a Universe of conceivable cases, and that any given Proposition is either true or false. Indeed the question of the existence of conceptions (*εἰ ἔσται*) is preliminary to any statement of their qualities or relations (*τί ἔσται*).—*Aristotle, Anal. Post.* lib. II. cap. 2.

It would appear from the above, that Propositions may be regarded as resting at once upon a positive and upon a negative foundation. Nor is such a view either foreign to the spirit of Deductive Reasoning or inappropriate to its Method; the latter ever proceeding by limitations, while the former contemplates the particular as derived from the general.

Demonstration of the Method of Indeterminate Multipliers, as applied to Simultaneous Elective Equations.

To avoid needless complexity, it will be sufficient to consider the case of three equations involving three elective symbols, those equations being the most general of the kind. It will be seen that the case is marked by every feature affecting the character of the demonstration, which would present itself in the discussion of the more general problem in which the number of equations and the number of variables are both unlimited.

Let the given equations be

$$\phi(xyz) = 0, \quad \psi(xyz) = 0, \quad \chi(xyz) = 0, \quad (1).$$

Multiplying the second and third of these by the arbitrary constants h and k , and adding to the first, we have

$$\phi(xyz) + h\psi(xyz) + k\chi(xyz) = 0, \quad (2);$$

and we are to shew, that in solving this equation with reference to any variable z by the general theorem (75), we shall obtain not only the general value of z independent of h and k , but also any subsidiary relations which may exist between x and y independently of z .

If we represent the general equation (2) under the form $F(xyz) = 0$, its solution may by (75) be written in the form

$$z = \frac{xy}{1 - \frac{F(111)}{F(110)}} + \frac{x(1-y)}{1 - \frac{F(101)}{F(100)}} + \frac{y(1-x)}{1 - \frac{F(011)}{F(010)}} + \frac{(1-x)(1-y)}{1 - \frac{F(001)}{F(000)}};$$

and we have seen, that any one of these four terms is to be equated to 0, whose modulus, which we may represent by M , does not satisfy the condition $M^n = M$, or, which is here the same thing, whose modulus has any other value than 0 or 1.

Consider the modulus (suppose M_1) of the first term, viz.

$$\frac{1}{1 - \frac{F(111)}{F(110)}}, \text{ and giving to the symbol } F \text{ its full meaning,}$$

$$\text{we have } M_1 = \frac{1}{1 - \frac{\phi(111) + h\psi(111) + k\chi(111)}{\phi(110) + h\psi(110) + k\chi(110)}}.$$

It is evident that the condition $M_1^n = M_1$ cannot be satisfied unless the right-hand member be independent of h and k ; and in order that this may be the case, we must have the function $\frac{\phi(111) + h\psi(111) + k\chi(111)}{\phi(110) + h\psi(110) + k\chi(110)}$ independent of h and k .

Assume then

$$\frac{\phi(111) + h\psi(111) + k\chi(111)}{\phi(110) + h\psi(110) + k\chi(110)} = c,$$

c being independent of h and k ; we have, on clearing of fractions and equating coefficients,

$$\phi(111) = c\phi(110), \quad \psi(111) = c\psi(110), \quad \chi(111) = c\chi(110);$$

whence, eliminating c ,

$$\frac{\phi(111)}{\phi(110)} = \frac{\psi(111)}{\psi(110)} = \frac{\chi(111)}{\chi(110)},$$

being equivalent to the triple system

$$\left. \begin{aligned} \phi(111) \psi(110) - \phi(110) \psi(111) &= 0 \\ \psi(111) \chi(110) - \psi(110) \chi(111) &= 0 \\ \chi(111) \phi(110) - \chi(110) \phi(111) &= 0 \end{aligned} \right\} (3);$$

and it appears that if any one of these equations is not satisfied, the modulus M_1 will not satisfy the condition $M_1^n = M_1$, whence the first term of the value of z must be equated to 0, and we shall have

$$xy = 0,$$

a relation between x and y independent of z .

Now if we expand in terms of z each pair of the primitive equations (1), we shall have

$$\phi(xy0) + \{\phi(xy1) - \phi(xy0)\} z = 0,$$

$$\psi(xy0) + \{\psi(xy1) - \psi(xy0)\} z = 0,$$

$$\chi(xy0) + \{\chi(xy1) - \chi(xy0)\} z = 0,$$

and successively eliminating z between each pair of these equations, we have

$$\phi(xy1) \psi(xy0) - \phi(xy0) \psi(xy1) = 0,$$

$$\psi(xy1) \chi(xy0) - \psi(xy0) \chi(xy1) = 0,$$

$$\chi(xy1) \phi(xy0) - \chi(xy0) \phi(xy1) = 0,$$

which express all the relations between x and y that are formed by the elimination of z . Expanding these, and writing in full the first term, we have

$$\{\phi(111) \psi(110) - \phi(110) \psi(111)\} xy + \&c. = 0,$$

$$\{\psi(111) \chi(110) - \psi(110) \chi(111)\} xy + \&c. = 0,$$

$$\{\chi(111) \phi(110) - \chi(110) \phi(111)\} xy + \&c. = 0;$$

and it appears from Prop. 1, that if the coefficient of xy in any of these equations does not vanish, we shall have the equation

$$xy = 0;$$

but the coefficients in question are the same as the first members of the system (3), and the two sets of conditions exactly agree. Thus, as respects the first term of the expansion, the method of indeterminate coefficients leads to the same result as ordinary elimination; and it is obvious that from their similarity of form, the same reasoning will apply to all the other terms.

Suppose, in the second place, that the conditions (3) are satisfied so that M_1 is independent of h and k . It will then indifferently assume the equivalent forms

$$M_1 = \frac{1}{1 - \frac{\phi(111)}{\phi(110)}} = \frac{1}{1 - \frac{\psi(111)}{\psi(110)}} = \frac{1}{1 - \frac{\chi(111)}{\chi(110)}}.$$

These are the exact forms of the first modulus in the expanded values of z , deduced from the solution of the three primitive equations singly. If this common value of M_1 is 1 or $\frac{1}{2} = v$, the term will be retained in z ; if any other constant value (except 0), we have a relation $xy = 0$, not given by elimination, but deducible from the primitive equations singly, and similarly for all the other terms. Thus in every case the expression of the subsidiary relations is a necessary accompaniment of the process of solution.

It is evident, upon consideration, that a similar proof will apply to the discussion of a system indefinite as to the number both of its symbols and of its equations.

POSTSCRIPT.

SOME additional explanations and references which have occurred to me during the printing of this work are subjoined.

The remarks on the connexion between Logic and Language, p. 5, are scarcely sufficiently explicit. Both the one and the other I hold to depend very materially upon our ability to form general notions by the faculty of abstraction. Language is an instrument of Logic, but not an indispensable instrument.

To the remarks on Cause, p. 12, I desire to add the following: Considering Cause as an invariable antecedent in Nature, (which is Brown's view), whether associated or not with the idea of Power, as suggested by Sir John Herschel, the knowledge of its existence is a knowledge which is properly expressed by the word *that* (τὸ ὅτι), not by *why* (τὸ διότι). It is very remarkable that the two greatest authorities in Logic, modern and ancient, agreeing in the latter interpretation, differ most widely in its application to Mathematics. Sir W. Hamilton says that Mathematics

exhibit only the *that* ($\tau\acute{o}\ \delta\tau\iota$): Aristotle says, The *why* belongs to mathematicians, for they have the demonstrations of Causes. *Anal. Post.* lib. 1., cap. xiv. It must be added that Aristotle's view is consistent with the sense (albeit an erroneous one) which in various parts of his writings he virtually assigns to the word Cause, viz. an antecedent in Logic, a sense according to which the premises might be said to be the cause of the conclusion. This view appears to me to give even to his physical inquiries much of their peculiar character.

Upon reconsideration, I think that the view on p. 41, as to the presence or absence of a medium of comparison, would readily follow from Professor De Morgan's doctrine, and I therefore relinquish all claim to a discovery. The mode in which it appears in this treatise is, however, remarkable.

I have seen reason to change the opinion expressed in pp. 42, 43. The system of equations there given for the expression of Propositions in Syllogism is *always* preferable to the one before employed—first, in generality—secondly, in facility of interpretation.

In virtue of the principle, that a Proposition is either true or false, every elective symbol employed in the expression of hypotheticals admits only of the values 0 and 1, which are the only quantitative forms of an elective symbol. It is in fact possible, setting out from the theory of Probabilities (which is purely quantitative), to arrive at a system of methods and processes for the treatment of hypotheticals exactly similar to those which have been given. The two systems of elective symbols and of quantity osculate, if I may use the expression, in the points 0 and 1. It seems to me to be implied by this, that unconditional truth (categoricals) and probable truth meet together in the constitution of contingent truth, (hypotheticals). The general doctrine of elective symbols and all the more characteristic applications are quite independent of any quantitative origin.

THE END.

by the durations of the eclipses, or intervals of darkness between the revolving lights, is excellent; but we are of opinion that coloured lights may form an admirable auxiliary, when the colours are obtained, in the manner we have tried them, from solid, fluid, and gaseous media; and as we know, from direct experiment, that a numerical character may be impressed optically upon the lights of our lighthouses, we have no doubt that a complete and scientific system of distinction will be obtained under a reformed management.

All attempts, however, at a partial change in the present barbarous system of illumination will prove entirely abortive. Hammered reflectors exclude all improvements: the old bottles cannot be accommodated to any new wine; and until these silvered idols of our British polytheists shall be torn down from their high places, and one brilliant vestal fire lighted on their altar, the winds will make havoc among our galleys, and the waves will devour their victims.

Such is a general view of the British lighthouse system, and of the valuable improvements which are ready to be introduced when our Boards shall undergo that renovation which is so loudly called for. The Legislature has lately pledged itself to a revision of the lighthouse code; and that it will act wisely and justly, we cannot doubt. In the preceding pages we have treated this great question as one of public economy, and of national honour; but we trust that a British House of Commons will never forget that the subject with which they have to deal is that of *human life*,—of the lives, too, of the industrious mariner whom they have severely taxed, and of the helpless seafaring stranger whom they have taxed without mercy. If they fail in this sacred duty, they will be answerable to a tribunal more solemn than that of their constituency—a tribunal where Benevolence will be their judge, science their accuser, and widows and orphans their jury.

(By Sir Wm Hamilton).
Edinburgh Review

- ART. IX.—1. *Artis Logicæ Rudimentd, with Illustrative Observations on each Section.* Fourth edition, with Additions. 12mo. Oxford: 1828.
2. *Elements of Logic.* By RICHARD WHATELY, D.D., Principal of St Alban's Hall, and late Fellow of Oriel College, Oxford. Third edition. 8vo. London: 1829.
3. *Introduction to Logic, from Dr Whately's Elements of Logic.* By the Rev. SAMUEL HINDS, M.A., of Queen's College, and Vice-Principal of St Alban's Hall, Oxford, 12mo. Oxford: 1827.
4. *Outline of a New System of Logic, with a Critical Examination of Dr Whately's 'Elements of Logic,'* by GEORGE BENTHAM, Esq. 8vo. London: 1827.
5. *An Examination of some Passages in Dr Whately's Elements of Logic.* By GEORGE CORNEWALL LEWIS, Esq., Student of Christ Church. 8vo. Oxford: 1829.
6. *A Treatise on Logic on the Basis of Aldrich, with Illustrative Notes* by the Rev. JOHN HUYSHE, M.A., Brazen-nose College, Oxford. 12mo. Second edition. Oxford: 1833.
7. *Questions on Aldrich's Logic, with References to the most Popular Treatises.* 12mo. Oxford: 1829.
8. *Key to Questions on Aldrich's Logic.* 12mo. Oxford: 1829.
9. *Introduction to Logic.* 12mo. Oxford: 1830.
10. *Aristotle's Philosophy.* (An Article in Vol. iii. of the Seventh Edition of the *Encyclopædia Britannica*, now publishing.) By the Rev. RENN DICKSON HAMPDEN, M.A., late Fellow of Oriel College, Oxford.) 4to. Edinburgh: 1832.

NOTHING, we think, affords a more decisive proof of the partial spirit in which philosophy has been cultivated in Britain, for the last century and a half, than the combined perversion and neglect which Logic—the science of the formal laws of thought—has experienced during that period. Since the time, and principally, we suspect, through the influence of Locke, (who, as Leibnitz observed, *sprevit logicam non intellexit*,) no country has been so poor in this department of philosophy, whether we estimate our dialectical literature by its mass or by its quality. Loath to surrender the subject altogether, yet unable, from their own misconception of its nature, to vindicate to logic, on the proper ground, its paramount importance to a science *a priori*, distinct, and independent; the few logical authors who appeared endeavoured, on the one hand, by throwing out all that belonged to it of a repulsive character, to obviate a taste, and, on the other, by interpolating what pertained to

Wi

other branches of knowledge—here a chapter of psychology, there a chapter of metaphysic, &c.—to conciliate to the declining study a broader interest than its own. The attempt was too irrational to succeed; and served only to justify the disregard it was meant to remedy. This was to convert the interest of knowledge with the interest of taste:—this was not to amplify logic, but to deform philosophy, by breaking down their boundaries, and running the different sciences into each other.

In the Universities, where Dialectic once reigned ‘The Queen of Arts,’ the failure of the study is more conspicuously remarkable.

In those of Scotland, the Chairs of Logic have for generations taught any thing rather than the science which they nominally profess;—a science by the way in which the Scots have not latterly maintained the reputation once established by them in all,* and still retained in other departments of philosophy. To the philosophers of our country, we must confess, that, in part at least, is to be attributed the prevalence of the erroneous notions on this subject promulgated by Locke. No system of logic deserving of notice ever appeared in Scotland; and for Scottish logical writers of any merit, we must travel back for more than two centuries to three contemporary authors, whose abilities, like those, indeed, of almost all the more illustrious scholars of their nation, were developed under foreign influence—to Robert Balfour, Mark Duncan, and William Chalmers, Professors in the Universities of Bourdeaux, Saumur, and Angers. In Cambridge the fortune of the study is indicated by the fact, that the *Elements of Logic* of William Duncan of Aberdeen, have long dispensed a muddy scantling of metaphysic, psychology, and dialectic, in the University where Downam taught; and *Murray’s Logic*, the Trinity College Compend, may show that matters are, if possible, at a lower pass in Dublin.

In Oxford, the fate of the science has been somewhat different, but, till lately, scarcely more favourable. And here it is necessary to be more particular, as this is the only British seminary where the study of logic proper can be said to have survived; and as, with one exception, the whole works under re-

* ‘Les Ecossois sont bons Philosophes,’ pronounced the Dictator of Letters (*Scaligerana Secunda*): and Servitus had previously testified to their character for logical subtilty;—‘Dialecticis argutiis sibi blandiuntur.’ (*Præf. in Ptolem. Geogr.* 1533.) For a considerable period, indeed, there was hardly to be found a continental University of any note, without the appendage of a Scottish Professor of Philosophy.

view* emanate from that University,—represent its character,—and are determined and modified by its circumstances. During the scholastic ages, Oxford was held inferior to no University throughout Europe; and it was celebrated, more especially, for its philosophers and dialecticians. But it was neither the recollection of old academical renown, nor any enlightened persuasion of its importance, that preserved to logic a place among the subjects of academical tuition, when the kindred branches of philosophy, with other statutory studies, were dropt from the course of instruction actually given. These were abandoned from no conviction of their inutility, nor even in favour of others of superior value: they were abandoned when the system under which they could be taught, was, for a private interest, illegally superseded by another under which they could not. When the College Fellows supplanted the University Professors, the course of statutory instruction necessarily fell with the statutory instruments by which it had been carried through. The same extensive, the same intensive, education which had once been possible when the work was distributed among a body of Professors, each chosen for his ability, and each concentrating his attention on a single study, could no longer be attempted when the collegial corporations, a fortuitous assemblage of individuals, in so far as literary qualification is concerned, had usurped the exclusive privilege of instruction; and when each of these individuals was authorized to become sole teacher of the whole academical encyclopædia. But while the one unqualified Fellow-tutor could not perform the work of a large body of qualified Professors; it is evident that, as he could not rise and expand himself to the former system, that the present, existing only for his behoof, must be contracted and brought down to him. This was accordingly done. The mode of teaching, and the subjects taught, were reduced to the required level and extent. The capacity of lecturing, that is, of delivering an original course of instruction, was not now to be expected in the tutor. The pupil, therefore, read to his tutor a lesson out of book; on this lesson the tutor might, at his discretion, interpose an observation, or preserve silence; and he was thus effectually guaranteed from all demands beyond his ability or inclination to meet. This reversed process was still denominated a *lecture*. In like manner, all subjects which required in the tutor more than the Fellows' average of learning or acuteness, were eschewed. Many of the most important branches of education in the legal system were thus discarded; and those

* These works, indeed, with one or two insignificant exclusions, comprise the *whole* recent logical literature of the kingdom.

which it was found necessary or convenient to retain in the intrusive, were studied in easier and more superficial treatises. This, in particular, was the case with logic.

By statute, the Professor of Dialectic was bound to read and expound the *Organon* of Aristotle twice a-week; and, by statute, regular attendance on his lectures was required from all under-graduates for their three last years. Until the statutory system was superseded, an energetic and improving exercise of mind from the intelligent study of the most remarkable monument of philosophical genius, imposed on all, was more especially secured in those who would engage in the subsidiary business of tuition. This, and the other conditions of that system, thus determined a far higher standard of qualification in the tutor when the tutor was still only a subordinate instructor, than remained when he had become the exclusive organ of academical education. When, at last, the voice of the Professors was silenced in the University, and in the Colleges the Fellows had been able to exclude all other graduates from the now principal office of tutor, the study of logic declined with the ability of those by whom the science was taught. The original treatises of Aristotle were now found to transcend the College complement of erudition and intellect. They were accordingly abandoned; and with these the various logical works previously in academical use, which supposed any reach of thought, or an original acquaintance with the *Organon*. The *Compend* of Sanderson stood its ground for a season, when the more elaborate treatises of Brerewood, Crackanthorpe, and Smiglecius, were forgotten. But this little treatise, the excellent work of an accomplished logician, was too closely relative to the books of the *Organon*, and demanded too frequently an inconvenient explanation, to retain its place, so soon as another text-book could be introduced, more accommodated to the fallen and falling standard of tutorial competency. Such a text-book was soon found in the *Compendium* of Aldrich. The dignity of its author, as Dean of Christ Church, and his reputation as an ingenious, and even learned, writer in other branches of knowledge, ensured it a favourable recommendation: it was even shorter than Sanderson; written in a less scholastic Latin; adopted an order wholly independent of the *Organon*; and made no awkward demands upon the tutor, as comprising only what was either plain in itself, or could without difficulty be expounded. The book—which, in justice to the Dean, we ought to mention was not originally written for the public—is undoubtedly a work of no inconsiderable talent; but the talent is, perhaps, principally shown in the author having performed so cleverly a task for which he was so indifferently

prepared. Absolutely considered, it has little or no value. It is but a slight eclectic epitome of one or two logical treatises in common use (that it is exclusively abridged from Wallis is incorrect); and when he wanders from, or mistakes his authorities, he displays a want of information to be expected, perhaps, in our generation, but altogether marvellous in his. It is clear that he knew nothing of the *Organon*, and very little of the modern logicians. The treatise likewise omits a large proportion of the most important matters; and those it does not exclude are treated with a truly unedifying brevity. As a slender introduction to the after-study of logic (were there not a hundred better) it is not to be despised; as a full course of instruction, as an independent system of the science, it is utterly contemptible. Yet, strange to say, the *Compend* of Aldrich having gradually supplanted the *Compend* of Sanderson, has furnished for above a century the little all of logic taught in these latter days by the University of Bradwardin and Scotus.*

Even the meliorations of the academical system have not proved beneficial to this study: perhaps, indeed, the reverse. Since the institution of honours, and of a real examination for the first degree in arts, a powerful stimulus has been applied to other studies—to that of logic none. Did a candidate make himself master of the *Organon*?—he would find as little favour from the dispensers of academical distinction, as he had previously obtained assistance from his tutor. For the public examiners could not be expected, either to put questions on what they did not understand, or to encourage the repetition of such overt manifestations of their own ignorance. The minimum of Aldrich, therefore, remained the maximum of the schools; and was ‘*got up*,’ not to obtain honour, but to avoid disgrace. But even this minimum was to be made less; there was ‘a lower deep beneath the lowest deep.’ The *Compendium*,

* Some thirty years ago, indeed, there was printed, *in usum academicæ juventutis*, certain *Excerpta ex Aristotelis Organo*. The execution of that work shows how inadequate its author was to the task he had undertaken. Nothing could be more conducive to the rational study of logic than a systematic condensation of the more essential parts of the different treatises of the *Organon*, with original illustrations, and selections from the best commentators, ancient and modern. As it is, this petty publication has exerted no influence on the logical studies of the University; we should like to know how many tutors have expounded it in their lectures, how many candidates have been examined on it in the schools. On the logical authors, at least, of the University, it has exerted none.

a meagre duodecimo of 180 pages, to be read in a day, and easily mastered in a week, was found too ponderous a volume for pupil, tutor, and examiner. It was accordingly subjected to a process of extenuation, out of which it emerged reduced to little more than a third of its original gracility—a skeleton without marrow or substance. ‘Those who go deep in dialectic,’ says Aristo Chius, ‘may be resembled to crab-eaters; for a mouthful of meat, they spend their time over a heap of shells.’ But your superficial student of logic loses his time without even a savour of this mouthful; and Oxford, in her old age, has proved herself no Alma Mater, in thus so unpiteously cramming her alumni with the shells alone. As Dr Whately observes, ‘a very small proportion even of distinguished students ever become proficient in logic; and by far the greater proportion pass through the University without knowing any thing at all of the subject. I do not mean that they have not learned by rote a string of technical terms, but that they understand absolutely nothing whatever of the principles of the science.’ The miracle would be, if they ever did. Logic thus degraded to an irksome but wholly unprofitable penance, the absurdity of its longer enforcement was felt by some intelligent leaders of the University. ‘It was proposed,’ says Dr Whately, ‘to leave the study of logic altogether to the option of the candidates;’ a proposal hailed with joy by the under-graduates, who had long prayed fervently with St Ambrose,—*A Dialectica Aristotelis libera nos, Domine.*

In these circumstances, when even the Heads could not much longer have continued obstinate, and logic seemed in Oxford on the eve of following metaphysic and psychology to an academic grave, a new life was suddenly communicated to the expiring study, and hope at least allowed for its ultimate convalescence under a reformed system.

This was mainly effected by the publication of the *Elements* of Dr Whately, then Principal of St Alban’s Hall, and recently (we rejoice) elevated to the Archbishopial See of Dublin. (No. 2 of the works at the head of this Article.) Somewhat previous, the *Rudimenta* (abbreviated *Compendium*) of Aldrich had been illustrated with English notes by an anonymous author, whom we find quoted in some of the subsequent treatises under the name of Hill, (No 1.) The success and ability of the ‘Elements’ prompted imitation and determined controversy. Mr Bentham (nephew of Mr Jeremy Bentham) published his *Outline and Examination*, in which Dr Whately is alternately the object of censure and encomium (No. 4); and the pamphlet of Mr Lewis (on two points only) is likewise controversial (No 5). The Principal, as becoming, was abridged and lauded by his Vice (No. 3); and the treatises by Mr Huyshe and others (Nos. 6,

7, 8, 9) are all more or less relative to Dr Whately's, and all so many manifestations of the awakened spirit of logical pursuit. The last decade, indeed, has done more in Oxford for the cause of this science than the whole hundred and thirty years preceding; for since the time of Wallis and Aldrich, until the works under review, we recollect nothing on the subject which the University could claim, except one or two ephemeral tracts;—the shallow *Reflections* of George Bentham, about the middle of the last century; and after the commencement of the present, a couple of clever pamphlets in vindication of logic, and in extinction of the logic of Kett—which last also was a mooncalf of Alma Mater.

It remains now to enquire at what value are we to rate these new logical publications. Before looking at their contents, and on a knowledge only of the general circumstances under which they were produced, we had formed a presumptive estimate of what they were likely to perform; and found our anticipation fully confirmed, since we recently examined what they had actually accomplished. None of the works are the productions of inferior ability; and though some of them propose only an humble end, they are all respectably executed. A few of them display talent rising far above mediocrity; and one is the effort of an intellect of great natural power. But when we look from the capacity of the author to his acquirements, our judgment is less favourable. If the writers are sometimes original, their matter is never new. They none of them possess,—not to say a superfluous erudition on their subject,—even the necessary complement of information. Not one seems to have studied the logical treatises of Aristotle; all are unread in the Greek Commentators on the *Organon*, in the Scholastic, Ramist, Cartesian, Wolfian, and Kantian Dialectic. In none is there any attempt at the higher logical philosophy: we have no preliminary determination of the fundamental laws of thought; no consequent evolution, from these laws, of the system itself. On the contrary, we find principle buried in detail; inadequate views of the science; a mere agglutination of its parts; of these some wholly neglected, and others, neither the most interesting nor important, elaborated out of bounds; and always, though in very different proportions, too much of the 'shell,' too little of the 'meat.' They are rarely, indeed, wise above Aldrich: his partial views of the order and comprehension of the science have determined theirs; his most egregious blunders are repeated; and sometimes when an attempt is made at a correction, either Aldrich is right, or a new error is substituted for the old. Even Dr Whately, who, in the teeth of every logician from Alexander to Kant, speaks of 'the boundless field within the

‘legitimate limits of the science,’ ‘walks in the trodden ways,’ and is guiltless of ‘removing the ancient landmark.’ His work, indeed, never transcends, and generally does not rise to, the actual level of the science; nor, with all its ability, can it justly pretend to more than a relative and local importance. Its most original and valuable portion is but the insufficient correction of mistakes touching the nature of logic, long exploded, if ever harboured, among the countrymen of Leibnitz, and only lingering among the disciples of Locke.

An articulate proof of the accuracy of these conclusions, on all the works under consideration, would far exceed our limits. Nor is this requisite. It will be sufficient to review that work, in chief, to which most of the others are correlative, and which stands among them all the highest in point of originality and learning; and the rest occasionally, in subordination to that one. Nor in criticising Dr Whately’s *Elements* can we attempt to vindicate all or even the principal points of our decision. To show the *deficiencies* in that work, either of principle or of detail, would, in the universal ignorance in this country of logical philosophy and of a high logical standard, require a preliminary exposition of what a system of this science ought to comprehend, far beyond our space, were we even to discuss these points to the exclusion of every other. We must, therefore, omitting imperfections, confine ourselves to an indication of some of Dr Whately’s positive *errors*. This we shall attempt, ‘though the ‘work,’ as its author assures us, ‘has undergone, not only the ‘close examination of himself and several friends, but the ‘severer scrutiny of determined opponents, without any material errors having been detected, or any considerable alterations found necessary.’ In doing this, nothing could be farther from our intention than any derogation from the merit of that eminent and excellent individual, whom, even when we differ most from his opinions, we admire and respect, both as a very shrewd and (what is a rarer phenomenon in Oxford) a very independent thinker. The interest of truth is above all personal considerations; and as Dr Whately, in vindication of his own practice, has well observed,—‘errors are the more carefully to be pointed out in proportion to the authority by which ‘they are sanctioned.’ ‘No mercy,’ says Lessing, ‘to a distinguished author.’ This, however, is not our motto; and if our ‘scrutiny’ be ‘severe,’ we are conscious that it cannot justly be attributed to ‘determined opposition.’

We find matter of controversy even in the first page of the ‘*Elements*.’ Dr Whately very properly opens by a statement, if not a definition, of the nature and domain of logic; and in no other part of his work have the originality and correctness of

his views been more applauded, than in the determination of this fundamental problem. 'Logic,' says he, 'in the most extensive sense which the name can with propriety be made to bear, may be considered as the Science, and also as the Art of Reasoning. It investigates the principles on which argumentation is conducted, and furnishes rules to secure the mind from error in its deductions. Its most appropriate office, however, is that of instituting an analysis of the process of the mind in reasoning; and in this point of view it is, as has been stated, strictly a science; while, considered in reference to the practical rules above mentioned, it may be called the art of reasoning. This distinction, as will hereafter appear, has been overlooked, or not clearly pointed out, by most writers on the subject; logic having been in general regarded as merely an art, and its claim to hold a place among the sciences having been expressly denied.'—*Elements*, p. 1.

Here the enquiry naturally separates into two branches;—the one concerns the *genus*, the other the *object-matter* of logic.

In regard to the former—Dr Whately's reduction of logic to the twofold category of *Art* and *Science*, has earned the praises of his Critical Examiner, but who, it must be acknowledged, is as often out in his encomium as in his censure. 'Dr Whately,' says Mr Bentham, 'has in particular brought to view one very important fact, overlooked by all his predecessors, though so obvious, when once exhibited, as to make us wonder that it should not have been remarked: viz. that logic is a *science* as well as an *art*. The universally prevailing error that human knowledge is divided into a number of parts, some of which are arts without science, and others sciences without art, has been fully exposed by Mr Bentham in his *Chrestomathia*. There also it has been shown, that there cannot exist a single art that has not its corresponding science, nor a single science which is not accompanied by some portion of art. The Schoolmen, on the contrary, have, with extraordinary effort, endeavoured to prove that logic is an art only, not a science; and in that particular instance, Dr Whately is, I believe, one of the first who has ventured to contradict this ill-founded assertion.' *Outline*, p. 12.—In all this there is but one statement with which we can agree. We should certainly 'wonder' with Mr Bentham, had any 'so obvious and important fact' been overlooked by all Dr Whately's predecessors; and knowing something of both, should assuredly be less disposed to presume a want of acuteness in the old logicians, than any ignorance of their speculations in the new. In the latter alternative, indeed, will be found a solution of the 'wonder.' Author and critic are equally in error.

In the first place, looking merely to the nomenclature, both are historically wrong. 'Logic,' says Dr Whately, 'has been in general regarded merely as an art, and its claim to hold a place among the sciences has been expressly denied.' The reverse is true. The great majority of logicians have regarded logic as a science, and expressly denied it to be an art. This is the oldest as well as the most general opinion. 'The Schoolmen,' says Mr Bentham, 'have with extraordinary effort endeavoured to prove that logic is an art only.' On the contrary, the Schoolmen have not only 'with extraordinary effort,' but with unexampled unanimity laboured in proving logic to be exclusively a science; and so far from 'Dr Whately being' (with Mr Jeremy Bentham) 'the first to contradict this ill-founded assertion,' the paradox of these gentlemen is only the truism of the world beside. This error is the more surprising, as the genus of logic is one of those vexed questions on which, as Ausonius has it,

— omnis certat Dialectica turba sophorum :

indeed, until latterly, no other perhaps stands so obtrusively forward during the whole progress of the study. Plato and the Platonists considered dialectic as a science; but with them dialectic was a real not a formal discipline, and corresponded rather to the metaphysic than to the logic of the Peripatetics. Logic is not defined by Aristotle. His Greek followers, and a considerable body of the most eminent Dialecticians since the revival of letters, deny it to be either science or art. The Stoics in general viewed it as a science. The Arabian and Latin schoolmen did the same. In this opinion Thomist and Scotist, Realist and Nominalist, concurred; an opinion adopted, almost to a man, by the Jesuit, Dominican, and Franciscan Cursualists. From the restoration of letters, however, and especially during the latter part of the 16th century, so many Aristotelians, with the whole body of Ramists, (to whom were afterwards to be added a majority of the Cartesians, and a large proportion of the Eclectics,) maintained that it was an art; that the error of Sanderson may be perhaps excused in attributing this opinion to 'almost all the more recent authors' at his time. Along with these, however, (so far is Dr Whately from having 'brought to view this important fact, overlooked by all his predecessors,') there was a very considerable party who anticipated the supposed novelty of this author in defining logic by the double genus of *art and science*.* In the schools of Wolf and Kant logic again obtained the name of science.

* To make reference to these would be *de trop*; we count above a dozen logicians of this class in our own collection. But independently of the older and less familiar authors, Mr Jeremy Bentham and Dr

But—to look beneath the name—as Dr Whately and his critic are wrong in imagining that there is any novelty in the observation, they are equally mistaken in attributing to it the smallest importance. The question never concerned logic itself, but merely the meaning of the terms by which it should be defined. The old logicians, however keenly they disputed whether logic were a science or an art—or neither—or both—a science speculative, or a science practical—or at once speculative and practical—never dreamt that the controversy possessed, in so far as logic was concerned, more than a verbal interest.* In regard to the essential nature of logic they were at one; and contended only, what was the comprehension of these terms in philosophical propriety, or rather what was the true interpretation of their Aristotelic definitions. Many intelligent thinkers denounced, with Vives, the whole problem as frivolous. ‘*Questioni locum dedit misera homonymia*,’ says Mark Duncan, among a hundred others. The most strenuous advocates of the several opinions regularly admit, that unless the terms are taken in the peculiar signification for which they themselves contend, that all and each of their adversaries may be correct; while, at the same time, it was recognised on all hands, that these terms were vulgarly employed in a vague or general acceptation, under which every opinion might be considered right, or rather no opinion could be deemed wrong.

Whately have no claim (the latter makes none) to originality in this observation. Even the last respectable writer on logic in the British Empire, previous to these gentlemen, Dr Richard Kirwan, whose popular and able volumes were published in 1807, defines logic as art and science; and this in terms so similar to those of Dr Whately, that we cannot hesitate in believing that this author had his predecessor’s definition (which we shall quote) immediately in view. ‘Logic is both a science and an art; it is a *science* inasmuch as, by analysing the elements, principles, and structure of arguments, it teaches us how to discover their truth or detect their fallacies, and point out the sources of such errors. It is an *art*, inasmuch as it teaches how to arrange arguments in such manner, that their truth may be most readily perceived, or their falsehood detected.’—Vol. i. p. 1.

* Father Buffier is unjust to the old logicians, but he places the matter on its proper footing in reference to the new.—‘*Si la logique est une science. Oui et non; selon l’idée qu’il vous plaît d’attacher au nom de science, &c. - - Si la logique est un art. Encore un fois, oui et non; - - Il plaît aux logiciens de disputer si la logique est, ou n’est pas un art; et il ne leur plaît pas toujours d’avouer ni d’enseigner a leurs disciples, que c’est une pure ou puerile question de nom.*’—*Cours des Sciences*, (Logique,) p. 887.

The preparatory step of the discussion was, therefore, an elimination of these less precise and appropriate significations, which, as they could at best only afford a remote genus and difference, were wholly incompetent for the purposes of a definition. But what the older logicians rejected as a useless truism, the recent embrace as a new and important observation. In regard to its novelty;—do Dr Whately and Mr Bentham imagine that any previous logician could ever have dreamt of denying that logic, in their acceptation of the terms, was at once an art and a science? Let them look into almost any of the older treatises, and they will find this explicitly admitted, even when the terms Art and Science are employed in senses far less vague and universal than is done by them. As to its importance;—do they suppose that a more precise and accurate conception of logic is thus obtained? The contrary is true. The term Science Dr Whately employs in its widest possible extension, for any knowledge considered absolutely, and not in relation to practice; in this acceptation every art in its doctrinal portion must be a science: and Art he defines the application of knowledge to practice; in which signification, ethics, politics, religion, and all other practical sciences, must be arts. Art and Science are thus distended till they run together. As philosophical terms they are now altogether worthless; too universal to define; too vacillating between identity and difference, to distinguish. In fact, their application to logic, or any other subject, is hereafter only to undefine, and to confuse; expressing, as they do, not any essential opposition between the things themselves, but only the different points of view under which the same thing may be contemplated by us;—every art being thus in itself also a science, every science in itself also an art. This Mr Bentham thinks the correction of a universal error,—the discovery of an important fact. If the question in the hands of the old logicians be frivolous, what is it in those of the new! *

* Such is the most favourable interpretation we can give of Dr Whately's meaning. But the language in which this meaning is conveyed is most ambiguous and inaccurate. *E. g.* he says, 'a science 'is conversant about knowledge only,'—p. 56. He cannot mean what the words express, that science has knowledge for its *object-matter*, for this is nonsense; and the words do not express, what, from the context, we must presume he means, that science has no end ulterior to the contemplative act of knowledge itself. Dr Whately thus means by *science*, what Aristotle meant by *speculative science*, but how different in the precision of their definitions! *Θεωρητικῆς μὲν (ἐπιστήμης) τέλος ἀλήθεια· πρακτικῆς δ' ἔργον*;—or, as Averroes has it, *Per speculativum scimus ut sciamus; per practicam scimus ut operemur*.—In like manner,

So much for the genus, now for the object-matter.

Of Dr Whately's *Elements*, Mr Hinds says, 'This treatise displays—and it is the only one that has clearly done so—the true nature and use of logic; so that it may be approached, no longer as a dark, curious, and merely speculative study; such as one is apt, in fancy, to class with astrology and alchymy.'—Pref. p. viii. These are strong words.

We are disposed to admit that Dr Whately is perhaps not far wrong with regard to the 'true nature and use of logic';—that he 'clearly displays' that nature and use, is palpably incorrect; and that his is the 'only treatise which has clearly done so,' is but another proof, that assertion is often in the inverse ratio of knowledge.

We shall say nothing of what we conceive a very partial conception of the science—that Dr Whately makes the process of *reasoning* not merely its principal, but even its adequate object; those of simple apprehension and judgment being considered not in themselves as constituent elements of thought, but simply as subordinate to argumentation. In this view logic is made convertible with syllogistic. This view, which may be allowed, in so far as it applies to the logic contained in the Aristotelic treatises now extant, was held by several of the Arabian and Latin

Dr Whately gives, without being aware of it, two very different definitions of the term Art. In one place (p. 1) it is said 'that logic may be called the *art* of reasoning, while, considered in reference to the practical rules, it furnishes to secure the mind from error in its deductions.' This is evidently the *Διαλεκτική χωρὶς πραγμάτων* of the Greek interpreters, the *logica docens* (*quæ tradit præcepta*) of the Arabian and Latin schools. Again, in another, (p. 56,) it is said, that 'an *art* is the application of knowledge to practice.' If words have any meaning, this definition (not to wander from logic) suits only the *Διαλεκτική ἐν χρήσει καὶ γυμνασία πραγμάτων* of the Greek, the *logica utens* (*quæ utitur præceptis*) of the Latin Aristotelians. The *L. docens*, and the *L. utens*, are, however, so far from being convertible, that by the great majority of philosophers, they have been placed in different genera. The Greek logicians denied the *L. docens* to be either science or art, regarding it as an instrument, not a part of philosophy; the *L. utens*, on the contrary, they admitted to be a science, and a part of philosophy, but not separable and distinct. The Latins, on the contrary, held in general the *L. docens* to be a science, and part of philosophy; the *L. utens* as neither, but only an instrument. Some, however, made the *docens* a science, the *utens* an art; while by others this opinion was reversed, &c. These distinctions are not to be confounded with the *pure* and *applied* logics of a more modern philosophy.

schoolmen: borrowed from them by the Oxford Crackanthorpe, it was adopted by Wallis, and from Wallis passed to Dr Whately. But, as applied to logic, in its own nature, this opinion has been long rejected, on grounds superfluously conclusive, by the immense majority even of the Peripatetic dialecticians; and not a single reason has been alleged by Dr Whately to induce us to waver in our belief, that the laws of thought, and not the laws of reasoning, constitute the adequate object of the science. This error, which we cannot now refute, would, however, be of comparatively little consequence, did it not—as is notoriously the case in Dr Whately's *Elements*—induce a perfunctory consideration of the laws of those faculties of thought which are viewed as only subsidiary to the process of reasoning.

In regard to the 'clearness' with which Dr Whately 'displays' the true nature and use of logic, we can only say, that, after all our consideration, we do not yet clearly apprehend what his notions on this point actually are. In the very passages where he formally defines the science, we find him indistinct, ambiguous, and even contradictory; and it is only by applying the most favourable interpretation to his words that we are able to allow him credit for any thing like a correct opinion.

He says, that 'the most appropriate office of logic (as science) is that of instituting an analysis of the *process of the mind in reasoning*,' (p. 1;) and again, that '*the process (operation) of reasoning* is alone the appropriate province of logic,' (pp. 13, 140.) The process or operation of reasoning is thus the object-matter about which the science of logic is conversant. Now, a definition which merely affirms that logic is the science which has the process of reasoning for its object, is not a definition of this science at all; it does not contain the differential quality by which logic is discriminated from other sciences; and it does not prevent the most erroneous opinions (it even suggests them) from being taken up in regard to its nature. Other sciences, as psychology and metaphysic, propose for their object (among the other faculties) the operation of reasoning, but this considered in its real nature: logic, on the contrary, has the same for its object, but only in its formal capacity; in fact, it has, in propriety of speech, nothing to do with the *process* or *operation*, but is conversant only with its *laws*. Dr Whately's definition is, therefore, not only incompetent, but delusive; it would identify logic and psychology and metaphysic—occasion those very misconceptions in regard to the nature of logic which other passages of the *Elements*, and indeed the general analogy of his work, show that it was not his intention to sanction.

But Dr Whately is not only ambiguous; he is contradictory.

We have seen, that, in some places, he makes the process of reasoning the adequate object of logic; what shall we think when we find, that, in others, he states that the total or adequate object of logic is *language*? But, as there cannot be two adequate objects, and as language and the operation of reasoning are not the same, there is therefore a contradiction. 'In introducing the mention of *language*, previously to the definition of logic, I have departed from established practice, in order that it may be clearly understood, that logic is *entirely conversant about language*; a truth which most writers on the subject, if indeed they were fully aware of it themselves, have certainly not taken due care to impress on their readers,'* (p. 56.) And again: 'Logic is *wholly* concerned in the use of language,' (p. 74.)

The term logic (as also dialectic) is of ambiguous derivation. It may either be derived from Λόγος (*ἐνδιάθετος*), reason, or our intellectual faculties in general; or from Λόγος (*προφορικὸς*), speech or language, by which these are expressed. The science of logic may, in like manner, be viewed either, 1. as adequately and essentially conversant about the former, (the internal λόγος, *verbum mentale*;) and partially and accidentally about the latter, (the external λόγος, *verbum oris*;) or, 2. as adequately and essentially conversant about the latter, partially and accidentally about the former.

The first opinion has been held by the great majority of logicians, ancient and modern. The second, of which some traces may be found in the Greek commentators of Aristotle, and in the more ancient Nominalists during the middle ages, (for the later scholastic Nominalists, to whom this doctrine is generally but falsely attributed, held in reality the former opinion,) was only fully developed in modern times by philosophers, of whom Hobbes may be regarded as the principal. In making the *analysis of the operation of reasoning the appropriate office of logic*, Dr Whately adopts the first of these opinions; in making *logic entirely conversant about language*, he adopts the second. We can hardly, however, believe that he seriously entertained this last. It is expressly contradicted by Aristotle, (*Analyt. Post.* i. 10, § 7); it involves a psychological hypothesis in regard to the absolute dependence of the mental faculties on language, once and again refuted, which we are confident that Dr Whately never could sanction; and, finally, it is at variance

* Almost all logicians, however, impress upon their readers, that logic is (not, indeed, *entirely*, but) partially and secondarily occupied with language as the vehicle of thought, about which last it is adequately and primarily conversant.

with sundry passages of the *Elements*, where a doctrine apparently very different is advanced. But, be his doctrine what it may, precision and perspicuity are not the qualities we should think of applying to it.

But if the Vice-Principal be an incompetent judge of what the Principal has achieved, he is a still more incompetent reporter of what all other logicians have not. If he has read even a hundredth part of the works it behoved him to have studied before being entitled to assert that Dr Whately's 'treatise is 'the *only* one that has clearly displayed the true use and nature 'of logic,' he has accomplished what not one of his brother dialecticians of Oxford has attempted. But the assertion betrays itself: *πάντοτε ἀμάθεια*. To any one on a level with the literature of this science, the statement must appear supremely ridiculous, that the notions held of the nature and use of logic in the Kantian and even in the Wolfian school are not so clear, adequate, and correct, as those promulgated by Dr Whately. A general survey, indeed, of the history of opinions on this subject would prove, that views essentially sound were always as frequent, as the carrying of these views into effect was rare. Many, speculatively, recognised principles of the science, which almost none practically applied to regulate its constitution. Even the scholastic logicians display, in general, more enlightened and profound conceptions of the nature of their science than any recent logician of this country. In their multifarious controversies on this matter, the diversity of their opinions on subordinate points is not more remarkable than their unanimity on principal. All their doctrines admit of a favourable interpretation; and some have, for truth and precision, been seldom equalled, never surpassed. Logic they all discriminated from psychology, metaphysic, &c. as a rational, not a real—as a formal, not a material science. The few who held the adequate object of logic to be *things in general*, held this, however, under the qualification, that things in general were considered by logic only as they stood under the general forms of thought imposed on them by the intellect, (*quatenus secundis intentionibus substabant.*) Those who maintained this object to be the higher processes of thought, (three, two, or one,) carefully explained, that the intellectual operations were not, in their own nature, proposed to the logician—that belonged to the psychologist—but only in so far as they were *dirigible*, or the subject of laws. The proximate end of logic was thus to analyze the canons of thought; its remote, to apply these to the intellectual acts.—Those, again, (and they formed the great majority,) who saw

this object in *second notions*,* did not allow that logic was concerned with these second notions abstractly and in themselves,—that was the province of metaphysic,—but only in concrete as applied to *first*, as the instruments and regulators of thought. It would require a longer exposition than we can afford to do justice to these opinions—especially the last; for, when properly understood, they will be found to contain, in principle, all that has been subsequently advanced of any value in regard to the object-matter and scope of logic.

Nothing can be more meagre and incorrect than Dr Whately's sketch of the history of logic. This part of his work, indeed, is almost wholly borrowed from the poverty of Aldrich. As specimens:

Archytas is, after Aldrich, set down as the inventor of the *Categories*; and this now exploded opinion is advanced with-

* The distinction (which we owe to the Arabians) of *first* and *second notions* (*notiones, conceptus, intentiones, intellecta prima et secunda*), is necessary to be known, not only on its own account, as a highly philosophical determination, but as the condition of any understanding of the scholastic philosophy, old and new, of which, especially the logic, it is almost the Alpha and Omega. Yet, strange to say, the knowledge of this famous distinction has been long lost in 'the (once) second school of the church.' Aldrich's definition is altogether inadequate, if not positively erroneous. Mr Hill and Dr Whately, followed by Mr Huyshe and the author of *Questions on Logic*, &c., misconceive Aldrich, who is their only authority, if Aldrich understood himself, and flounder on from one error to another, without even a glimpse of the light. (*Hill*, pp. 30—33; *Whately*, pp. 173—175; *Huyshe*, pp. 18, 19; *Questions*, pp. 10, 11, 71.) (Of a surety, no calumny could be more unfounded, as now applied to Oxford, than the 'clamour,' of which Dr Whately is apprehensive, 'against confining the human mind in the trammels of the schoolmen!') The matter is worth some little illustration; we can spare it none, and must content ourselves with a definition of the terms. A *first notion* is the conception of a thing as it exists of itself, and independently of any operation of thought; as, John, Man, Animal, &c. A *second notion* is the conception, not of an object as it is in reality, but of the mode under which it is conceived by the mind itself; as, Individual, Species, Genus, &c. The former is the conception of a thing—real—immediate—direct: the latter the conception of a conception—formal—mediate—reflex. For elucidation of this distinction, and its applications, it is needless to make references. The subject is copiously treated by several authors in distinct treatises, but will be found competently explained in almost all the older systems of logic and philosophy.

out a suspicion of its truth. The same unacquaintance with philosophical literature and Aristotelic criticism is manifested by every recent Oxford writer who has alluded to the subject. We may refer to the *Excerpta ex Organo, in usum academicae Juventutis*—to the *Oxonia Purgata* of Dr Tatham—to Mr Hill's *Notes on Aldrich*—to Mr Huyshe's *Logic*—and to the *Philosophy of Aristotle* by Mr Hampden. This last even makes the Stagirite derive his moral system from the Pythagoreans, although the forgery of the fragments preserved by Stobæus, under the name of *Theages*, and other ethical writers of that school, has now been for half a century fully established. They stand likewise without an obelus in Dr Gaisford's respectable edition of the *Florilegium*. Aristotle would be, indeed, the sorriest plagiarist on record, were the thefts believed of him by his Oxford votaries not false only, but ridiculous. By Aldrich it is stated, as on indisputable evidence, that, while in Asia, he received a great part of his philosophy from a learned Jew; and this silly fable stands uncontradicted in the *Compendium* to the present day: while, by the Oxford writers at large, he is still supposed to have stolen his *Categories* and *Ethic* from the Pythagoreans. What would Schleiermacher or Creuzer think of this!

In discriminating Aristotle's merits in regard to logic, Dr Whately, we are sorry to say, is vague and incorrect. 'The greatest mistakes have always prevailed respecting the nature of logic; and its province has, in consequence, been extended by many writers to subjects with which it has no proper connexion. Indeed, with the exception of Aristotle, (who is himself not entirely exempt from the errors in question,) hardly a writer on logic can be mentioned who has clearly perceived, and steadily kept in view throughout, its real nature and object.' (p. 2.)—So far is Aristotle—so far at least are his logical treatises which still remain, (and these are few to the many that are lost,) from meriting this comparative eulogium, that nine-tenths—in fact, more than nineteen-twentieths,—of these treat of matters, which, if logical at all, can be viewed as the objects, not of *pure*, but only of an *applied* logic; and we have no hesitation in affirming, that the incorrect notions which have prevailed, and still continue to prevail, in regard to the 'nature and province of logic,' are, without detracting from his merits, mainly to be attributed to the example and authority of the Philosopher himself. The book of *Categories*, as containing an objective classification of real things, is metaphysical not logical. The two books of *Posterior Analytics*, as solely conversant about demonstrative or necessary

matter, transcend the limits of the formal science; and the same is true of the eight books of *Topics*, as wholly occupied with probable matter, its accidents and applications. Even the two books of the *Prior Analytics*, in which the pure syllogism is considered, are swelled with extralogical discussions. Such, for example, is the whole doctrine of the modality of syllogisms as founded on the distinction of pure, necessary, and contingent matter;—the consideration of the real truth or falsehood of propositions, and the power so irrelevantly attributed to the syllogism of inferring a true conclusion from false premises;—the distinction of the enthymeme through the extraformal character of its premises, as a reasoning from signs and probabilities;—the physiognomic syllogism, &c. &c. The same is true of the book *Περὶ Ἐγερνείας*; and matters are even worse with that on *Fallacies*. If Aristotle, therefore, did more than any other philosopher for the progress of the science; he also did more than any other to overlay it with extraneous lumber, and to impede its developement under a precise and elegant form. Many of his successors had the correctest views of the object and scope of logic; and even among the schoolmen there were minds who could have purified the science from its adventitious sediment, had they not been prevented from applying their principles to details, by the implicit deference then exacted to the precept and practice of Aristotle.

‘It has been remarked,’ says Dr. Whately, after Aldrich, ‘that the logical system is one of those few theories which have been begun and perfected by the same individual. The history of its discovery, as far as the main principles of the science are concerned, properly commences and ends with Aristotle,’ (p. 6.) In so far as ‘the main principles of the science are concerned,’ this cannot be denied. It ought, however, to have been stated with greater qualification. Aristotle left to his successors much to reject,—a good deal to supply,—and the whole to simplify, digest, and arrange. In regard to the deficiencies:—if Dr Whately and the other Oxford logicians are right, (we think decidedly otherwise,) in adding the fourth syllogistic figure, (which, by the way, none of them, from Aldrich downwards, ever hint to the under-graduates not to be of Aristotelic origin,) the Stagirite is wrong in recognising the exclusive possibility of the other three (*Analyt. Pr.* i. 23, § 1;) and so far his system can hardly be affirmed by them to have been perfected by himself. To say nothing of the five moods subsequently added by Theophrastus and Eudemus, the extensive and important doctrine of hypotheticals—a doctrine, in a great measure, peculiar

and independent—was, probably, an original supplement by these philosophers; previous to which, the logical system remained altogether defective.

‘The writings of Aristotle,’ says Dr Whately, ‘were not only absolutely lost to the world for about two centuries, [not all,] but seem to have been but little studied for a long time after their recovery. An art, however, of logic, derived from the principles traditionally preserved by his disciples, seems to have been generally known, and to have been employed by Cicero in his philosophical works; but the pursuit of the science seems to have been abandoned for a long time. Early in the Christian era the Peripatetic doctrines experienced a considerable revival; and we meet with the names of Galen and Porphyry as logicians; but it is not till the fifth [sixth] century that Aristotle’s logical works were translated into Latin by the celebrated Boethius. Not one of these seems to have made any considerable advances in developing the theory of reasoning. Of Galen’s labours little is known; and Porphyry’s principal work is merely on the *Pre-dicables*. We have little of the science till the revival of learning among the Arabians, by whom Aristotle’s treatises on this as well as on other subjects were eagerly studied,’ (p. 7.)—In this sketch of the fortune of logic from Aristotle to the schoolmen, Dr Whately closely follows Aldrich; and how utterly incompetent was Aldrich for a guide, is significantly shown by his incomparable (but still uncorrected) blunder of confounding Galen with Alexander of Aphrodisias! ‘Circa annum Christi 140,’ says he, ‘interpretum princeps Galenus floruit, Ἐξηγητής, sive Expositor, κατ’ ἐξοχὴν, dictus.’ Galen, who thus flourished at nine years old, never deserved, never received the title of *The Commentator*. This designation, as every tyro ought to know, was exclusively given to Alexander, the oldest and ablest of the Greek interpreters of Aristotle, until it was afterwards divided with him by Averroes. The names of Theophrastus and Eudemus, the great founders of logic after Aristotle, do not appear. We say nothing of inferior logicians, but the Aphrodisian and Ammonius Hermiæ were certainly not less worthy of notice than Porphyry. Of Galen’s logical labours, some are preserved, and of others we know not a little from his own information and that of others. Why is it not stated, here or elsewhere, that the fourth figure is to be attributed to Galen, and on what authority? Nothing is said of the original logical treatises of Boethius, though his work on Hypotheticals is the most copious we possess. Had Dr Whately studied the subject for himself, he would hardly have failed to do greater justice to the Greek logicians. What does he mean by saying, ‘we have

‘little of the science till the revival of learning among the Arabians?’ Are Averroes and Avicenna so greatly superior to Alexander and Ammonius?

Speaking of the Schoolmen, he says, ‘It may be sufficient to observe, that their fault did not lie in their diligent study of logic, and the high value they set upon it, but in their utterly mistaking the true nature and object of the science; and by the attempt to employ it for the purpose of physical discoveries, involving every subject in a mist of words, to the exclusion of sound philosophical investigation. Their errors may serve to account for the strong terms in which Bacon sometimes appears to censure logical pursuits; but that this censure was intended to bear against the extravagant perversions, not the legitimate cultivation, of the science, may be proved from his own observations on the subject, in his *Advancement of Learning*,’ (p. 8.) It has been long the fashion to attribute every absurdity to the schoolmen; it is only when a man of talent like Dr Whately follows the example that a contradiction is worth while. The schoolmen, (we except always such eccentric individuals as Raymond Lully,) had correcter notions of the domain of logic than those who now condemn them, without a knowledge of their works: they certainly did not ‘attempt to employ it for the purpose of physical discoveries.’ We pledge ourselves to refute the accusation whenever an effort is made to prove it; till then we must be allowed to treat it as a groundless though a common calumny. As to Bacon, we recollect no such reproach directed by him either against logic or against the scholastic logicians. On the contrary, ‘Logic,’ he says, ‘doth not pretend to invent sciences, or the axioms of sciences, but passeth it over with a *cuique in sua arte credendum*.*’ And so say the Schoolmen; and so says Aristotle.

We are not quite satisfied with Dr Whately’s strictures on Locke, Watts, &c., but cannot afford the space necessary to explain our views. One mistake in relation to the former we

* *Advancement of Learning*:—and similar statements, frequently occur in the *De Augmentis* and *Novum Organum*. The censure of Bacon, most pertinent to the point, is in the *Organum*, Aph. 63. It is, however, directed, not against the Schoolmen, but exclusively against Aristotle; it does not reprobate any false theory of the nature and object of logic, but certain practical misapplications of it; and, at any rate, it only shows that Bacon gave the name of *Dialectic* to *Ontology*. Aristotle did not corrupt physics by logic, but by metaphysic. The Schoolmen have enough to answer for, without imputing to them sins they did not commit.

shall correct, as it can be done in a few words. After speaking of Locke's animadversions on the syllogism, he says: 'He (Locke) presently after inserts an encomium upon Aristotle, in which he is equally unfortunate; he praises him for the "*invention of syllogisms*," to which he certainly had no more claim than Linnæus to the creation of plants and animals, or Hervey,' &c. (p. 19.) In the first place, Locke's words are, '*invention of forms of argumentation*,' which is by no means convertible with '*invention of syllogisms*,' the phrase attributed to him. But if syllogism had been the word, in one sense it is right, in another wrong. 'Aristotle,' says Dr Gillies, '*invented the syllogism*,' &c.; and in that author's (not in Dr Whately's) meaning, this may be correctly affirmed. But, in the second place, Dr Whately is wrong in thinking that the word '*invention*' is used by Locke, in the restricted sense in which it is now exclusively employed, as opposed to *discovery*. In Locke and his contemporaries, to say nothing of the older writers, to *invent* is currently used for to *discover*. An example occurs in the sentence of Bacon just quoted; and in this signification we may presume that '*invention*' is here employed by Locke.

But to proceed to the science itself: turning over a few pages, we come to an error not peculiar to Dr Whately, but shared with him by all logicians—we mean the *modality* of propositions and syllogisms; in other words, the *necessity, possibility*, &c., of their *matter*, as an object of logical consideration.

It has always been our wonder, how the integrity of logic has not long ago been purified from this metaphysical admixture. Kant, whose views of the nature and province of the science were peculiarly correct, and from whose acuteness, after that of Aristotle, every thing might have been expected, so far from ejecting the modality of propositions and syllogisms, again sanctioned its right of occupancy, by deducing from it, as an essential element of logical science, the last of his four generic categories, or fundamental forms of thought. Nothing, however, can be clearer, than that this modality is no object of logical concernment. Logic is a formal science; it takes no consideration of real existence, or of its relations, but is occupied solely about that existence and those relations which arise through, and are regulated by, the conditions of thought itself. Of the truth or falsehood of propositions, in themselves, it knows nothing, and takes no account: all in logic may be held true that is not conceived as contradictory. In reasoning, logic guarantees neither the premises nor the conclusion, but merely the *consequence* of the latter from the former; for a syllogism

is nothing more than the explicit assertion of the truth of one proposition on the hypothesis of other propositions being true in which that one is implicitly contained. A conclusion may thus be true in reality (as an assertion), and yet logically false (as an inference.)

But if truth or falsehood, as a material quality of propositions and syllogisms is extralogical, so also is their modality. Necessity, Possibility, &c., are circumstances which do not affect the logical copula or the logical inference. They do not relate to the connexion of the subject and predicate of the antecedent and consequent, as terms in thought, but as realities in existence; they are metaphysical, not logical conditions. The syllogistic inference is always necessary; it is modified by no extralogical condition; is equally apodictic in contingent as in necessary matter.

If such introduction of metaphysical notions into logic is once admitted, there is no limit to the intrusion. This is indeed shown in the vacillation or indefinitude of Aristotle himself in regard to the number of the modes. In one passage (*De Interp.* c. 12, § 1), he enumerates four—the *necessary*, the *impossible*, the *contingent*, the *possible*; and this determination has been generally received among logicians. In another (*Ibid.* § 9) he adds to these four modes two others, viz. the *true*, and, consequently, the *false*. Some logicians have accordingly admitted, but exclusively, these six modes; his Greek interpreters, however, very properly observe, (though they made no use of the observation,) that Aristotle did not mean by these enumerations to limit the number of modes to *four* or *six*, but thought only of signaling the more important. Modes may be conceived without end;—as the *certain*, the *probable*, the *useful*, the *good*, the *just*,—and what not? All, however, must be admitted into logic if any are: the line of distinction attempted to be drawn is futile. Such was the confusion and intricacy occasioned by the *four* modes alone, that the doctrine of modals long formed, not only the most useless, but the most difficult and disgusting branch of logic. It was at once the *criterium et crux ingeniorum*. ‘*De modali non gustabit asinus*,’ said the schoolmen; ‘*De modali non gustabit logicus*,’ say we. This subject was only perplexed because different sciences were jumbled in it together; and modals ought entirely on principle, as they have almost entirely in practice, to be relegated from the domain of logic, and consigned to the grammarian and metaphysician. This was, indeed, long ago obscurely perceived by a profound but now forgotten thinker. ‘*Pronunciata illa*,’ says Vives, ‘*quibus additur modus, non dialecticam sed grammati-*

'cam quæstionem habent,' etc.; and Ramus also felt the propriety of their exclusion, though he was equally unable to explicate its reasons.

Dr Whately has very correctly stated, that 'it belongs exclusively to a syllogism, properly so called, (*i. e.* a valid argument, so stated that its conclusiveness is evident from the mere form of the expression,) that if letters, or any other unmeaning symbols, be substituted for the several terms, the validity of the argument shall still be evident,' (p. 37.) Here logic appears, in Dr Whately's exposition, as it is in truth, a distinct and self-sufficient science. What, then, are we to think of the following:—'Should there be *no sign* at all to the common term, the quantity of the proposition (which is called an *indefinite* proposition) is ascertained by the *matter*—*i. e.* the nature of the connexion between the extremes, which is either necessary, impossible, or contingent,' &c. (p. 64.) 'As it is evident that the truth or falsity of any proposition (its quantity and quality being known) must depend on the *matter* of it, we must bear in mind, that *in necessary matter all affirmatives are true, and negatives false; in impossible matter vice versa; in contingent matter, all universals false and particulars true: e. g.* "all islands, (or some islands,) are surrounded by water," must be true, because the *matter is necessary*: to say, "no islands, or some—not," &c., would have been false: again, "some islands are fertile, some are not fertile," are both true, because it is contingent matter: put "*all*" or "*no*" instead of "*some*," and the propositions will be false,' (p. 67.)—In these passages logic is reduced from an independent science to a scientific accident. Necessary, impossible, and contingent matter, are terms expressive of certain lofty generalizations from an extensive observation of real existence; and logic, inasmuch as it postulates a knowledge of these generalizations, postulates its own degradation into a precarious appendage—a fortuitous sequel, of all the sciences from which that knowledge must be borrowed. If in syllogisms, 'unless unmeaning symbols can be substituted for the several terms, the argument is either unsound or sophistical';—why does not the same hold good in propositions, of which syllogisms are but the complement? But A, B, and C, know nothing of the necessary, impossible, and contingent. Is logic a formal science in one chapter, a real science in another? Is it independent, as a constituted whole; dependent in its constituent parts?

We cannot pass without notice Dr Whately's employment of the term *argument*. This word he defines, and professes to use in a 'strict logical sense;' and gives us, moreover,

under a distinct head, a formal enumeration of its other various significations in ordinary discourse. The *true* logical acceptation of the term, he, however, not only does not employ, but even absolutely overlooks; while, otherwise, his list of meanings is neither well discriminated, nor at all complete. We shall speak only of the logical omission and mistake.—‘Reasoning (or discourse) expressed in words is *argument*; and ‘an argument stated *at full length*, and in its *regular form*, is ‘called a *syllogism*; the third part of logic, therefore, treats of ‘the *syllogism*. Every argument consists of two parts; that ‘which is *proved*; and that *by means of which* it is proved,’ &c. And on this, in a note, he adds; ‘I mean, in the strict technical ‘sense; for, in popular use, the word Argument is often employed to denote the latter of these two parts alone: *e. g.* ‘this is an *argument* to prove so and so,’ &c., p. 72.—Now, the signification here (not quite correctly) given as the ‘popular use’ of the term is nearer to the ‘strict technical sense’ than that which Dr Whately supposes to be such. In technical propriety *argument* cannot be used for *argumentation*, as is done by Dr Whately—but exclusively for its *middle term*. In this meaning the word (though not with uniform consistency) was employed by Cicero, Quintilian, Boethius, &c.; it was thus subsequently used by the Latin Aristotelians, from whom it passed even to the Ramists;* and this is the meaning which the expression always first and most naturally suggests to a logician. Of the older dialecticians, Crackanthorpe is the only one we recollect, who uses, and professes to use, the word not in its strict logical signification, but with the vulgar as convertible with Reasoning. In vindicating his innovation, he, however, misrepresents his authorities. Sanderson is, if we remember, rigidly correct. The example of Crackanthorpe, and of some French Cartesians, may have seduced Wallis; and Wallis’s authority, with his own ignorance of logical propriety, determined the usage of Aldrich—and of Oxford. We say again Aldrich’s ignorance; and the point in question supplies a significant example. ‘Terminus tertius ‘(says he) cui quæstionis extrema comparantur, Aristotelî *Argumentum*, vulgo *Medium*.’ The reverse would be correct;—

* Ramus, in his definitions, indeed, abusively extends the word to both the other terms; the middle he calls the *tertium argumentum*. Throughout his writings, however—and the same is true of those of his friend Taleus—*argumentum*, without an adjective, is uniformly the word used for the middle term of a syllogism; and in this he is followed by the Ramists and Semi-Ramists in general.

Aristoteli Medium, vulgo Argumentum. This elementary blunder of the Dean, corrected by none, is repeated by nearly all his epitomators, expositors, and imitators. It stands in *Hill* (p. 118)—in *Huyshe* (p. 84)—in the *Questions on Logic* (p. 41)—and in the *Key to the Questions* (p. 101); and proves emphatically, that, for a century and a half at least, the *Organon* (to say nothing of other logical works) could have been as little read in Oxford as the Targum or Zendavesta.

A parallel to this error is Dr Whately's statement, that 'the 'major premiss is often called the *Principle*,' (p. 25.) The major premiss is often called the *Proposition*; never the *Principle*. A principle may, indeed, be a major premiss; but we make bold to say, that no logician ever employed the term Principle as a synonyme for major premiss.

'Most, if not all, writers,' says Dr Whately, 'on this point, 'either omit to tell, whether the Dilemma is a kind of *conditional* 'or of *disjunctive* argument, or else refer it to the latter class, on 'account of its having one disjunctive premiss; though it clearly 'belongs to the class of conditionals.' (P. 100.) Most, if not all, logical writers, do not omit to tell this, but Dr Whately, we fear, has omitted to consult them; and the opinion he himself adopts, so far from being held by few or none, has been, in fact, long the catholic doctrine. For every one logician, during the last century, who does not hold the dilemma to be a conditional syllogism, we could produce ten who do.

Dr Whately—indeed all the Oxford logicians—adopt the inelegant division of the *Hypothetical* proposition and syllogism into the *Conditional* and *Disjunctive*. This is wrong in itself. The name of the genus should not, without necessity, be confounded with that of a species. But the terms Hypothetical and Conditional are in sense identical, differing only in the language from which they are taken. It is likewise wrong on the score of authority; for the words have been used as synonymous by those logicians who, independently of their natural identity, were best entitled to regulate their conventional use. Boethius, the first among the Latins who elaborated this part of logic, employs indifferently the terms *hypotheticus, conditionalis, non simplex*, for the genus, and as opposed to *categoricus* or *simplex*; and this genus he divides into the *Propositio et Syllogismus conjunctivi* (called also *conjuncti, connexi, per connexionem*) equivalent to Dr Whately's Conditionals; and into the *Propositio et Syllogismus disjunctivi* (also *disjuncti, per disjunctionem*.) Other logicians have employed other, never better, terms of distinction; but, in general, all who had freed themselves of the

scholastic slime, avoided the needless confusion to which we object.

'Aldrich,' says our author, 'has stated, through a mistake, that Aristotle utterly despised hypothetical syllogisms, and thence made no mention of them; but he did indicate his intention to treat of them in some part of his work, which either was not completed by him according to his design, or else (in common with many of his writings) has not come down to us,' (p. 104.) Any ignorance of Aristotle on the part of Aldrich is conceivable, but in his censure Dr Whately is not himself correct. With the other Oxford logicians he never doubts the *Συλλογισμοὶ ἐξ ὑποθέσεως* of Aristotle and our hypothetical syllogisms to be the same. In this error, which is natural enough, he is not without associates even of distinguished name. Those versed in Aristotelic and logical literature are, however, aware, that this opinion has been long, if not exploded, at least rendered extremely improbable. We cannot at present enter on the subject, and must content ourselves with stating that hypothetical syllogisms, in the present acceptation, were first expounded, and the name first applied to them by Theophrastus and Eudemus. The latter, indeed, clearly discriminated such hypothetical syllogisms from those of Aristotle; and, what has not, we believe, been observed, even Boethius expressly declares the *Συλλογισμὸς ἐξ ὁμολογίας* of the philosopher to be really categorical, while in regard to the *Συλλογισμὸς εἰς τὸ ἀδύνατον*, there is no ground of doubt. The only reason for hesitation arises from the passage (*Analyt. Pr.* i. 44, § 4,) in which it is said, that there are *many* other syllogisms concluding *by hypothesis*, and these the philosopher promises to discuss. Of what nature these were, we have now no means even of conjecture. If we judge from Aristotle's notion of hypothesis, and from the syllogisms he calls by that name, we should infer that they had no analogy to the hypotheticals of Theophrastus; and it will immediately be seen, that a complete revolution in the nomenclature of this branch of logic was effected subsequently to Aristotle. We may add, that no reliance is to be placed in the account given by Pacius of the Aristotelic doctrine on this point: he is at variance with his own authorities, and has not attentively studied the Greek logicians.

So far we state only the conclusions of others. The following observation, as farther illustrating this point, will probably surprise those best qualified to judge, by its novelty and paradox. It must appear, indeed, at first sight ridiculous to talk at the present day of discoveries in the *Organon*. The certainty

of the fact is, however, equal to its improbability. The term *Categorical* (κατηγορικὸς), applied to proposition or syllogism, in contrast to *Hypothetical* (ὑποθετικὸς), we find employed in all the writings extant of the Peripatetic School, subsequent to those of its founder. In this acceptance it is universally applied by the interpreters of Aristotle up to the Aphrodisian, and previous to him we certainly know that it was so used by Theophrastus and Eudemus. Now, no logician, ancient or modern, has ever remarked that it was not understood in this signification by the philosopher himself. The Greek commentators on the *Organon*, indeed, once and again observe, in particular places, that the term *Categorical* is there to be interpreted *affirmative*; but none has made the general observation, that it was *never* applied by Aristotle in the sense in which it was exclusively usurped by themselves. But so it is. Throughout the *Organon* there is not to be found a single passage in which *categorical* stands opposed to *hypothetical*, (ἐξ ὑποθέσεως); there is not a single passage in which it is not manifestly used in the meaning of *affirmative*, as convertible with καταφατικὸς, and opposed to ἀποφατικὸς and εἰρητικὸς. Nor is the induction scanty. In the *Prior Analytics* alone the word occurs at least *eighty-five* times.—Nay, farther, as this never was, so there is another term always employed by Aristotle in contrast to his syllogisms by hypothesis. The syllogisms of this class, (whether they conclude by *agreement*, or through a *reductio ad absurdum*,) he uniformly opposes to those which conclude δεικτικῶς, *ostensively*; and the number of passages in which this opposition occurs are not a few.—*Categorical*, in our signification, is thus not of Aristotelic origin. The change in the meaning of the term was undoubtedly, we think, introduced by Theophrastus. The marvel is, that no logician or commentator has hitherto signalized the contrast between the Aristotelic signification of the word, and that which has subsequently prevailed.

We may allude (we can do no more) to another instance, in which Aristotle's meaning has been almost universally mistaken; and to the authority of this mistake we owe the introduction of an illogical absurdity into all the systems of logic. We refer to the *Enthymeme*. On the vulgar doctrine this is a species of reasoning, distinguished from the syllogism proper, by having one or other of its premises not expressed but understood; and this distinction, without a suspicion either of its legitimacy or origin, is fathered on the Stagirite. The division of syllogism and enthymeme, in *this* sense, would involve nothing less than a discrimination of species between the reasoning of logic and the reasoning of ordinary discourse; syllogism being the form

peculiar to the one, enthymeme that appropriate to the other. Nay, even this distinction, if admitted, would not avail; syllogism and enthymeme being distinguished as two intralogical forms of argumentation. Those who defend the distinction are thus driven back on the even greater absurdity—of establishing an essential difference of form, on an accidental variety of expression—of maintaining that logic regards the accident of the external language, and not the necessity of the internal thought. This, at least, is not the opinion of Aristotle. ‘Syllogism and ‘Demonstration,’ (says he,) ‘belong not to the outward discourse, but to the discourse that passes in the mind;’ ‘Ὁν πρὸς τὸν ἔξω λόγον ἢ ἀποδείξεις, ἀλλὰ πρὸς τὸν ἐν τῇ ψυχῇ· ἐπεὶ οὐδὲ συλλογισμός. (*Analyt. Post.* i. 10, § 7.) But if the distinction, in its general nature, is unphilosophical, it is still more irrational at the hands of its reputed author. For Aristotle distinguishes the enthymeme from the pure syllogism, as a reasoning of a peculiar *matter*—from *signs* and *likelihoods*; so that if he over-and-above discriminated these by an accident of *form*, he would divide the genus by two differences, and differences also of a merely contingent association. Yet, strange to say, this improbability has been believed;—believed without any cogent evidence;—believed from the most ancient times; and even when the opinion was at last competently refuted, the refutation was itself so immediately forgotten, that we do not believe there is at present a logical author—not to say in England, but—in Europe, who is even aware of the existence of the controversy.*

A discussion of the question would exceed our limits. For those who may wish to study the point—it would be a pretty subject for an Oxford pamphlet—we may briefly indicate the sources of information. Our references, though few, will be found to exhaust the subject.

Towards the conclusion of the fifteenth century, the celebrated Rodolphus Agricola, († 1485,) in his posthumous book, *De Inventione Dialectica*, recognises it as doubtful, whether Aristotle meant to discriminate the Enthymeme from the Syllogism, by any peculiarity of *form*; and Phrissemius in his *Scholia* on that book, (1523,) shows articulately that the common opinion was at variance with the statements of the Philosopher. Without, it is probable, any knowledge of Phrissemius, the matter was

* In this country, some years ago, the question was stated in a popular miscellany, with his usual ability, by a learned friend to whom we pointed out the evidence; but none of the subsequent writers have profited by the information.

discussed by Majoragius, in his *Reprehensiones contra Nizolium*, and his *Explanaciones in Aristotelis Rhetoricam*—the latter in 1572. Twenty-five years thereafter, Julius Pacius (who was not apparently aware of either) argued the whole question on far broader grounds; and, in particular, on the authority of four Greek MSS., ejected as a gloss the term ἀτελής, (*Analyt. Pr. ii. 27, § 3*), on which the argument for the common doctrine mainly rests; which has been also silently done by the Berlin Academicians, in their late splendid edition of Aristotle's works, on two of the three MSS. of the *Organon* they collated. We may notice that the Masters of Louvain, in their commentary on the logical treatises of Aristotle, (1547,) observe that the word *imperfectus* (translation of ἀτελής) is not to be found in many MSS. of the old Latin version. Scaynus, in his *Paraphrasis in Organum*, (1599,) adopts the opinion without arguing the question; and he does not seem to have been aware even of the Commentary of Pacius, published three years before. About 1620, Corydaleus, bishop of Mitylene, who had studied in Italy, maintained in his *Logic* the opinion of Pacius, but without additional corroboration. In his *Rhetoric*, (reprinted by Fabricius, in the *Bibliotheca Græca*) he adheres to the vulgar doctrine. A century thereafter, Facciolati expanded the argument of Pacius—for he, as the others, was ignorant of Majoragius, and Phrissemius, and adds nothing of his own except an error or two—into a special *Acroama*: but his eloquence was not more effective than the reasoning of his predecessors; and the question again fell into complete oblivion. Any one who competently reargues the point, will have both to supply and to correct.*

* For example:—Pacius (whom Facciolati, by rhetorical hyperbole, pronounces ‘Aristotelis Interpres, quot sunt, quotque fuerunt, quotque futuri sunt, longe præstantissimus,’) establishes as one of the main pillars of his argument, that the Greek interpreters did not acknowledge the term ἀτελής,—‘quoniam Johannes Grammaticus hic ‘nullam ejus mentionem facit; et tam ipse, quam Alexander superior libro explicantes definitionem syllogismi ab Aristotele traditam, ac distinguentes syllogismum ab argumentatione constante ex una propositione, non vocant hanc argumentationem *enthymema*, sed ‘syllogismum *μονολήμματον*.’ (*Comm. in Analyt. Pr. ii. 27, § 3*).—Pacius is completely wrong. Philoponus, on the place in question (*Anal. Pr. ii. c. 27, § 3*), states, indeed, (as far as we recollect, for our copy of his Commentary is not at hand,) nothing to the point; but the fallacy of such negative evidence is shown in his exposition of the *Posterior Analytics*, where he says, ‘Ενθύμημα δι εἰρηται, ἀπὸ τοῦ καταλημπάνει τῇ τῇ ἐνθυμίσσεται τὴν μίαν προτάσιν. (f. 4. a. Edit. Ald. 1534.) How inac-

We proceed to consider a still more important subject—the nature of the *Inductive* inference; and regret that we cannot echo the praises that have been bestowed on Dr Whately's analysis of this process. We do not, indeed, know the logician who has clearly defined the proper character of dialectical induction, and there are few who have not in the attempt been guilty of the grossest blunders. Aristotle's doctrine on this point, though meagre, is substantially correct; but succeeding logicians, in attempting to improve upon their master, have only corrupted what they endeavoured to complete. As confusion is here a principal cause of error, we must simplify the question by some preliminary distinctions and exclusions.

The term Induction (*ἐπαγωγή*) has been employed to denote three very different things:—1. The objective process of investigating particular facts as preparatory to illation;—2. A material illation of the universal from the singular, warranted either by the general analogies of nature, or by special presump-

curate also Pacius is in regard to Alexander, (whose interpretation of the *second* book of the *Prior Analytics*, which contains the passage in question, is still in MS., and probably spurious,) may be seen by referring to his Commentary on the first book of the *Prior Analytics*, (f. 7. a. b. Edit. Ald. 1534,) compared with his Commentary on the *Topics*, (pp. 6, 7, Edit. Ald. 1513.) This last we shall quote. He is speaking of Aristotle's definition of the Syllogism:—*Τιθέντων δὲ ἑπὶ πέντε ἄλλ' οὐ τιθέντος, ὥς τις ἀξιῶσιν, αἰτιώμενοι τὸν λόγον, ὅτι μηδὲν συλλογιστικῶς δι' ἐνὸς τιθέντος δίκνυται ἀλλ' ἐκ δύο τό ελάχισον. Οὐδὲ γὰρ οἱ περὶ Ἀντίπατρον (Tarsensem Tyriumve?) μενολημέματους συλλογισμοὺς λέγουσιν, οὐκ εἰσὶ συλλογισμοί, ἀλλ' ἐνδεῶς ἐρωτῶνται. - - - - - Τοιοῦτοι δὲ εἰσὶ καὶ οἱ ῥητορικοὶ συλλογισμοί, οὓς ἐνθυμήματα λέγομεν καὶ γὰρ ἐν ἐκείνοις δοκεῖ γίνεσθαι διὰ μιᾶς προτάσεως συλλογισμὸς, τῷ τὴν ἑτέραν γινώσκοντι οὖσαν ὑπὸ δίκασιν, ἢ τῶν ἀκροατῶν προσέβησθαι οἷον, κ. τ. λ. - - - - - Διὸ οὐδὲ οἱ τοιοῦτοι κυρίως συλλογισμοί, ἀλλὰ τὸ ὅλον, ῥητορικοὶ συλλογισμοί. 'Εφ' ᾧ οὖν μὴ γινώσκοντι ἐστὶ τὸ παραλειπόμενον, οὐκ ἔστι ἐπὶ τούτων οἷον τε τὸν δι' ἐνθυμήματος γίνεσθαι συλλογισμὸν καὶ γὰρ καὶ ἀπ' αὐτοῦ τῷ ὀνόματι συλλογισμὸς συνθεσὶν τινα λόγον ἔοικε σημαίνειν ὥσπερ καὶ ὁ συμφητισμὸς, ψήφων.—From these passages, it is manifest against Pacius,—1. That the *Ἐνθύμημα* was used by the oldest commentators on Aristotle in the modern signification, as a syllogism of one expressed premiss; and, 2. That the *συλλογισμὸς μονολήμματος* was not a term of the Aristotelian, but Stoical School. Boethius, and all the later Greek logicians, favour the common opinion. Their authority is, however, of little weight, and the general result of the argument stands unaffected.—In these errors, it is needless to say that Pacius is followed by Corydaleus and Faccioplati.*

tions afforded by the object-matter of any real science;—3. A formal illation of the universal from the individual, as legitimated solely by the laws of thought, and abstracted from the conditions of any particular matter.

That the first of these, an inventive process, is beyond the sphere of a critical science, is manifest; nor has Induction, in this abusive application of the term, been ever arrogated to Logic. By logicians, however, the second and third have been confounded into one, and, under every phasis of misconception, treated as a simple and purely logical operation. Yet nothing can be clearer than that these constitute two separate acts, and that the second is not properly a logical process at all. In logic, all inference is determined *ratione formæ*, the conclusion being necessarily implied in the very conception of the premises. In this second Induction, on the contrary, the illation is effected *vi materiæ*, on grounds not involved in the notion of its antecedent. To take, for example, Dr Whately's instance: The naturalist who, from the proposition—'ox, sheep, deer, goat, (*i. e. some*) horned animals, 'ruminates,' infers the conclusion—'all horned animals ruminate,' may be warranted in this procedure by the material probabilities of his science; but his illation is logically vicious. Here the inference is not necessitated by the laws of thought; the *some* of the antecedent, as it is not *thought* either to contain or constitute, so it does not mentally determine, the *all* of the consequent; and the reasoner must transcend the sphere of logic if he would attempt to vindicate the truth of his conclusion. And yet, this has by logicians been almost universally done. Induction they have distinguished into *perfect* and *imperfect*, according as the *whole* concluded was inferred from *all*, or from *some* only of its constituent parts. They thus involved themselves in a twofold absurdity. For, on the one hand, they recognised the consequence of the imperfect Induction to be legitimate, though, admitting it to be not necessarily cogent; as if logic could infer with a degree of certainty inferior to the highest; and, on the other, they attempted to corroborate this imbecility, by calling in real presumptions—physical, psychological, metaphysical—which logic could neither, as a formal science, know, nor, as an apodictic science, take into account. This was a corollary of the fundamental error to which we have already alluded—the non-exclusion of all material modality from the domain of logic. Thus, it was maintained, that, in necessary matter, the imperfect Induction was necessarily conclusive; as if logic could be aware of what was necessary matter—as if, indeed, this were not itself the frequent point of controversy in the objective sciences themselves.

The two first processes to which the name of Induction has been given, being thus excluded, it remains only to say a few words in explanation of that Induction, with which alone logic is concerned, but the nature of which has, by almost all logicians, been wholly misrepresented.

Logic does not consider things as they exist really and in themselves, but only the general forms of thought under which the mind conceives them; in the language of the schools, logic is conversant, not about *first*, but about *second notions*. Thus a logical inference is not determined by any objective relation of Causality subsisting between the terms of the premises and conclusion, but solely by the subjective relation of Reason and Consequence, under which they are construed to the mind in thought. The notion conceived as determining, is the *reason* or *antecedent*; the notion conceived as determined, is the *consequent*. Now, the mind can think two notions under the formal relation of reason and consequence, only in one or other of two modes. Either the determining notion must be conceived as a *whole*, *containing*, and therefore necessitating, the determined notion, conceived as its *contained part* or *parts*;—or the determining notion must be conceived as the *parts constituting*, and, therefore, necessitating the determined notion, conceived as their *constituted whole*. Considered, indeed, absolutely and in themselves, the *whole* and *all the parts* are identical. Relatively, however, to us, they are not; for in the order of thought, (and logic is only conversant with the laws of thought,) the whole may be conceived first, and then by mental analysis separated into its parts; or the parts may be conceived first, and then by mental synthesis collected into a whole. Logical inference is thus of two, and only of two, kinds:—it must proceed either from the whole to the parts, or from the parts to the whole; and it is only under the character of a constituted or containing whole, or of a constituting or contained part, that any thing can become the term of a logical argumentation.

Before proceeding, we must, however, allude to the nature of the whole and part, about which logic is conversant. These are not real or essential existences, but creations of the mind itself, in secondary operation on the primary objects of its knowledge. Things may be conceived the *same*, inasmuch as they are conceived the subjects of the same attribute, or collection of attributes, (*i. e.* of the same nature): inasmuch as they are conceived the same, they must be conceived as the parts constituent of, and contained under, a *whole*: and as they are conceived the same, only as they are conceived to be the subjects of the same nature, this common nature must

be convertible with that whole. A logical or universal whole is called a *genus* when its parts are also containing wholes or species; a *species* when its parts are only contained parts or individuals.

Such being the nature and relations of a logical whole and parts, it is manifest what must be the conditions under which the two kinds of logical inference are possible. The one of these, the process from the whole to the parts, is *Deductive* reasoning, (or Syllogism proper); the other, the process from the parts to the whole, is *Inductive* reasoning. The former is governed by the rule—What belongs (or does not belong) to the containing whole, belongs (or does not belong) to each and all of the contained parts. The latter by the rule—What belongs (or does not belong) to *all* the constituent parts, belongs (or does not belong) to the constituted whole. These rules exclusively determine all formal inference; whatever transcends or violates them, transcends or violates logic. Both are equally absolute. It would be not less illegal to infer by the Deductive syllogism an attribute, belonging to the whole, of something it was not conceived to contain as a part; than by the Inductive, to conclude of the whole, what is not conceived as a predicate of all its constituent parts. In either case, the consequent is not *thought* as determined by the antecedent;—the premises do not involve the conclusion.

The Deductive and Inductive processes are elements of logic equally essential. Each requires the other. The former is only possible through the latter; and the latter is only valuable as realizing the possibility of the former. As our knowledge commences with the apprehension of singulars, every universal whole is consequently only a knowledge at second-hand. Deductive reasoning is thus not an original and independent process. The universal major proposition, out of which it develops the conclusion, is itself necessarily the conclusion of a foregone Induction, and, mediately or immediately, an inference—a collection, from individual objects of perception, and consciousness. Logic, therefore, as a definite and self-sufficient science, must equally vindicate the formal purity of the synthetic illation, by which it ascends to its wholes, as the analytic illation by which it re-descends to their parts.*

Not only is the Deductive thus, in a general way, dependent for its possibility on the Inductive syllogism; the former is, what has not been observed, in principle and detail, in whole

* See Note, page 236.

and in part, in end and in means, in perfection and imperfection, precisely an inverted counterpart of the latter. The attempts that have been made by almost every logician, except (perhaps?) Aristotle, to assimilate and even identify the two processes, by reducing the Inductive syllogism to the schematic proprieties of the Deductive—proceeding as they do on a total misconception of their analogy and differences, have contributed to involve the doctrine of Logical Induction in a cloud of error and confusion. The Inductive inference is equally independent, and, though far less complex, equally worthy of analysis as the Deductive; it is governed by its own laws; and, if judged aright, must be estimated by its own standard. The correlation of the two processes is best exemplified by employing the same symbols in our ascent through an Inductive, and our re-descent through a Deductive syllogism.

<i>Inductive.</i>		<i>Deductive.</i>	
	x, y, z are A ;		B is A ;
	x, y, z are (<u>whole</u>) B ;		x, y, z are (<u>under</u>) B ;
Therefore, B is A.		Therefore, x, y, z are A.	
	<i>or</i>		<i>or</i>
	A contains x, y, z ;		A contains B ;
	x, y, z constitute B ;		B contains x, y, z ;
Therefore, A contains B.		Therefore, A contains x, y, z.	

These two syllogisms exhibit, each in its kind, the one natural and perfect figure. This will be at once admitted of the Deductive which is in the first. But the Inductive, estimated, as it has always been, by the standard of the Deductive, will appear a monster. It appears on that standard only in the third figure;* and then, contrary to the rule of that figure it has an universal conclusion. (V. *Analyt. Pr.* i. 22, § 8.) But when we look less partially and more profoundly into the matter, our con-

* We say 'it appears,' &c., because, though so held by logicians, it is not. The mistake arose from the ambiguity of the copula or substantive verb, which in different relations expresses either 'are contained' 'under' or 'constitute.' Thus, taking Aristotle's example :

Man, Horse, Mule, are long lived ;

Man, Horse, Mule, are the whole class of animals wanting bile ;

Therefore, the whole class of animals wanting bile are long lived.

Now here it is evident that the subject stands in a very different relation to its predicate in the major and in the minor premise ; though in both cases the connexion is expressed by the same copula. In the former the 'are' expresses that the predicate determines the

clusion will be very different. In the first place, we find that the two syllogisms present so systematic a relation of contrast and similarity, that, the perfection of the one being admitted, we are analogically led to presume the perfection of the other. In the *propositions*, the order of the terms remains unchanged: but the order of the propositions themselves are reversed; the conclusion of the one syllogism forming the major premise of the other. Of the *terms* the major is common to both; but the middle term of the one is the minor of the other. In the common minor premise, the terms, though identical, have, with the different nature of the process, changed their relation in thought. In the Inductive, the parts being conceived as constituting the whole, are the determining notion; whereas, in the Deductive, the parts being conceived as contained under the whole, are the determined.—But, in the second place, however apparently dissimilar in figure and proportion may be the two syllogisms on this partial standard, it will be found, if we ascend to a higher, that a common general principle regulates a similar, nay, a one exclusive perfection in each. The perfection of figure in *all* syllogisms is this—that the middle term should be the determined notion in the proposition, the determining notion in the assumption. This condition is realized in the first figure of the Deductive syllogism. There the middle term is the subject (contained, determined notion) in the proposition; and the predicate (containing, determining notion) in the assumption. In like manner, in our Inductive syllogism, the middle term is the subject (contained, determined notion) of the proposition, and the constituent (determining notion) of the assumption. Thus, not only are the Inductive and Deductive syllogisms, in a general sense, reversed processes; the perfect figure of the one is the exact evolution or involution of the perfect figure of the other.—The same analogy holds with their imperfections. Taking,

subject as a contained part; in the latter, that the subject determines the predicate by constituting it a whole. Explicitly thus:

Long-lived—contains—Man, horse, mule;

Man, horse, mule—constitute—animal wanting bile;

Therefore, Long-lived—contains—animal wanting bile.

That the logicians have neglected to analyze the Inductive inference as an independent process, and attempted to reduce it to the conditions of the Deductive; is the cause or the effect of a primary deficiency in their technical language. They have no word to express the *synthesis* of a logical whole. The word *constitute*, &c., which we have, from necessity, employed in this sense, belongs properly to the relations of an *Essential* (Physical or Metaphysical) whole, and parts.

for example, what logicians have in general given as the perfected figure, but which is, in fact, an unnatural perversion of the Inductive syllogism, (*i. e.* its reduction to the first figure, by converting the terms of the minor premise,) we shall find that its reversal into a Deductive syllogism affords, as we should have anticipated, only a kindred imperfection (in the third figure.)

<i>Inductive.</i>	<i>Deductive.</i>
x, y, z are A ;	B is A ;
B is x, y, z ;	B is x, y, z ;
Therefore, B is A.	Therefore, x, y, z are A.
<i>or</i>	
A contains x, y, z ;	A contains B ;
x, y, z contain B ;	x, y, z contain B ;
Therefore, A contains B.	Therefore, A contains x, y, z.

We call this reduction of the Inductive syllogism an *unnatural perversion*; because in the converted minor premise the constituent parts are perverted into a containing whole, and the containing whole into a subject, contained under its constituent parts.—After these hints of what we deem the true nature of logical Induction, we return to our author.

Dr Whately's account of Induction is principally given in two passages. We shall quote them both. The first:—‘Logic takes no cognisance of *Induction*, for instance, or of a *priori* reasoning, &c., as distinct *forms* of argument; for when thrown into the syllogistic form, and when letters of the alphabet are substituted for the terms, (and it is thus that an argument is properly to be brought under the cognisance of logic,) there is no distinction between them; *e. g.* a “Property which belongs to the ox, sheep, deer, goat, and antelope, belongs to all horned animals; rumination belongs to these; therefore to all.” This, which is an inductive argument, is evidently a syllogism in *Barbara*. The essence of an inductive argument (and so of the other kinds which are distinguished from it) consists not in the *form of the argument*, but in the relation which the *subject-matter* of the premises bears to the conclusion,’ (p. 110.) The second:—‘In the process of reasoning by which we deduce, from our observation of certain known cases, an inference with respect to unknown ones, we are employing a syllogism in *Barbara* with the major premiss suppressed; that being always substantially the same, as it asserts, that, “what belongs to the individual or individuals we have examined, belongs to the whole class under which they come,”’ (p. 216.)—This statement is consistent neither with the Aristotelic doctrine nor with truth.

We must presume, from his silence, that our author, in his analysis of the inductive process, was not aware of any essential deviation from the doctrine of Aristotle. This he does not seem to have studied either in the *Organon* or in any of its authentic expositors; and nothing can be conceived more contradictory than the statements of the philosopher on this subject and those of Dr Whately. Aristotle views the Inductive and the Deductive syllogisms as in certain respects similar in form; in others, as diametrically opposed. Dr Whately regards them as formally identical, and only discriminated by a material difference, *i. e.* logically considered, by no difference at all. Aristotle regards the Deductive syllogism as the analysis of a logical whole into its parts,—as a descent from the (more) general to the (more) particular; the Inductive as a synthesis of logical parts into a logical whole,—as an ascent from the (more) particular to the (more) general. Dr Whately, on the other hand, virtually annihilates the latter process, and identifies the Inductive with the Deductive inference. Aristotle makes Deduction necessarily dependent on Induction; he maintains that the highest or most universal axioms which constitute the primary and immediate propositions of the former, are all conclusions previously furnished by the latter. Whately, on the contrary, implicitly asserts the independence of the syllogism proper, as he considers the conclusions of Induction to be only inferences evolved from a more universal major. Aristotle recognises only a perfect Induction, *i. e.* an enumeration (actual or presumed) of all the parts; Whately only an imperfect, *i. e.* an enumeration professedly only of some. To Aristotle Induction is a syllogism, *apparently*, of the third figure; to Whately a syllogism of the first. If Whately be right, Aristotle is fundamentally wrong; wrong in admitting Inductive reasoning within the sphere of logic at all; wrong in discriminating Induction from syllogism; wrong in all the particulars of the contrast.

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 and extra logical

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But that the Philosopher is not in error is evident at once; the Archbishop's doctrine is palpably suicidal. On that doctrine the Inductive reasoning is 'a syllogism in Barbara, the major premiss being always substantially the same—"What belongs to the individual or individuals we have examined, belongs to the whole class under which they come."—Now, we ask, in what manner do we obtain this major, in the evolution of which all Induction consists? To this question there are only four possible answers:—1. This proposition, (like the *dictum de omni et nullo*, and the axiom of the convertibility of the whole and its parts,) it may be said is (analytically) self-evident, its negation implying a contradiction. This answer is manifestly false; for so far from

being necessitated by the laws of thought, it is in opposition to them; the *whole* of the consequent not being determined in thought by the *some* of the antecedent.—2. It may be said to be acquired by Induction. That, however, would be absurd; inasmuch as Induction itself is, *ex hypothesi*, only possible through and after the principle it is thus adduced to construct. This of the proposition as a whole. The same is also true of its parts. ‘Class’ is a notion, itself the result of an Induction; it cannot, therefore, be postulated as a pre-requisite or element of that process itself. A similar remark applies to ‘property.’—3. It may be said to be deduced from a higher axiom. What then is such axiom? That has not been declared. And if such existed, the same questions would remain to be answered regarding the higher proposition which are now required in relation to the lower.—4. It may be said to be (as Kant would say, synthetically) given as an ultimate principle of our intellectual constitution. This will not do. In the first place, if such principle exist, it only inclines, it does not necessitate. In the second, by appealing to it, we should transcend our science, confound the logical and formal with the metaphysical and material. In the third, we should thus attempt to prove a logical law from a psychological observation; *i. e.* establish an *a priori*, necessary science on a precarious experience,—an experience admitted perhaps by the disciples of Reid and Royer-Collard; but scouted by those of Aristotle and Locke.* Logicians, we already observed, have been guilty of a fundamental error in bringing the distinction of perfect and imperfect Induction within the sphere of their science, as this distinction proceeds on a material, consequently on an extralogical, difference. In this error, however, Dr Whately exceeds all other logicians, recognising, as he does, exclusively, that Induction, which is only precariously valid, and valid only through an extralogical presumption. This common major premise, if stated as necessary, is (formally and materially) false; if stated as probable, it is (formally) illegitimate, even if not (materially) untrue, both because an inferior degree of certainty is incompatible with an apodictic science, and because the amount of certainty itself must, if not capriciously assumed, be borrowed from evidence

* ‘It is by *induction* that all axioms are known, such as, ‘Things that are equal to the same are equal to one another;’ ‘A whole is greater than its parts;’ and all other mathematical axioms.’ *Huyshe*, p. 132. The same doctrine is held by *Hill*, p. 176. Is such the *Oxford Metaphysic*?

dependent on material conditions beyond the purview of a formal science.

Dr Whately is not less unfortunate in refuting the opinions of other logicians touching induction, than in establishing his own. 'In this process,' he says, 'we are employing a syllogism in Barbara with the major premiss suppressed; not the minor, as Aldrich represents it. The instance he gives will sufficiently prove this:—"This and that, and the other magnet, attract iron; therefore so do all." If this were, as he asserts, an enthymeme whose *minor* is suppressed, the only premise which we could supply to fill it up would be, "all magnets are this, that, and the other;" which is manifestly false,' (p. 217.) Aldrich has faults sufficient of his own, without taking burden of the sins of others. He is here singly reprehended for saying only what, his critic seems not aware, had been said by all logicians before him. The suppressed minor even obtained in the schools the name of the *constantia*; and it was not until the time of Wolf that a new-fangled doctrine, in this respect the same as Whately's, in some degree superseded the older and correcter theory. 'In the example of Aldrich,' says our author, 'the suppressed minor premiss, "all magnets are this, that, and the other," is manifestly false.' Why? Is it because the proposition affirms that a certain three magnets ('this, that, and the other') are all magnets? Even admitting this, the objection is null. The logician has a perfect right to suppose this or any other *material* falsity for an example; all that is required of him is, that his syllogism should be *formally* correct. Logic only proves on the *hypothetical* truth of its antecedents. As Magentinus notices, Aristotle's example of Induction is physiologically false; but it is not on that account a whit the worse as a dialectical illustration. The objection is wholly extralogical. But this is not in fact the meaning of the proposition. The words (in the original ('hic, et ille, et iste magnes')) are intended to denote *every* several magnet. Aldrich borrows the instance from Sanderson, by whom it is also more fully expressed:—"Iste magnes trahit ferrum, et ille, et hic, et pariter se habet in reliquis," &c. Perhaps, however, and this is the only possible alternative, Dr Whately thinks the assumption 'manifestly false,' on the ground that no extent of observation could possibly be commensurate with 'all magnets.' This objection likewise lies beyond the domain of the science. The logician, *qua* logician, knows nothing of material possibility and impossibility. To him all is possible that does not involve a contradiction in terms. At the same time, the present is merely the logical manner of wording the proposition. The physical observer asserts on the analogy

of his science, 'This, that, the other magnet, &c., represent, all 'magnets;' which the logician *accepting*, brings under the conditions, and translates into the language of his—This, that, the other magnet, &c. *are* all magnets—i. e. *are conceived as constituting* the whole—Magnet.

Dr Whately's errors relative to Induction are, however, surpassed by those of another able writer, Mr Hampden, in regard to that process, and the Aristotelic exposition of its nature;—errors the more inconceivable, as he professes to have devoted peculiar attention to the subject, which, he says, 'deserves a more particular notice, as throwing light on Aristotle's whole method of philosophising, while it shows how far 'he approximated to the Induction of modern philosophy.'

'To obtain,' (says Mr Hampden,) 'an accurate notion of the being of any thing, we require a definition of it. A definition of the thing corresponds, in dialectic, with the essential notion of it in metaphysics. This abstract notion, then, according to Aristotle, constituting the true scientific view of a thing—and all the real knowledge consequently of the properties of the thing depending on the right limitation of this notion—some exact method of arriving at definitions which should express these limitations, and serve as the principles of sciences, became indispensable in such a system of philosophy. But in order to attain such definitions, a process of induction was required,—not merely an induction of that kind, which is only a peculiar form of syllogism, enumerating all the individuals implied in a class instead of the whole class collectively, but an induction of a philosophical character, and only differing from the induction of modern philosophy so far as it is employed about language. We shall endeavour to show this more fully. There are, then, two kinds of induction treated of by Aristotle. The first, that of simple enumeration.—[After explaining with ordinary accuracy the first, in fact the only, species of induction, he proceeds.]—But there is also a higher kind of induction employed by Aristotle, and pointed out by him expressly in its suberviency to the exact notions of things, by its leading to the right definitions of them in words. As it appears that words, in a dialectical point of view, are classes more or less comprehensive of observations on things, it is evident that we must gradually approximate towards a definition of any individual notion, by assigning class within class, until we have narrowed the extent of the expression as far as language will admit.* The first definitions of any object are vague, founded on some obvious resemblance which it exhibits compared with other objects. This point of resemblance we abstract in thought, and it becomes, when expressed in language, a genus or class, under which

* 'Analyt. Post. ii. c. 13.—Ζητειν δι δὴ ἐπιλείποντα ἐπὶ τα ὅμοια καὶ ἀδιαφορεα, πρῶτον τι ἅπαντα ταῦτοι ἔχουσι, κ. τ. λ. p. 175, Du Val.'

we regard the object as included. A more attentive examination suggests to us less obvious points of resemblance between this object and some of those with which we had classed it before. Thus carrying on the analysis—and by the power of abstraction giving an independent existence to those successive points of resemblance—we obtain subaltern genera or species, or subordinate classes included in that original class with which the process of abstraction commenced. As these several classifications are relative to each other, and dependent on the class with which we first commenced, the definition of any notion requires a successive enumeration of the several classes in the line of abstraction, and hence is said technically to consist of genus and differentia; the genus being the first abstraction, or class to which the object is first referred, and the differentia being the subordinate classes in the same line of abstraction. Now, the process by which we discover these successive genera, is strictly one of philosophical induction. As in the philosophy of nature in general, we take certain facts as the basis of enquiry, and proceed by rejection and exclusion of principles involved in the enquiry, until at last—there appearing no ground for further rejection—we conclude that we are in possession of the true principle of the object examined; so, in the philosophy of language, we must proceed by a like rejection and exclusion of notions implied in the general term with which we set out, until we reach the very confines of that notion of it with which our enquiry is concerned. This exclusion is effected in language, by annexing to the general term denoting the class to which the object is primarily referred, other terms not including under them those other objects or notions to which the general term applies. For thus, whilst each successive term in the definition, in itself, extends to more than the object so defined,—yet all viewed together do not; and this their relative bearing on the one point constitutes the being of the thing. This is thus illustrated by Aristotle:—"If we are enquiring," he says, "what magnanimity is, we must consider the instances of certain magnanimous persons whom we know, what one thing they all have so far forth as they are such; as, if Alcibiades was magnanimous, or Achilles, or Ajax;—what one thing they all have; say, "impatience under insult;" for one made war, another raged, the other slew himself. Again, in the instances of others, as of Lysander or Socrates,—if here it is, "to be unaltered by prosperity or adversity;"—taking these two cases, I consider, what this "apathy in regard to events," and "impatience under insult," have the same in them. "If, now, they have nothing the same, there must be two species of magnanimity." (P. 513.)

Mr Hampden afterwards states, *inter alia*, that the induction of Aristotle, 'having for its object to determine accurately in words the notion of the being of things, proceeds, according to the nature of language, from the general, and ends in the particular; whereas the investigation of a law of nature proceeds from the particular, and ends in the general. Dialectical induction is synthetical, whilst philosophical induction is analytical in the result.' On this ground, he explains the

meaning of the term (*ἐπαγωγή*), and defends the Induction of Aristotle against its disparagement by Lord Bacon.

We had imagined that every compend of logic explained the two grand methods of investigating the definition; but upon looking into the Oxford treatises on this science, we were surprised to find, that this, among other important matters, had in all of them been overlooked. This may, in part, enable us to surmise how Mr Hampden could have so misconceived so elementary a point as to have actually reversed the doctrine, not only of Aristotle, but of all other philosophers. A few words will be sufficient to illustrate the nature of the error.

In the thirteenth chapter (Pacian division) of the second book of the *Posterior Analytics*, Aristotle treats of the manner of hunting out, as he terms it, the essential nature (*τὸ τί ἐστι, quidditas*) of a thing, the enunciation of which nature constitutes its definition. This may be attempted in two contrary ways. By the one, we may descend from the category, or higher genus of the thing to be defined, dividing and subdividing it through the opposite differences till we reach the genus under which it is proximately contained; and this last genus, along with the specific difference by which it is divided, will be the definition required. By the other, we may ascend from the singulars contained under the thing to be defined (which is necessarily an universal) by an exclusion of their differences, until we attain an attribution common to them all, which attribution will supply the definition sought. The former of these is, after Plato, called by Aristotle, and logicians in general, the method of *Division*; the higher genus being regarded as the (universal) *whole*, the subaltern genera and species as the (subjective) *parts* into which it is divided. The *extension* here determines the totality. The latter, which is described but not named by Aristotle, is variously denominated by his followers. Some, as his Greek commentators, taking the totality as determined by the *comprehension*, view the singulars as so many (essential) wholes, of which the common attribute or definition is a part, and accordingly call this mode of hunting up the essence the *Analytic*; others again, regarding the genus as the whole, the species and individuals as the parts, style it the *Compositive*, or *Synthetic*, or *Collective*; * while others, in fine, looking

* 'In one respect,' says Aristotle, 'the Genus is called a part of the Species; in another, the Species a part of the Genus.' (*Metaph. L. v. c. 25.*) In like manner, the same method, viewed in different relations, may be styled either *Analysis* or *Synthesis*. This, however, has not

simply to the order of the process itself, from the particular to the general, name it the *Inductive*. These last we shall imitate.

Now, in the chapter referred to, Aristotle considers and contrasts these two methods. In regard to Division (§ 8—20) he shows on the one hand, (against Plato, who is not named,) that this process is not to be viewed as having any power of demonstration or argument; * and on the other, (against Speusippus, as we learn from Eudemus, through the Greek expositors,) that it is not wholly to be rejected as worthless, being useful, in subservience always to the other method of induction, to ensure—that none of the essential qualities are omitted—that these qualities alone are taken—and that they are properly subordinated and arranged. In reference to the Inductive method, which is to be considered as the principal, he explains its nature, and delivers various precepts for its due application, (§ § 7, 21, etc.)

This summary will enable the reader to understand Mr Hampden's perversion of Aristotle's doctrine. In the first place, that gentleman is mistaken in supposing that the philosopher applies the term Induction to any method of investigating the definition discussed by him in the chapter in question. The word does not once occur. In the second place, he is still farther deceived in thinking that Aristotle there bestows that name on a descent from the universal to the particular, whereas in his philosophy—indeed in all philosophies—it exclusively pertains to an ascent from the particular to the universal. In the third place, he is wrong in imagining that Aristotle there treats only of a single method, for he considers and contrasts two methods, not only different, but opposed.† In the fourth place, he is mistaken, in understanding as applied to one contrary, the observations which Aristotle applies, and which are only applicable

been acknowledged; nor has it even attracted notice, that different logicians and philosophers, though severally applying the terms only in a single sense, are still at cross purposes with each other. One calls Synthesis, what another calls Analysis; and this both in ancient and modern times. We ourselves think it best to regulate the use of these terms by reference to the notion of a whole and parts, of *any* kind. This we do, and do professedly. Mr Hampden, but probably without intending it, does the same: in one part of the passage we have quoted, speaking of Division, (his logical induction,) as an 'analysis;' in another, describing it as 'synthetical.'

* This he had elsewhere done; *Pr. Analyt.* l. i. c. 31. *Post. Analyt.* l. ii. c. 5. et alibi.

† Mr Hampden's error, we suspect, originates in the circumstance that Pacius (whom Du Val follows in the *Organon*) speaks, in his

in expounding the reverse. For example, he quotes in the note as pertinent to Division words of the original relative to Induction; and the instance (from the definition of Magnanimity) adduced to illucidate the one method, is in reality employed by Aristotle to explain the other. In the fifth place, his error is enhanced by seeing in his single method the subordinate of Aristotle's two; and in lauding as a peculiarly important part of the Aristotelic philosophy, a process in the exposition of which Aristotle has no claim to originality, and to which he himself, here and elsewhere, justly attributes only an inferior importance. In the sixth place, in contradiction equally of his whole philosophy and of the truth of nature, the Stagirite is made to hold that our highest abstractions are first in the order of time; that our process of classification is encentric not eccentric; that a child generalizes *substance* and *accident* before *egg* and *white*.—Mr Hampden's statement of the Inductive method being thus the reverse of truth, it is needless to say that the etymological explanation he has hazarded of the term (*ἐπαγωγή*) must be erroneous. But even more erroneous is the pendant by which he attempts to illustrate his interpretation of that term. 'The *ἀπαγωγή*, Abduction, spoken of by Aristotle, (*Anal. Prior.* ii. c. 25,) is just the reverse,—a leading away, by the terms successively brought from the more accurate notion conveyed by a former one.' The *abduction* here referred to is no more such a 'leading away' than it is a theft. It is a kind of syllogism,—of what nature we cannot longer trespass on the patience of our readers by explaining. For the same reason we say nothing of some other errors we had remarked in Mr Hampden's account of that branch of the Aristotelic philosophy which we have been now considering.

analytic argument of the chapter, of a *methodus divisiva*, and a *methodus inductiva*; and that Mr H., in his extemporaneous study of the subject, not previously aware that there were two opposite methods of investigating the definition, took up the notion that these were merely a twofold expression for the same thing. Mr Hampden is an able man; but to understand Aristotle in any of his works, he must be understood in all; and to be understood in all, he must be long and patiently studied by a mind disciplined to speculation, and familiar with the literature of philosophy.

- ART. X.—1. *The Penny Magazine of the Society for the Diffusion of Useful Knowledge.* 1832.
 2. *The Saturday Magazine of the Society for Propagating Christian Knowledge.* 1832.
 3. *Chambers's Edinburgh Journal.* 1832.

WE have prefixed the titles of these popular publications to the few remarks which their object suggests, because their success is the fact from which those remarks chiefly spring; and it is a fact of the most cheering nature at the present moment. Differing in other respects, in this they all agree, that neither personal slander, nor any species of scurrility—neither party discussion, whether political or religious, nor invective of any description against men or things—nothing to excite the passions, to influence or corrupt—finds its way into their pages; and yet they are by far the most extensively circulated of any periodical works that issue from the press. When we say that there is nothing of attack or of a controversial description in these works, we might unfortunately make an exception; and that is in some of the unworthy remarks of the Society for Propagating the Gospel of Charity—that Gospel which teaches ‘to think no evil.’ We perceive insinuations of the most unwarrantable kind against the other cheap publications, as unfavourable, if not positively hostile, to religion; and no exception made in favour of the very work which gave rise to the Society’s own Magazine. The High Church party on this, as upon every other occasion, suffered the lead to be taken by the Low Church and the Sects, or, at least, by those persons who love to convey instruction to all classes alike, without distinction of religious denomination. As soon as their labours proved successful, the clerical party came into the field; and we should not be surprised to find them hereafter assuming the merit of first printing cheap works, as they habitually, we perceive, take the credit of having begun the Education of the Poor! But at least those who preceded them, and whose pages are in no one line hostile to religion, but throughout most friendly to its interests, have a right to expect that they shall not be (though only in the way of insinuation) slandered on this score by their imitators. This Society’s Magazine, we repeat, is a useful work, and it is executed with ability, both as to writing and embellishments: that it is a religious work, no one who reads it can pretend to fancy.

There is hardly one article in twenty that bears at all upon religious topics. But our present object is to consider the great circulation of these works.

We have before us the preface to the Penny Magazine. What may be the numbers printed of the two others we are not informed. But in the preface, the Society for the Diffusion of Useful Knowledge state, that of their Magazine two hundred thousand are sold. We have understood that about sixty thousand are sold of each of the other two. The sale of the Penny Magazine and of Chambers's Journal is entirely voluntary, and not forced by any fund whatever. The yearly published accounts of the Useful Knowledge Society prove this as to the Penny Magazine; and Messrs Chambers, being booksellers, of course can only sell their work on the ordinary terms of the trade. Whether any portion of the other Society's funds has been appropriated to give their work a forced circulation we know not; but any such impulse could only be temporary, and it may be reasonably assumed either that none was given, or that it has ceased, and that the natural circulation is what we have stated. Here then are *three hundred and twenty thousand* of these weekly publications actually sold to persons who can only purchase them in order to gratify a thirst for useful and pleasing information,—for that which may improve their minds, and afford innocent and refined entertainment, without gratifying any feeling of a debasing, or unamiable, or even private kind.

The price paid by the vast numbers who thus buy, deserves to be considered. The works are indeed cheap beyond all former example; the cheapness is such as could only be attained by means of the extraordinary circulation; nevertheless, two of the works (the Magazines) cost six shillings a-year, exclusive of binding, and the Journal (being sold at three-halfpence, but we believe without the supplements which the others have,*) may cost six and sixpence. So that near three hundred thousand pounds a-year, (accurately L.296,000,) are thus expended by persons chiefly of the middle and poorer classes in gratifying their desire for knowledge.

It is a most delightful reflection for the friend of human improvement, to think, that at the low price of a few shillings, the poor may obtain, and with all the accommodation which periodical publication affords, a volume of five or six hundred pages,

* Let us not be understood as in the least degree intending to state Messrs Chambers's work as costing more than it ought. It is well worth the money, and was published *before* either of the others.

FIRST NOTIONS

OF

LOGIC

(PREPARATORY TO THE STUDY OF GEOMETRY).

BY

AUGUSTUS DE MORGAN,

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The root of all the mischief in the sciences, is this ; that falsely magnifying and admiring the powers of the mind, we seek not its real helps. — BACON.

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*SECOND EDITION.*  
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28 UPPER GOWER STREET.

M.DCCC.XL.

* * This Tract contains no more than the author has found, from experience, to be much wanted by students who are commencing with Euclid. It will ultimately form an Appendix to his Treatise on Arithmetic.

The author would not, by any means, in presenting the minimum necessary for a particular purpose, be held to imply that he has given enough of the subject for all the ends of education. He has long regretted the neglect of logic ; a science, the study of which would shew many of its opponents that the light esteem in which they hold it arises from those habits of inference which thrive best in its absence. He strongly recommends any student to whom this tract may be the first introduction of the subject, to pursue it to a much greater extent.

University College, Jan. 8, 1839.

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FIRST NOTIONS

OF

LOGIC.

WHAT we here mean by *Logic* is the examination of that part of reasoning which depends upon the manner in which inferences are formed, and the investigation of general maxims and rules for constructing arguments, so that the conclusion may contain no inaccuracy which was not previously asserted in the premises. It has nothing to do with the truth of the facts, opinions, or presumptions, from which an inference is derived ; but simply takes care that the inference shall certainly be true, if the premises be true. Thus, when we say that all men will die, and that all men are rational beings, and thence infer that some rational beings will die, the *logical* truth of this sentence is the same whether it be true or false that men are mortal and rational. This logical truth depends upon the structure of the sentence, and not on the particular matters spoken of. Thus,

Instead of,

All men will die.

All men are rational beings.

Therefore some rational beings will die.

Write,

Every A is B.

Every A is C.

Therefore some Cs are Bs.

The second of these is the same proposition, logically considered, as the first ; the consequence in both is virtually contained in, and rightly inferred from, the premises. Whether the premises be true or false, is not a question of logic, but of morals, philosophy, history, or any other knowledge to which their subject-matter belongs : the question of logic is, does the conclusion certainly follow if the premises be true ?

Every act of reasoning must mainly consist in comparing together different things, and either finding out, or recalling from previous knowledge, the points in which they resemble or differ from each other. That particular part of reasoning which is called *inference*, consists in the comparison of several and different things with one and the same other thing; and ascertaining the resemblances, or differences, of the several things, by means of the points in which they resemble, or differ from, the thing with which all are compared.

There must then be some propositions already obtained before any inference can be drawn. All propositions are either assertions or denials, and are thus divided into *affirmative* and *negative*. Thus, A is B, and A is not B, are the two forms to which all propositions may be reduced. These are, for our present purpose, the most simple forms; though it will frequently happen that much circumlocution is needed to reduce propositions to them. Thus, suppose the following assertion, 'If he should come to-morrow, he will probably stay till Monday;' how is this to be reduced to the form A is B? There is evidently something spoken of, something said of it, and an affirmative connexion between them. Something, if it happen, that is, the happening of something, makes the happening of another something probable; or is one of the things which render the happening of the second thing probable.

A	is	B
The happening of his arrival to-morrow	} is {	an event from which it may be inferred as probable that he will stay till Monday.

The forms of language will allow the manner of asserting to be varied in a great number of ways; but the reduction to the preceding form is always possible. Thus, 'so he said' is an affirmation, reducible as follows:

What you have just said (or whatever else 'so' refers to)	} is {	the thing which he said.
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By changing 'is' into 'is not,' we make a negative proposition;

but care must always be taken to ascertain whether a proposition which appears negative is really so. The principal danger is that of confounding a proposition which is negative with another which is affirmative of something requiring a negative to describe it. Thus 'he resembles the man who was not in the room,' is affirmative, and must not be confounded with 'he does not resemble the man who was in the room.' Again, 'if he should come to-morrow, it is probable he will not stay till Monday,' does not mean the simple denial of the preceding proposition, but the affirmation of the directly opposite proposition. It is,

A	is	B
The happening of his arrival to-morrow,	is	{ an event from which it may be inferred to be <i>improbable</i> that he will stay till Monday :

whereas the following,

The happening of his arrival to-morrow,	is <i>not</i>	{ an event from which it may be inferred as probable that he will stay till Monday,
--	---------------	---

would be expressed thus : 'If he should come to-morrow, that is no reason why he should stay till Monday.'

Moreover, the negative words not, no, &c., have two kinds of meaning which must be carefully distinguished. Sometimes they deny, and nothing more : sometimes they are used to affirm the direct contrary. In cases which offer but two alternatives, one of which is necessary, these amount to the same thing, since the denial of one, and the affirmation of the other, are obviously equivalent propositions. In many idioms of conversation, the negative implies affirmation of the contrary in cases which offer not only alternatives, but degrees of alternatives. Thus, to the question, 'Is he tall?' the simple answer, 'No,' most frequently means that he is the contrary of tall, or considerably under the average. But it must be remembered, that, in all logical reasoning, the negation is simply negation, and nothing more, never implying affirmation of the contrary.

The common proposition that two negatives make an affirmative, is

true only upon the supposition that there are but two possible things, one of which is denied. Grant that a man must be either able or unable to do a particular thing, and then *not unable* and able are the same things. But if we suppose various degrees of performance, and therefore degrees of ability, it is false, in the common sense of the words, that two negatives make an affirmative. Thus, it would be erroneous to say, 'John is able to translate Virgil, and Thomas is not unable; therefore, what John can do Thomas can do,' for it is evident that the premises mean that John is so near to the best sort of translation that an affirmation of his ability may be made, while Thomas is considerably lower than John, but not so near to absolute deficiency that his ability may be altogether denied. It will generally be found that two negatives imply an affirmative of a weaker degree than the positive affirmation.

Each of the propositions, 'A is B,' and 'A is not B,' may be subdivided into two species: the *universal*, in which every possible case is included; and the *particular*, in which it is not meant to be asserted that the affirmation or negation is universal. The four species of propositions are then as follows, each being marked with the letter by which writers on logic have always distinguished it.

A	<i>Universal Affirmative</i>	Every A is	B
E	<i>Universal Negative</i>	No A is	B
I	<i>Particular Affirmative</i>	Some A is	B
O	<i>Particular Negative</i>	Some A is not	B

In common conversation the affirmation of a part is meant to imply the denial of the remainder. Thus, by 'some of the apples are ripe,' it is always intended to signify that some are not ripe. This is not the case in logical language, but every proposition is intended to make its amount of affirmation or denial, and no more. When we say, 'Some A is B,' or, more grammatically, 'Some As are Bs,' we do not mean to imply that some are not: this may or may not be. Again, the word *some* means, 'one or more, possibly all.' The following table will shew the bearing of each proposition on the rest.

Every A is B affirms and contains *Some A is B* and denies $\begin{cases} \text{No } A \text{ is } B \\ \text{Some } A \text{ is not } B \end{cases}$

No A is B affirms and contains *Some A is not B* and denies $\begin{cases} \text{Every } A \text{ is } B \\ \text{Some } A \text{ is } B \end{cases}$

Some A is B does not contradict $\begin{cases} \text{Every } A \text{ is } B \\ \text{Some } A \text{ is not } B \end{cases}$ but denies *No A is B*

Some A is not B does not contradict $\begin{cases} \text{No } A \text{ is } B \\ \text{Some } A \text{ is } B \end{cases}$ but denies *Every A is B*

Contradictory propositions are those in which one denies *any thing* that the other affirms; *contrary* propositions are those in which one denies *every thing* which the other affirms, or affirms *every thing* which the other denies. The following pair are contraries,

Every A is B and No A is B

and the following are contradictories,

Every A is B	to	Some A is not B
No A is B	to	Some A is B

A contrary, therefore, is a complete and total contradictory; and a little consideration will make it appear that the decisive distinction between contraries and contradictories lies in this, that contraries may both be false, but of contradictories, one must be true and the other false. We may say, 'Either P is true, or *something* in contradiction of it is true;' but we cannot say, 'Either P is true, or *every thing* in contradiction of it is true.' It is a very common mistake to imagine that the *denial* of a proposition gives a right to *affirm* the contrary; whereas it should be, that the *affirmation* of a proposition gives a right to *deny* the contrary. Thus, if we deny that Every A is B, we do not affirm that No A is B, but only that Some A is not B; while, if we affirm that Every A is B, we deny No A is B, and also Some A is not B.

But, as to contradictories, affirmation of one is a denial of the other, and denial of one is affirmation of the other. Thus, either Every A is B, or Some A is not B: affirmation of either is denial of the other, and *vice versâ*.

Let the student now endeavour to satisfy himself of the following. Taking the four preceding propositions, A, E, I, O, let the simple letter

signify the affirmation, the same letter in parentheses the denial, and the absence of the letter, that there is neither affirmation nor denial.

From A follow (E), I, (O)	From (A) follow O
From E (A), (I), O	From (E) I
From I (E)	From (I) (A), E, O
From O (A)	From (O) A, (E), I

These may be thus summed up : The affirmation of a universal proposition, and the denial of a particular one, enable us to affirm or deny all the other three ; but the denial of a universal proposition, and the affirmation of a particular one, leave us unable to affirm or deny two of the others.

In such propositions as ' Every A is B,' ' Some A is not B,' &c., A is called the *subject*, and B the *predicate*, while the verb ' is ' or ' is not,' is called the *copula*. It is obvious that the words of the proposition point out whether the subject is spoken of universally or partially, but not so of the predicate, which it is therefore important to examine. Logical writers generally give the name of *distributed* subjects or predicates to those which are spoken of universally ; but as this word is rather technical, I shall say that a subject or predicate enters wholly or partially, according as it is universally or particularly spoken of.

1. In A, or ' Every A is B,' the subject enters wholly, but the predicate only partially. For it obviously says, ' Among the Bs are all the As,' ' Every A is part of the collection of Bs, so that all the As make a part of the Bs, the whole it *may* be.' Thus, ' Every horse is an animal,' does not speak of all animals, but states that all the horses make up a portion of the animals.

2. In E, or ' No A is B,' both subject and predicate enter wholly. ' No A whatsoever is any one out of all the Bs,' ' search the whole collection of Bs, and *every* B shall be found to be something which is not A.'

3. In I, or ' Some A is B,' both subject and predicate enter partially. ' Some of the As are found among the Bs, or make up a part (the whole possibly, but not known from the preceding) of the Bs.'

4. In O, or 'Some A is not B,' the subject enters partially, and the predicate wholly. 'Some As are none of them any whatsoever of the Bs; every B will be found to be no one out of a certain portion of the As.'

It appears then that,

In affirmatives, the predicate enters partially.

In negatives, the predicate enters wholly.

In contradictory propositions, both subject and predicate enter differently in the two.

The *converse* of a proposition is that which is made by interchanging the subject and predicate, as follows :

The proposition.	Its converse.
A Every A is B	Every B is A
E No A is B	No B is A
I Some A is B	Some B is A
O Some A is not B	Some B is not A

Now, it is a fundamental and self-evident proposition, that no consequence must be allowed to assert more widely than its premises ; so that, for instance, an assertion which is only of some Bs can never lead to a result which is true of all Bs. But if a proposition assert agreement or disagreement, any other proposition which asserts the same, to the same extent and no further, must be a legitimate consequence ; or, if you please, must amount to the whole, or part, of the original assertion in another form. Thus, the converse of A is not true : for, in 'Every A is B,' the predicate enters partially ; while in 'Every B is A,' the subject enters wholly. 'All the As make up a part of the Bs, then a part of the Bs are among the As, or some B is A.' Hence, the only *legitimate* converse of 'Every A is B' is, 'Some B is A.' But in 'No A is B,' both subject and predicate enter wholly, and 'No B is A' is, in fact, the same proposition as 'No A is B.' And 'Some A is B' is also the same as its converse 'Some B is A : ' here both terms enter partially. But 'Some A is not B' admits of no converse whatever ; it is perfectly consistent with all assertions upon B

and A in which B is the subject. Thus neither of the four following lines is inconsistent with itself.

Some A is not B and Every B is A
 Some A is not B and No B is A
 Some A is not B and Some B is A
 Some A is not B and Some B is not A.

We find then, including converses, which are not identical with their direct propositions, *six* different ways of asserting or denying, with respect to agreement or non-agreement, total or partial, between A and, say X: these we write down, designating the additional assertions by U and Y.

		Identical.		Identical.	
A Every A is X	E	{ No A is X }	I	{ Some A is X }	O Some A is not X
U Every X is A					{ No X is A }

We shall now repeat and extend the table of page 8 (A), &c., meaning, as before, the denial of A, &c.

From A or (O) follow A, (E), I (O)
 From E or (I) (A), E, (I), O, (U), Y
 From I or (E) (E) I
 From O or (A) (A), O
 From U or (Y) (E) I, U (Y)
 From Y or (U) (U) Y

Having thus discussed the principal points connected with the simple assertion, we pass to the manner of making two assertions give a third. Every instance of this is called a syllogism, the two assertions which form the basis of the third are called premises, and the third itself the conclusion.

If two things both agree with a third in any particular, they agree with each other in the same; as, if A be of the same colour as X, and B of the same colour as X, then A is of the same colour as B. Again, if A differ from X in any particular in which B agrees with X, then A and B differ in that particular. If A be not of the same colour as X,

and B be of the same colour as X, then A is not of the colour of B. But if A and B both differ from X in any particular, nothing can be inferred; they may either differ in the same way and to the same extent, or not. Thus, if A and B be both of different colours from X, it neither follows that they agree, nor differ, in their own colours.

The paragraph preceding contains the essential parts of all inference, which consists in comparing two things with a third, and finding from their agreement or difference with that third, their agreement or difference with one another. Thus, Every A is X, every B is X, allows us to infer that A and B have all those qualities in common which are necessary to X. Again, from Every A is X, and 'No B is X,' we infer that A and B differ from one another in all particulars which are essential to X. The preceding forms, however, though they represent common reasoning better than the ordinary syllogism, to which we are now coming, do not constitute the ultimate forms of inference. Simple *identity* or *non-identity* is the ultimate state to which every assertion may be reduced; and we shall, therefore, first ask, from what identities, &c., can other identities, &c., be produced? Again, since we name objects in species, each species consisting of a number of individuals, and since our assertion may include all or only part of a species, it is further necessary to ask, in every instance, to what extent the conclusion drawn is true, whether of all, or only of part?

Let us take the simple assertion, 'Every living man respires;' or every living man is one of the things (however varied they may be) which respire. If we were to inclose all living men in a large triangle, and all respiring objects in a large circle, the preceding assertion, if true, would require that the whole of the triangle should be contained in the circle. And in the same way we may reduce any assertion to the expression of a coincidence, total or partial, between two figures. Thus, a point in a circle may represent an individual of one species, and a point in a triangle an individual of another species: and we may express that the whole of one species is asserted to be contained or not contained in the other by such forms as, 'All the \triangle is in the \circ ;' 'None of the \triangle is in the \circ '.

Any two assertions about A and B, each expressing agreement or disagreement, total or partial, with or from X, and leading to a conclusion with respect to A or B, is called a syllogism, of which X is called the *middle term*. The plainest syllogism is the following:—

Every A is X	All the Δ is in the \bigcirc
Every X is B	All the \bigcirc is in the \square
Therefore Every A is B	Therefore All the Δ is in the \square

In order to find all the possible forms of syllogism, we must make a table of all the elements of which they can consist; namely—

A and X		B and X
Every A is X	A	Every B is X
No A is X	E	No B is X
Some A is X	I	Some B is X
Some A is not X	O	Some B is not X
Every X is A	U	Every X is B
Some X is not A	Y	Some X is not B

Or their synonyms,

Δ and \bigcirc		\square and \bigcirc
All the Δ is in the \bigcirc	A	All the \square is in the \bigcirc
None of the Δ is in the \bigcirc	E	None of the \square is in the \bigcirc
Some of the Δ is in the \bigcirc	I	Some of the \square is in the \bigcirc
Some of the Δ is not in the \bigcirc	O	Some of the \square is not in the \bigcirc
All the \bigcirc is in the Δ	U	All the \bigcirc is in the \square
Some of the \bigcirc is not in the Δ	Y	Some of the \bigcirc is not in the \square

Now, taking any one of the six relations between A and X, and combining it with either of those between B and X, we have six pairs of premises, and the same number repeated for every different relation of A and X. We have then thirty-six forms to consider: but, thirty of these (namely, all but (A, A) (E, E), &c.) are half of them repetitions of the other half. Thus, 'Every A is X, no B is X,' and 'Every B is X, no A is X,' are of the same form, and only differ by changing A into B and B into A. There are then only 15 + 6, or 21 distinct

forms, some of which give a necessary conclusion, while others do not. We shall select the former of these, classifying them by their conclusions ; that is, according as the inference is of the form A, E, I, or O.

I. In what manner can a universal affirmative conclusion be drawn ; namely, that one figure is entirely contained in the other ? This we can only assert when we know that one figure is entirely contained in the circle, which itself is entirely contained in the other figure. Thus,

Every A is X		All the \triangle is in the \bigcirc	A
Every X is B		All the \bigcirc is in the \square	A
\therefore Every A is B		\therefore All the \triangle is in the \square	A

is the only way in which a universal affirmative conclusion can be drawn.

II. In what manner can a universal negative conclusion be drawn ; namely, that one figure is entirely exterior to the other ? Only when we are able to assert that one figure is entirely within, and the other entirely without, the circle. Thus,

Every A is X		All the \triangle is in the \bigcirc	A
No B is X		None of the \square is in the \bigcirc	E
\therefore No A is B		None of the \triangle is in the \square	E

is the only way in which a universal negative conclusion can be drawn.

III. In what manner can a particular affirmative conclusion be drawn ; namely, that part or all of one figure is contained in the other ? Only when we are able to assert that the whole circle is part of one of the figures, and that the whole, or part of the circle, is part of the other figure. We have then two forms.

Every X is A		All the \bigcirc is in the \triangle	A
Every X is B		All the \bigcirc is in the \square	A
\therefore Some A is B		\therefore Some of the \triangle is in the \square	I
Every X is A		All the \bigcirc is in the \triangle	A
Some X is B		Some of the \bigcirc is in the \square	I
Some A is B		Some of the \triangle is in the \square	I

B

The second of these contains all that is strictly necessary to the conclusion, and the first may be omitted. That which follows when an assertion can be made as to some, must follow when the same assertion can be made of all.

IV. How can a particular negative proposition be inferred; namely, that part, or all of one figure, is not contained in the other? It would seem at first sight, whenever we are able to assert that part or all of one figure is in the circle, and that part or all of the other figure is not. The weakest syllogism from which such an inference can be drawn would then seem to be as follows.

Some A is X		Some of the Δ is in the \bigcirc	
Some B is not X		Some of the \square is not in the \bigcirc	
\therefore Some B is not A		\therefore Some of the Δ is not in the \square	

But here it will appear, on a little consideration, that the conclusion is only thus far true; that those As which are Xs cannot be *those* Bs which are not Xs; but they may be *other* Bs, about which nothing is asserted when we say that *some* Bs are not Xs. And further consideration will make it evident, that a conclusion of this form can only be arrived at when one of the figures is entirely within the circle, and the whole or part of the other without; or else when the whole of one of the figures is without the circle, and the whole or part of the other within; or lastly, when the circle lies entirely within one of the figures, and not entirely within the other. That is, the following are the distinct forms which allow of a particular negative conclusion, in which it should be remembered that a particular proposition in the premises may always be changed into a universal one, without affecting the conclusion. For that which necessarily follows from "some," follows from "all."

Every A is X		All the Δ is in the \bigcirc	A
Some B is not X		Some of the \square is not in the \bigcirc	O
\therefore Some B is not A		Some of the \square is not in the Δ	O

No A is X	None of the Δ is in the \bigcirc	E
Some B is X	Some of the \square is in the \bigcirc	I
\therefore Some B is not A	Some of the \square is not in the Δ	O
Every X is A	All the \bigcirc is in the Δ	A
Some X is not B	Some of the \bigcirc is not in the \square	O
\therefore Some A is not B	Some of the Δ is not in the \square	O

It appears, then, that there are but six distinct syllogisms. All others are made from them by strengthening one of the premises, or converting one or both of the premises, where such conversion is allowable; or else by first making the conversion, and then strengthening one of the premises. And the following arrangement will shew that two of them are universal, three of the others being derived from them by weakening one of the premises in a manner which does not destroy, but only weakens, the conclusion.

1. Every A is X Every X is B <hr/> Every A is B	3. Every A is X No B is X <hr/> No A is B
2. Some A is X Every X is B <hr/> Some A is B	4. Some A is X No B is X <hr/> Some A is not B	5. Every A is X Some B is not X <hr/> Some B is not A
		6. Every X is A Some X is not B <hr/> Some A is not B

We may see how it arises that one of the partial syllogisms is not immediately derived, like the others, from a universal one. In the preceding, AEE may be considered as derived from AAA, by changing the term in which X enters universally into its contrary. If this be done with the other term instead, we have

No A is X } from which universal premises we cannot deduce a
Every X is B } universal conclusion, but only Some B is not A.

If we weaken one and the other of these premises, as they stand, we obtain

Some A is not X	No A is X
Every X is B	Some X is B
<hr/> No conclusion	<hr/> Some B is not A

equivalent to the fourth of the preceding: but if we convert the first premiss, and proceed in the same manner,

	No	X is A		Some	X is not A
From	Every	X is B	we obtain	Every	X is B
	Some	B is not A		Some	B is not A

which is legitimate, and is the same as the last of the preceding list, with A and B interchanged.

Before proceeding to shew that all the usual forms are contained in the preceding, let the reader remark the following rules, which may be proved either by collecting them from the preceding cases, or by independent reasoning.

1. The middle term must enter universally into one or the other premiss. If it were not so, the one premiss might speak of one part of the middle term, and the other of the other; so that there would, in fact, be no middle term. Thus, 'Every A is X, Every B is X,' gives no conclusion: it may be thus stated;

All the As make up a *part* of the Xs

All the Bs make up a *part* of the Xs

And, before we can know that there is any common term of comparison at all, we must have some means of shewing that the two parts are the same; or the preceding premises by themselves are inconclusive.

2. No term must enter the conclusion more generally than it is found in the premises; thus, if A be spoken of partially in the premises, it must enter partially into the conclusion. This is obvious, since the conclusion must assert no more than the premises imply.

3. From premises both negative no conclusion can be drawn. For it is obvious, that the mere assertion of disagreement between each of two things and a third, can be no reason for inferring either agreement or disagreement between these two things. It will not be difficult to reduce any case which falls under this rule to a breach of the first rule: thus, No A is X, No B is X, gives

Every A is (something which is not X)

Every B is (something which is not X)

in which the middle term is not spoken of universally in either. Again, 'No X is A, Some X is not B,' may be converted into

Every A is (a thing which is not X)

Some (thing which is not B) is X

in which there is no middle term.

4. From premises both particular no conclusion can be drawn. This is sufficiently obvious when the first or second rule is broken, as in 'Some A is X, Some B is X.' But it is not immediately obvious when the middle term enters one of the premises universally. The following reasoning will serve for exercise in the preceding results. Since both premises are particular in form, the middle term can only enter one of them universally by being the predicate of a negative proposition; consequently (Rule 3) the other premiss must be affirmative, and, being particular, neither of its terms is universal. Consequently both the terms as to which the conclusion is to be drawn enter partially, and the conclusion (Rule 2) can only be a particular *affirmative* proposition. But if one of the premises be negative, the conclusion must be *negative* (as we shall immediately see). This contradiction shews that the supposition of particular premises producing a legitimate result is inadmissible.

5. If one premiss be negative, the conclusion, if any, must be negative. If one term agree with a second and disagree with a third, no agreement can be inferred between the second and third.

6. If one premiss be particular, the conclusion must be particular. This is not very obvious, since the middle term may be universally spoken of in a particular proposition, as in Some B is not X. But this requires one negative proposition, whence (Rule 3) the other must be affirmative. Again, since the conclusion must be negative (Rule 5) its predicate is spoken of universally, and, therefore, must enter universally; the other term A must enter, then, in a universal affirmative proposition, which is against the supposition.

In the preceding set of syllogisms we observe one form only which produces A, or E, or I, but three which produce O.

Let an assertion be said to be weakened when it is reduced from universal to particular, and strengthened in the contrary case. Thus, 'Every A is B' is called stronger than 'Some A is B.'

Every form of syllogism which can give a legitimate result is either one of the preceding six, or another formed from one of the six, either by changing one of the assertions into its converse, if that be allowable, or by strengthening one of the premises without altering the conclusion, or both. Thus,

$$\left. \begin{array}{l} \text{Some A is X} \\ \text{Every X is B} \end{array} \right\} \text{ may be written } \left\{ \begin{array}{l} \text{Some X is A} \\ \text{Every X is B} \end{array} \right.$$

$$\text{What follows will still follow from } \left\{ \begin{array}{l} \text{Every X is A} \\ \text{Every X is B} \end{array} \right.$$

for all which is true when 'Some X is A,' is not less true when 'Every X is A.'

It would be possible also to form a legitimate syllogism by weakening the conclusion, when it is universal, since that which is true of all is true of some. Thus, 'Every A is X, Every X is B,' which yields 'Every A is B,' also yields 'Some A is B.' But writers on logic have always considered these syllogisms as useless, conceiving it better to draw from any premises their strongest conclusion. In this they were undoubtedly right; and the only question is, whether it would not have been advisable to make the premises as weak as possible, and not to admit any syllogisms in which more appeared than was absolutely necessary to the conclusion. If such had been the practice, then

Every X is A, Every X is B, therefore Some A is B

would have been considered as formed by a spurious and unnecessary excess of assertion. The minimum of assertion would be contained in either of the following,

Every X is A, Some X is B, therefore Some A is B
 Some X is A, Every X is B, therefore Some A is B

In this tract, syllogisms have been divided into two classes: first,

those which prove a universal conclusion ; secondly, those which prove a partial conclusion, and which are (all but one) derived from the first by weakening one of the premises, in such manner as to produce a legitimate but weakened conclusion. Those of the first class are placed in the first column, and the other in the second.

Universal.		Particular.	
A	Every A is X	Some A is X	I
A	<u>Every X is B</u>	<u>Every X is B</u>	A
A	Every A is B	Some A is B	I
		Some A is X	I
		No X is B	E
A	Every A is X	Some A is not B	O
E	<u>No X is B</u>	Every A is X	A
E	No A is B	Some B is not X	O
		Some B is not A	O
		<u>Every X is A</u>	A
		<u>Some X is not B</u>	O
		Some A is not B	O

In all works on logic, it is customary to write that premiss first which contains the predicate of the conclusion. Thus,

Every X is B		Every A is X
Every A is X	would be written, and not	Every X is B
<u>Every A is B</u>		<u>Every A is B</u>

The premises thus arranged are called major and minor ; the predicate of the conclusion being called the major term, and its subject the minor. Again, in the preceding case we see the various subjects coming in the order X, B ; A, X ; A, B : and the number of different orders which can appear is four, namely —

X B	B X	X B	B X
A X	A X	X A	X A
<u>A B</u>	<u>A B</u>	<u>A B</u>	<u>A B</u>

which are called the four figures, and every kind of syllogism in each figure is called a mood. I now put down the various moods of each figure, the letters of which will be a guide to find out those of the preceding list from which they are derived. Co means that a premiss of the preceding list has been converted; + that it has been strengthened; Co +, that both changes have taken place. Thus,

<i>Barbati</i>	A Every X is B		A Every X is B	
	I <u>Some A is X</u>	becomes	A <u>Every X is A : (Co +)</u>	
	I Some A is B		I Some A is B	<i>Barbati</i>

And Co + abbreviates the following: If some A be X, then some X is A (Co); and all that is true when Some X is A, is true when Every X is A (+); therefore the second is legitimate, if the first be so.

First Figure.

A Every X is B	A Every X is B
A <u>Every A is X</u>	I <u>Some A is X</u>
A Every A is B	I Some A is B
E No X is B	E No X is B
A <u>Every A is X</u>	I <u>Some A is X</u>
E No A is B	O Some A is not B

Second Figure.

E No B is X (Co)	E No B is X (Co)
A <u>Every A is X</u>	I <u>Some A is X</u>
E No A is B	O Some A is not B
A Every B is X	A Every B is X
E <u>No A is X (Co)</u>	O <u>Some A is not X</u>
E No A is B	O Some A is not B

Third Figure.

A	Every X is B	E	No X is B
A	<u>Every X is A (Co +)</u>	A	<u>Every X is A (Co +)</u>
I	Some A is B	O	Some A is not B
I	Some X is B (Co)	O	Some X is not B
A	<u>Every X is A</u>	A	<u>Every X is A</u>
I	Some A is B	O	Some A is not B
A	Every X is B	E	No X is B
I	<u>Some X is A (Co)</u>	I	<u>Some X is A (Co)</u>
I	Some A is B	O	Some A is not B

Fourth Figure.

A	Every B is X (+)	I	Some B is X
A	<u>Every X is A</u>	A	<u>Every X is A</u>
I	Some A is B	I	Some B is A
A	Every B is X	E	No B is X (Co)
E	<u>No X is A</u>	A	<u>Every X is A (Co +)</u>
E	No A is B	O	Some A is not B
E	No B is X (Co)		
I	<u>Some X is A (Co)</u>		
O	Some A is not B		

The above is the ancient method of dividing syllogisms ; but, for the present purpose, it will be sufficient to consider the six from which the rest can be obtained. And since some of the six have A in the predicate of the conclusion, and not B, we shall join to them the six other syllogisms which are found by transposing B and A. The complete list, therefore, of syllogisms with the weakest premises and the strongest conclusions, in which a comparison of A and B is obtained by comparison of both with X, is as follows :

Every A is X	Every B is X	Some A is X	Some B is X
Every X is B	Every X is A	No X is B	No X is A
Every A is B	Every B is A	Some A is not B	Some B is not A
Every A is X	Every B is X	Every A is X	Every B is X
No X is B	No X is A	Some B is not X	Some A is not X
No A is B	No B is A	Some B is not A	Some A is not B
Some A is X	Some B is X	Every X is A	Every X is B
Every X is B	Every X is A	Some X is not B	Some X is not A
Some A is B	Some B is A	Some A is not B	Some B is not A

In the list of page 19, there was nothing but recapitulation of forms, each form admitting a variation by interchanging A and B. This interchange having been made, and the results collected as above, if we take every case in which B is the predicate, or can be made the predicate by allowable conversion, we have a collection of all possible *weakest* forms in which the result is one of the four 'Every A is B,' 'No A is B,' 'Some A is B,' 'Some A is not B;' as follows. The premises are written in what appeared the most natural order, without distinction of major or minor; and the letters prefixed are according to the forms of the several premises, as in page 10.

A Every A is X					
U Every X is B					
A Every A is B					
I Some A is X			I Some B is X		
U Every X is B			U Every X is A		
I Some A is B			I Some A is B		
A Every A is X			A Every B is X		
E No B is X			E No A is X		
E No A is B			E No A is B		
I Some A is X	A Every B is X		U Every X is A		
E No B is X	O Some A is not X		Y Some X is not B		
O Some A is not B	O Some A is not B		O Some A is not B		

Every assertion which can be made upon two things by comparison with any third, that is, every simple inference, can be reduced to one of the preceding forms. Generally speaking, one of the premises is omitted, as obvious from the conclusion; that is, one premiss being named and the conclusion, that premiss is implied which is necessary to make the conclusion good. Thus, if I say, "That race must have possessed some of the arts of life, for they came from Asia," it is obviously meant to be asserted, that all races coming from Asia must have possessed some of the arts of life. The preceding is then a syllogism, as follows:

'That race' is 'a race of Asiatic origin:'

Every 'race of Asiatic origin' is 'a race which must
have possessed some of the arts of life:'

Therefore, That race is a race which must have possessed
some of the arts of life.

A person who makes the preceding assertion either means to imply, antecedently to the conclusion, that all Asiatic races must have possessed arts, or he talks nonsense if he asserts the conclusion positively. 'A must be B, for it is X,' can only be true when 'Every X is B.' This latter proposition may be called the suppressed premiss; and it is in such suppressed propositions that the greatest danger of error lies. It is also in such propositions that men convey opinions which they would not willingly express. Thus, the honest witness who said, 'I always thought him a respectable man—he kept his gig,' would probably not have admitted in direct terms, 'Every man who keeps a gig must be respectable.'

I shall now give a few detached illustrations of what precedes.

"His imbecility of character might have been inferred from his proneness to favourites; for all weak princes have this failing." The preceding would stand very well in a history, and many would pass it over as containing very good inference. Written, however, in the form of a syllogism, it is,

	All weak princes are prone to favourites	
	He	was prone to favourites
	<hr/>	
Therefore	He	was a weak prince

which is palpably wrong. (Rule 1.) The writer of such a sentence as the preceding might have meant to say, 'for all who have this failing are weak princes;' in which case he would have inferred rightly. Every one should be aware that there is much false inference arising out of badness of style, which is just as injurious to the habits of the untrained reader as if the errors were mistakes of logic in the mind of the writer.

'A is less than B; B is less than C: therefore A is less than C.' This, at first sight, appears to be a syllogism; but, on reducing it to the usual form, we find it to be,

	A is (a magnitude less than B)
	B is (a magnitude less than C)
Therefore	A is (a magnitude less than C)

which is not a syllogism, since there is no middle term. Evident as the preceding is, the following additional proposition must be formed before it can be made explicitly logical. 'If B be a magnitude less than C, then every magnitude less than B is also less than C.' There is, then, before the preceding can be reduced to a syllogistic form, the necessity of a deduction from the second premiss, and the substitution of the result instead of that premiss. Thus,

	A is less than B
	Less than B is less than C: following from B is less than C.
	<hr/>
Therefore	A is less than C

But, if the additional argument be examined — namely, if B be less than C, then that which is less than B is less than C — it will be found to require precisely the same considerations repeated; for the original inference was nothing more. In fact, it may easily be seen as follows, that the proposition before us involves more than any simple syllogism

can express. When we say that A is less than B, we say that if A were applied to B, every part of A would match a part of B, and there would be parts of B remaining over. But when we say, 'Every A is B,' meaning the premiss of a common syllogism, we say that every instance of A is an instance of B, without saying any thing as to whether there are or are not instances of B still left, after those which are also A are taken away. If, then, we wish to write an ordinary syllogism in a manner which shall correspond with 'A is less than B, B is less than C, therefore A is less than C,' we must introduce a more definite amount of assertion than was made in the preceding forms. Thus,

Every A is B, and there are Bs which are not As
 Every B is C, and there are Cs which are not Bs

Therefore Every A is C, and there are Cs which are not As

Or thus :

The Bs contain all the As, and more
 The Cs contain all the Bs, and more

The Cs contain all the As, and more

The most technical form, however, is,

From Every A is B ; [Some B is not A]
 Every B is C ; [Some C is not B]
 Follows Every A is C ; [Some C is not A]

This sort of argument is called *à fortiori* argument, because the premisses are more than sufficient to prove the conclusion, and the extent of the conclusion is thereby greater than its mere form would indicate. Thus, 'A is less than B, B is less than C, therefore, *à fortiori*, A is less than C,' means that the extent to which A is less than C must be greater than that to which A is less than B, or B than C. In the syllogism last written, either of the bracketted premisses might be struck out without destroying the conclusion ; which last would, however, be weakened. As it stands, then, the part of the conclusion, 'Some C is not A,' follows *à fortiori*.

The argument *à fortiori* may then be defined as a universally

c

affirmative syllogism, in which both of the premises are shewn to be less than the whole truth, or greater. Thus, in 'Every A is X, Every X is B, therefore Every A is B,' we do not certainly imply that there are more Xs than As, or more Bs than Xs, so that we do not know that there are more Bs than As. But if we are at liberty to state the syllogism as follows,

All the As make up part (and part only) of the Xs
Every X is B ;

then we are certain that

All the As make up part (and part only) of the Bs.

But if we are at liberty further to say that

All the As make up part (and part only) of the Xs
All the Xs make up part (and part only) of the Bs

then we conclude that

All the As make up *part of part* (only) of the Bs

and the words in *Italics* mark that quality of the conclusion from which the argument is called *à fortiori*.

Most syllogisms which give an affirmative conclusion are generally meant to imply *à fortiori* arguments, except only in mathematics. It is seldom, except in the exact sciences, that we meet with a proposition, 'Every A is B,' which we cannot immediately couple with 'Some Bs are not As.'

When an argument is completely established, with the exception of one assertion only, so that the inference may be drawn as soon as that one assertion is established, the result is stated in a form which bears the name of an *hypothetical* syllogism. The word hypothesis means nothing but supposition ; and the species of syllogism just mentioned first lays down the assertion that a consequence will be true if a certain condition be fulfilled, and then either asserts the fulfilment of the condition, and thence the consequence, or else denies the consequence, and thence denies the fulfilment of the condition. Thus, if we know that

When A is B, it follows that P is Q ;

then, as soon as we can ascertain that A is B, we can conclude that P is Q; or, if we can shew that P is not Q, we know that A is not B. But if we find that A is not B, we can infer nothing; for the preceding does not assert that P is Q *only* when A is B. And if we find out that P is Q, we can infer nothing. This conditional syllogism may be converted into an ordinary syllogism, as follows. Let K be any 'case in which A is B,' and Z a 'case in which P is Q;' then the preceding assertion amounts to 'Every K is Z.' Let L be a particular instance, the A of which may or may not be B. If A be B in the instance under discussion, or if A be not B, we have, in the one case and the other,

	Every K is Z	Every K is Z
	<u>L is a K</u>	<u>L is not a K</u>
Therefore	L is a Z	No conclusion

Similarly, according as a particular case (M) is or is not Z, we have

Every K is Z	Every K is Z
<u>M is a Z</u>	<u>M is not a Z</u>
No conclusion	M is not a K

That is to say: The assertion of an hypothesis is the assertion of its necessary consequence, and the denial of the necessary consequence is the denial of the hypothesis; but the assertion of the necessary consequence gives no right to assert the hypothesis, nor does the denial of the hypothesis give any right to deny the truth of that which would (were the hypothesis true) be its necessary consequence.

Demonstration is of two kinds: which arises from this, that every proposition has a contradictory; and of these two, one must be true and the other must be false. We may then either prove a proposition to be true, or its contradictory to be false. 'It is true that Every A is B,' and, 'it is false that there are some As which are not Bs,' are the same proposition; and the proof of either is called the indirect proof of the other.

But how is any proposition to be proved false, except by proving a

contradiction to be true? By proving a necessary consequence of the proposition to be false. But this is not a complete answer, since it involves the necessity of doing the same thing; or, so far as this answer goes, one proposition cannot be proved false unless by proving another to be false. But it may happen, that a necessary consequence can be obtained which is obviously and self-evidently false, in which case no further proof of the falsehood of the hypothesis is necessary. Thus the proof which Euclid gives that all equiangular triangles are equilateral is of the following structure, logically considered.

(1.) If there be an equiangular triangle not equilateral, it follows that a whole can be found which is not greater than its part.*

(2.) It is false that there can be any whole which is not greater than its part (self evident).

(3.) Therefore it is false that there is any equiangular triangle which is not equilateral; or all equiangular triangles are equilateral.

When a proposition is established by proving the truth of the matters it contains, the demonstration is called *direct*; when by proving the falsehood of every contradictory proposition, it is called *indirect*. The latter species of demonstration is as logical as the former, but not of so simple a kind; whence it is desirable to use the former whenever it can be obtained.

The use of indirect demonstration in the Elements of Euclid is almost entirely confined to those propositions in which the converses of simple propositions are proved. It frequently happens that an established assertion of the form

Every A is B (1)

may be easily made the means of deducing,

Every (thing not A) is not B ... (2)

which last gives

Every B is A (3)

* This is the proposition in proof of which nearly the whole of the demonstration of Euclid is spent.

The conversion of the second proposition into the third is usually made by an indirect demonstration, in the following manner. If possible, let there be one B which is not A, (2) being true. Then there is one thing which is not A and is B; but every thing not A is not B; therefore there is one thing which is B and is not B: which is absurd. It is then absurd that there should be one single B which is not A; or, Every B is A.

The following proposition contains a method which is of frequent use.

HYPOTHESIS.—Let there be any number of propositions or assertions,—three for instance, A, B, and C,—of which it is the property that one or the other must be true, *and one only*. Let there be three other propositions, P, Q, and R, of which it is also the property that one, and one only, must be true. Let it also be a connexion of those assertions, that

When A is true, P is true

When B is true, Q is true

When C is true, R is true

CONSEQUENCE: then it follows that

When P is true, A is true

When Q is true, B is true

When R is true, C is true

For, when P is true, then Q and R must be false; consequently, neither B nor C can be true, for then Q or R would be true. But either A, B, or C must be true, therefore A must be true; or, when P is true, A is true. In a similar way the remaining assertions may be proved.

Case 1. If When P is Q, A is B

When P is not Q, A is not B

It follows that

When A is B, P is Q

When A is not B, P is not Q

Case 2. If $\left\{ \begin{array}{ll} \text{When A is greater than B, P is greater than Q} \\ \text{When A is equal to B, P is equal to Q} \\ \text{When A is less than B, P is less than Q} \end{array} \right.$

It follows that	{	When P is greater than Q, A is greater than B	
		When P is equal to Q, A is equal to B	
		When P is less than Q, A is less than B	

We have hitherto supposed that the premises are actually true; and, in such a case, the logical conclusion is as certain as the premises. It remains to say a few words upon the case in which the premises are probably, but not certainly, true.

The probability of an event being about to happen, and that of an argument being true, may be so connected that the usual method of measuring the first may be made to give an easy method of expressing the second. Suppose an urn, or lottery, with a large number of balls, black or white; then, if there be twelve white balls to one black, we say it is twelve to one that a white ball will be drawn, or that a white ball is twelve times as probable as a black one. A certain assertion may be in the same condition as to the force of probability with which it strikes the mind: that is, the questions

Is the assertion true?

Will a white ball be drawn?

may be such that the answer, 'most probably,' expresses the same degree of likelihood in both cases.

We have before explained that logic has nothing to do with the truth or falsehood of assertions, but only professes, supposing them true, to collect and classify the legitimate methods of drawing inferences. Similarly, in this part of the subject, we do not trouble ourselves with the question, How are we to find the probability due to premises? but we ask: Supposing (happen how it may) that we *have* found the probability of the premises, required the probability of the conclusion. When the odds in favour of a conclusion are, say 6 to 1, there are, out of every 7 possible chances, 6 in favour of the conclusion, and 1 against it. Hence $\frac{6}{7}$ and $\frac{1}{7}$ will represent the proportions, for and against, of all the possible cases which exist.

Thus we have the succession of such results as in the following table:—

Odds in favour of an event	Probability for	Probability against
1 to 1	$\frac{1}{2}$	$\frac{1}{2}$
2 to 1	$\frac{2}{3}$	$\frac{1}{3}$
3 to 1	$\frac{3}{4}$	$\frac{1}{4}$
3 to 2	$\frac{3}{5}$	$\frac{2}{5}$
4 to 1	$\frac{4}{5}$	$\frac{1}{5}$
4 to 3	$\frac{4}{7}$	$\frac{3}{7}$
5 to 1	$\frac{5}{6}$	$\frac{1}{6}$
&c.	&c.	&c.

Let the probability of a conclusion, as derived from the premises (that is on the supposition that it was never imagined to be possible till the argument was heard), be called the *intrinsic probability* of the argument. This is found by multiplying together the probabilities of all the assertions which are necessary to the argument. Thus, suppose that a conclusion was held to be impossible until an argument of a single syllogism was produced, the premises of which have severally five to one and eight to one in their favour. Then $\frac{5}{6} \times \frac{8}{9}$, or $\frac{40}{54}$, is the intrinsic probability of the argument, and the odds in its favour are 40 to 14, or 20 to 7.

But this intrinsic probability is not always that of the conclusion; the latter, of course, depending in some degree on the likelihood which the conclusion was supposed to have before the argument was produced. A syllogism of 20 to 7 in its favour, advanced in favour of a conclusion which was beforehand as likely as not, produces a much more probable result than if the conclusion had been thought absolutely false until the argument produced a certain belief in the possibility of its being true.

14/11/75 in may be admitted

The change made in the probability of a conclusion by the introduction of an argument (or of a new argument, if some have already preceded) is found by the following rule.

From the sum of the existing probability of the conclusion and the intrinsic probability of the new argument, take their product; the remainder is the probability of the conclusion, as reinforced by the argument. Thus, $a + b - ab$ is the probability of the truth of a conclusion after the introduction of an argument of the intrinsic probability b , the previous probability of the said conclusion having been a .

Thus, a conclusion which has at present the chance $\frac{2}{3}$ in its favour, when reinforced by an argument whose intrinsic probability is $\frac{3}{4}$, acquires the probability $\frac{2}{3} + \frac{3}{4} - \frac{2}{3} \times \frac{3}{4}$ or, $\frac{2}{3} + \frac{3}{4} - \frac{1}{2}$, or $\frac{11}{12}$; or, having 2 to 1 in its favour before, it has 11 to 1 in its favour after, the argument.

When the conclusion was neither likely nor unlikely beforehand (or had the probability $\frac{1}{2}$), the shortest way of applying the preceding rule (in which $a + b - ab$ becomes $\frac{1}{2} + \frac{1}{2}b$) is to divide the sum of the numerator and denominator of the intrinsic probability of the argument by twice the denominator. Thus, an argument of which the intrinsic probability is $\frac{3}{4}$, gives to a conclusion on which no bias previously existed, the probability $\frac{7}{8}$ or $\frac{3+4}{2 \times 4}$.

THE END.

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Barbara, Selagrent, Darie, Torigo que prioris
 Cesareq Camestres, Festinog, Baroko secundae
 Jentia Darapgi Disagmis, Datigsi, Telapton
 Bokardo, Terson, habet. Quarta usque addit
 Bramantigp, Camegnes, Dimarigs, Tegsapo, Tregison

Z Y
 Y X ✓
 X Z

Sam Zs represent Ys

~~Sam Ys represent Zs~~ by Xs

All Ys represent Xs

Some Xs represent Zs

Sam Xs are now $\frac{1}{2}$ Z

As in the
 example

Note: the above is by Mr. de Morgan - and
 the character "g" represents the case
 in which the word is formed
 by dropping many be common

STATEMENT

IN ANSWER TO AN ASSERTION

MADE BY

SIR WILLIAM HAMILTON, BART.,

Professor of Logic in the University of Edinburgh;

BY AUGUSTUS DE MORGAN, *Sec. R.A.S., F.C.P.S.,*

*Of Trinity College, Cambridge, Professor of Mathematics
in University College, London.*

IF any one should be surprised at my printing this statement, on account of the smallness of the pretext for the things alleged against me, I have only to tell him that I am as well aware of the insignificance of that pretext as he is, and that the cogent necessity, as I believe it to be, for taking immediate notice of it, arises from the unquestioned literary celebrity and private worth of a gentleman who was rash enough to tax me with conduct which, could his words be made good, ought to drive me from the society of honest men. The learning and character of the accuser, while they force this publication, make me desirous that it should contain the least possible amount of counter-charge, and cause the least possible annoyance.

This statement arises out of an assertion made by Sir William Hamilton*, Professor of Logic in the University of Edinburgh. He charges me with wilfully and knowingly (for so I interpret the words *breach of confidence* and *false dealing*) attempting to appropriate as my own certain ideas on the subject of logic, which he avers he communicated to me privately. It is true that he has desired to withdraw the letter in which these imputations were made. But this, as will be pointed out, was done in a manner which amounts only to a proposal of suspension followed by private inquiry, and that after a month's persistence in the charge. I prefer a public inquiry. To the assertion I reply, *first*, that the substance of all which I have claimed was deposited at Cambridge, and out of my hands, before I ever saw his handwriting; *secondly*, that no *intelligible* communication was ever made by him to me; *thirdly*, that so soon as I received anything which induced me to suppose that we must have had something in common, I gave him full information of what I then had by return of post, even to the means of identifying the papers on which all that

* To prevent all mistake, I must desire the reader to avoid a confusion which frequently occurs between the eminent professor of logic in the University of Edinburgh, and my friend Sir William Rowan Hamilton of Dublin, Astronomer Royal for Ireland, and so widely known as one of the first of our mathematicians.

development was written which was not already at Cambridge; *fourthly*, that I have strong subsequent reason to suppose we really have nothing in common; *fifthly*, that his mode of conducting his own case, subsequently to the production of his charge, is enough to satisfy any reasonable person that he never had any confidence in his own power of proving it.

I shall offer proof of the first, second, and third of the above points. The fourth I can so far establish as to throw upon Sir William Hamilton the onus of proving that he ever published in his lectures or otherwise any of the notions which I have advanced; indeed, he must in any case furnish this proof. The fifth is matter of opinion, which I must leave to the reader.

There is something to say which is not evidence, except to those who know me. I think I can show that I never had the *opportunity* of copying Sir William Hamilton's ideas; but, be this how it may, I declare upon my honour that I never derived a single hint of any kind from his communications. I declare also, that I remember (and what is more to the purpose, that I remembered in February, when I printed it,) that the papers which I gave the means of identifying in January, were written before I received any communication from Sir William Hamilton, except a civil note, promising to answer certain questions on the *history* of logic when he returned to Edinburgh. In any question of mere priority which may arise, these papers of course can only date, at the earliest, from the time at which I announced their existence. But as regards my own integrity in this matter, the two declarations above made will, as I said, be enough with those who know me. I will now proceed with my statement.

Early in the last summer I began to collect matters, which had suggested themselves to me at different times, connected with the theory of the syllogism in logic. My intention was, and is, to enlarge a slight treatise on that subject, which I published a few years ago, into a work on formal logic. In the course of my investigations I fell upon various matters which I thought worth communicating to the Cambridge Philosophical Society. I accordingly sent to that body a Memoir headed, 'On the Structure of the Syllogism, and on the Application of the Theory of Probabilities to questions of Argument and Authority.' That paper is now printed, and will appear in vol. viii. part 3 of the Cambridge Transactions. It is dated *October 3, 1846*. The Master of Trinity College (to whom I sent it in the first instance, and from whose hands it passed into those of the Secretary of the Society) testifies that he received it on the 6th. The manuscript, and the first proof taken from it, are in existence and can be produced; neither has any corrections worth noting. By comparison of the dates of my communications from Sir William Hamilton (the first is dated *October 7*, and the first professing to speak of logic is of *November 2*), it will be seen that I have no occasion to say any more about the body of the paper.

But when the printed proof came into my hands in February last, I made an *addition* at the end. This addition I assert to be a mere development of what is in the body of the paper; and on the truth of this the question turns whether I *can* have borrowed, knowingly or unknowingly, from Sir William Hamilton. If there *be* any thing new in principle in the addition, then comes the question whether Sir William Hamilton's communication *could* have given me a hint of *that new principle*. To enable those who can and will to judge for themselves, I shall presently print—

1. The part of my paper which I assert to contain the principles developed in the addition. 2. The only communication which Sir William Hamilton can make a pretext for his charge, as will distinctly appear in my statement

of our correspondence. 3. All* the addition to my paper which was in any way mentioned in those letters of mine to Sir William Hamilton on which he wrote his charge.

But as those who are not inclined to go into such matters will judge by the manner in which the imputation was made and met, I begin by a statement of all that has taken place between us.

Sir William Hamilton of Edinburgh stands very high among the cultivators of mental philosophy in these kingdoms, and is well known to have paid profound attention to the history of that science in all ages. Of his character as a man I never heard anything but the highest praise; and I most distinctly say, that I am perfectly satisfied he believed the charge when he made it, and that, more or less doubtfully, he believes it up to this moment. At the end of September I wrote to him, without previous acquaintance, to gain some information on existing sources as to the *history* of technical logic. I think I mentioned my own investigations in general terms. The answer, dated Oct. 7 (received by me four days after my paper had been sent to Cambridge), contains the following:—

“I have for many years taught in my class, what, I think, affords a full extension and, thereby, a complete simplification, of the syllogistic theory; and through the Notes and Essays of my students, this development of the doctrine has obtained considerable publicity, though I have not yet given it to the world through the press. Should you feel any curiosity on this matter; or should you wish for a list of the professed Histories of Logic, I will do what I can to satisfy you, on my return home, in about a fortnight hence, . . .”

In my reply, I accepted Sir William Hamilton's offer as to his system of syllogism. The next communication I received was dated Nov. 2; it is (why will presently appear) the pretext for the charge made against me, and is printed in the sequel *entire*, between the two extracts from my paper. I shall reserve my comment on it, and shall point out the reason why it was as unintelligible to me as I predict it will be to the reader. I replied to this letter, with some remarks on parts of the *requisites* for the prize essay. The next communication is dated Dec. 28, and consisted of—1. A letter. 2. A printed prospectus of Sir William Hamilton's intended work on logic. Nothing turns on this, for the simple reason that my answer contained the most express and formal proof that, come by it how I might, I was *then* in the most complete *written* possession of all I have since published. The following is an extract from the letter:—

“I send you what I promised—a more articulate statement of my syllogistic theory. I should have made it however less summary, had the prize Essays (for which I sent you the Requisites) been given in; but the day of delivery is not till the 1st of January. I have appended this statement to my edition of Reid's Works, which has just appeared, and should have sent it to you sooner, but waited for an answer to my application to Mr. ———, which ought to have come some ten days earlier. That gentleman was a pupil of mine six years ago, and obtained one of the highest honours of the class; he was therefore fully competent to afford you information, which I begged † him to do, in regard to my logical doctrines

* To the best of my knowledge and belief: for I kept no copies of my letters previous to March 13. I have no doubt Sir William Hamilton would have supplied them; but I did not think it necessary to ask, as it is on *his* communications that the charge was made.

† Since that time, Mr. ———, who was once my own pupil, called on me one evening after a lecture at University College, and left with me his notes of Sir William Hamilton's lectures. My knowledge of these notes has nothing to do with the present question, since it dates from some weeks after the 1st of January. I shall allude to them again presently. Not to omit mention of any communication which has passed, I should state that Sir William Hamilton, about the time I received these notes, sent me part of his supplement to Reid, containing notes into which an enormous amount of digested learning is thrown, but not having

as they were taught so far back. When he attended me, besides the twofold wholes in which the syllogism proceeds, the quantification of the predicate, and the effect of that on the doctrine of conversion, on the doctrine of syllogistic moods, on the special syllogistic rules, &c., were topics discussed, and partly given out for exercises. They were, in fact, then mere common-place."

The prospectus which accompanied this letter is very full on the *results* which Sir William Hamilton can produce from his principles; but gives nothing, I think, certainly nothing intelligible to me, on those principles themselves. There is a curious circumstance about this prospectus. Of the two communications to me, it is avowedly the fuller, the more developed, the "more articulate statement." It begins, moreover, with a profession, that it was drawn up to secure the rights of authorship. And yet Sir William Hamilton could not wait till after January 1 (when the prize essays would have been sent in), but must publish it in December, though thereby obliged, as he says, to make it *summary*, and, as he implies, so *summary*, that those who had been and were his own pupils might not get help from it for their prize essays. Why this haste? I take it to have been because Sir William Hamilton was prepared to imagine that I, or any one who ventured into his science, might possibly appropriate his ideas. But be this as it may, I hold it rather a strong fact, that the *more developed* of his two communications was meant to be such as would recal no information for a prize essay to those who had heard the system discussed at many hours' length.

As soon as I saw these *results*, I instantly saw that many of them agreed with my own. I had then no doubt that we possessed something in common; and I said so very distinctly in my reply. As the reader will presently see, this first impression has not been confirmed. Feeling it now time to secure whatever of independent discovery might belong to me, I answered Sir William Hamilton in two letters, dated December 31 and January 1. In these letters—

1. I returned the printed prospectus with the results underlined which *my* system would produce.

2. I stated that I had a system * written on certain sheets of paper, which I described as to number, size, &c., adding the head words of each page. I felt inclined to get the signature of some good witness put upon these papers; but at the same time I felt reluctant that Sir William Hamilton should see, if it ever became necessary to produce these papers, that I had been taking *precautions* against him. I therefore determined to make himself my witness.

3. I stated distinctly the first principles of *both* my systems, and the syllogistic formulæ to which they lead.

I heard no more, till by note of *March* 3, Sir William Hamilton informed me that he had been ill for two months, and asked, as I thought, for a sight of the requirements for the prize essay which had been forwarded on *Nov.* 2. I therefore sent them, and received another note, dated the 8th, correcting my mistake, returning them, and asking for *his own letter of Nov.* 2. This I sent; and it was returned with another letter (the cause of this statement), dated *Edinburgh, March* 13; as follows:—

"DEAR SIR,—Your note of the 10th, with its enclosure of my letter, I received this morning. Having again read over the whole correspondence and being now sufficiently recovered, I

reference to any system of formal logic, except in the prospectus already mentioned, which is attached to them, and of which I then received a second copy.

* This is my second system, described, I assert, in all its essentials, in my paper then at Cambridge, but more developed from these very sheets, in the *Addition*. I had then quite forgotten how nearly the papers safe at Cambridge contained the matters which I was proving to be in my possession by thus describing them.

proceed briefly to answer your communications of the 31st Dec. and 1st Jan. ult., which reached me after I was laid up by a tedious inflammatory attack. From these (if I am not mistaken) it appears that you claim for yourself the independent re-discovery of the fundamental doctrine of syllogism, which I privately communicated to you, and of many of its most important consequences more fully developed in the printed prospectus. This claim, though it be only to secondary originality, I am altogether unable to admit. To me, it is manifest, that for the principle of the doctrine you are wholly indebted to my information; and I cannot but think that if you (though recognising always my priority) give forth that doctrine as a speculation of your own, you will be guilty,—pardon the plain speaking,—both of an injurious breach of confidence towards me and of false dealing towards the public. Am I therefore correct in my understanding of your letters? I shall be glad to find that I am wrong.

“I remain, dear Sir, your most obedient Servant,

“W. HAMILTON.”

There is nothing so pardonable as plain speaking; in the present instance I owe to it the opportunity of strangling this imputation in its cradle. I replied by return of post, that I saw the propriety of abstaining from further private correspondence upon the subject in question; that, when my paper was published, he should have some copies; and that the hasty manner in which he had expressed his suspicions of an odious charge did not diminish the respect in which I held him. In a few days I received copies of my paper in the Cambridge Transactions, of which I forwarded a few copies to Edinburgh in a bookseller's parcel. On the 25th of March, judging they had been received, I addressed Sir William Hamilton again, still in terms of respect (indeed, I never have used any others), informing him that I intended to bring his charge to a speedy, and if necessary, a public issue; and giving him till the 10th of April (or any not unreasonably later date which he should propose) either to retract, or to announce the time and manner in which he would maintain his assertions by public proof: failing his acceptance of either alternative, I informed him that I should draw up a statement myself. The answer, dated the 27th, informed me that my paper had not been received, but that the expressions bore exclusive reference to my letters, and that there was no occasion to wait. That (I quote descriptively, in the third person) it would afford him sincere pleasure to be enabled to retract what was written both painfully and unwillingly, but which he could not but write under the conviction of its truth (if he had said, under the conviction of his ability to prove it, it had been more to the purpose). That there were only two ways in which this retraction would be for him possible: he must be satisfied either that the meaning of my letters was not what he supposed them unambiguously to express; or that I had been acting under the influence of some intellectual error (that he himself should be in such a predicament is virtually declared impossible, except only by misunderstanding my letters). That it would truly gratify him to be allowed, on either alternative, to cancel all derogatory expressions and sentiments towards one whom otherwise, on many accounts, he was so much inclined to respect. That he cordially sympathised with my feelings on the occasion (I wonder how; being sure that he believed his own charge, I should have thought he would have taken me to be putting a false front of integrity over a consciousness of wilful theft): and that assuredly his co-operation should not be wanting to lay the *whole* relevant documents fairly and faithfully before the public,—on the supposition, always, that my book preferred the same claim as my correspondence. That I might depend on a statement from him, of the grounds on which he felt himself constrained to advance the obnoxious allegations; and that if his health continued as it was, I should receive it (*in manuscript, mind*) before the time I specified. But, as his public statement was thus to be communicated to me, of course mine, in like manner, would be communicated to him, before their joint publication; and

either, each made independent of the other, and final; or, if I were allowed to answer him, he should equally be allowed to answer me. He would therefore, he concluded, before writing, await my determination upon this point.

If I had felt any anger at the hasty and utterly groundless imputation which has given me the trouble of making this statement, I think the preceding letter would have removed it. There is something racy about the idea of private declaration and answer before going into court, circumstanced as we were. In cases of amicable literary controversy it would be an excellent plan, and would save a good deal of printing. But the notion of the prosecutor and the accused thief quietly interchanging their notes of evidence, and settling its balance before trial, would, if carried into effect, have vastly tickled the public, never indisposed to find jokes in controversies of this sort. I replied, *March 30*, by totally declining the arrangement, and calling upon Sir William, as he valued his own honour, either forthwith to retract, or publicly to attempt the proof of his accusation. I begged him to understand distinctly, that when I proposed to him to retract, it was because such a mark of consideration was usual and proper: but that as far as I was concerned, I had much rather that he should attempt publicly to maintain his words. I added, that I should give no explanations except in print: that had they been asked for in the first instance I would have given them; but certainly not to a person who had preferred to make facts of his surmises, and inferences of his suspicions. I now certainly thought I should draw him out.

From that time to the limit fixed, the 10th of April, I heard no more directly from Sir William Hamilton. But, in a circuitous way, I was offered a sort of retraction on terms. Sir William, I am certain, did not believe me when I said that I would rather he should attempt to maintain than retract. A friend of mine, to whom I had mentioned the whole matter, numbers among his friends one of Sir William Hamilton's, a gentleman of high consideration at Edinburgh. To this gentleman my friend wrote, expressing his regret at the quarrel that was taking place. Some correspondence ensued, to which, as I was no party to it (though I heard extracts read), I should not have alluded, had it not been that Sir William Hamilton became a party to it. And in the course of it he suggested (with a full knowledge, I believe, that it would be conveyed to me), that if I could manage to remember that the papers I had described in one of my letters of Dec. 31 and Jan. 1 were not written till *after* I had seen his letter, &c. of November 2, he, on his part, could say that—I forget the words, but the substance was that—I was probably acting only under intellectual confusion, and not under intention to appropriate his ideas. This I simply refused, for two reasons; I forget whether I gave them or not, but here they are. In the first place, I could not remember what did not happen: though, as I will show presently, it has nothing to do with the matter, yet those papers *were* written before the letter of November arrived. And in the second place, nothing should ever induce me to buy the withdrawal of such a charge by any concession whatever. I could only meet it by every possible defiance, expressed in the most peremptory terms which would consist with the respect I wished (and wish) to preserve towards great learning, high character, and ill health.

When the 10th of *April* had passed over, I thought it necessary to make one more effort. Accordingly, on the 12th, I wrote to Sir William Hamilton, asking first, whether he was prepared to give me his assurance that he would immediately proceed to the public proof of his charge of *March 13*; and secondly, when I might expect his publication. I added, that on failure of an immediate and explicit answer to both questions, I should consider him as

shrinking from the attempt to show cause for his assertions, and should proclaim him as so shrinking in the public statement which I should immediately draw up.

On the 15th of *April* I received, dated the 13th, what at first I thought (the course of post making it possible) was the answer: and what Sir William Hamilton allowed to stand in place of an answer for several days:—

"DEAR SIR,—As I find that my letter of the 13th March ult. prevents you from favouring me with an answer to the inquiries which I would beg leave to propose; and as I am anxious that there should be no possible misunderstanding in reference to your claim of being an original excogitator of the doctrine of syllogism founded on the expressed quantity of the predicate; I request your permission to withdraw that letter—to hold it unconditionally *pro non scripto*; expressing also my regret that the allegations it contains, though only hypothetical*, were stated before you had an opportunity of explanation. This being done, on my part, I trust on yours, that you will not deny me the satisfaction of answering the questions which I am desirous to put; in the hope that the whole question may be solved by the detection of a mistake on the one side or the other. I remain, dear Sir,

"Your most obedient Servant,

"W. HAMILTON."

I acknowledged the receipt of this letter, referring to my statement now before the reader for the further answer, which I proceed to give.

In the first place, I never will have anything to do with the doctrine of *scriptum pro non scripto*. The only Latin I hold by on such a matter is *Litera scripta manet*. I understand a man when he says he has changed his opinion, or that he will not or cannot maintain what he formerly said he could maintain; but I do not understand holding what was written as not having been written. If Sir William Hamilton had said in effect, "I have made a charge, which, though I thought it true at the time, I now do not believe," or "which, though I thought I could prove at the time, I do not find I can prove it now," he would have put me in a difficult position. I am not at all prepared to say that even in such a case, I should, looking at all that has passed, have judged it safe to withhold my present statement. But, with reference to the letter above given, I am firmly and clearly of opinion, that I owe this publication to myself, to the Cambridge Society, and to the college in which I teach. In the first place, the withdrawal is made, not because the accuser has found out his mistake, but because he finds the accused will not enter on the matter with him while it stands, except with the whole public to judge. Secondly, it is a *suspension*, and a *suspension* only: Sir William Hamilton, who now sees that he came to a decision too hastily, invites me into the court in which he proposes to *hear* the case. I am to defend myself upon interrogatories, and a hope is expressed that it will turn out to be all a mistake. But if that hope should be disappointed (as ten to one it would be), what must be supposed likely to follow? a renewal of the charge; the suspension will be taken off. Thirdly, the acceptance of a withdrawal of the charge, accompanied by regret expressed for having made it before I could explain, would be an admission on my part, that there was some reasonable *prima facie* case against me. Now, if Sir William Hamilton should complain that I would not give him grounds for retracting his assertion, I ask him, *what were his grounds for making it?* I say he had *none* but what were utterly futile: I say this before I absolutely know what they were; and once

* So far hypothetical that they depend on my "book preferring the same claim as my correspondence," and upon Sir William Hamilton's correctly understanding my letters. Now the book (that is, the Memoir) *does* prefer that claim; and my correspondence *does*, as Sir William Hamilton correctly understands it, make a full declaration of my having independently arrived at everything new which I did arrive at. Besides, he afterwards informs me that he was constrained to write the derogatory expressions under a conviction of their truth.

again I challenge him to produce them, and to let them undergo my public handling.

My reader will see that I have been most anxious to be put on my defence, or as Sir William says, to give explanations—refutations I should call them. But not to my respected accuser himself: I have preferred to appeal to Cæsar; and Sir William would have done better if he had answered, *Unto Cæsar shalt thou go.*

[This statement was put into the printer's hands on the 19th of April, and on the 23rd I received the following note, dated the 20th, being the seventh day after the receipt of the letter in which I had asked for an immediate and explicit answer to the two questions above:

“Sir William Hamilton presents his compliments to Professor De Morgan, whose note of the 12th he delayed answering, until he received the reply to his own of the 13th. Sir Wm. Hamilton is surprised, that Mr. De Morgan could think the intimation, he required, even possible, so long as the conciliatory intervention of Mr. De Morgan's friend Dr. ——— was not ended, and ended, in effect*, it was not, until Mr. De Morgan's last note terminated all hope of the difference being resolved into a mistake, on the one side or the other: Sir William Hamilton has now therefore no alternative†, but to state the evidence on which he disputes Mr. De Morgan's claim to an independent rediscovery‡ of the new principle of syllogism; and is too confident of the strength of his position, to deny his opponent the advantage of seeing the one case, before publishing the other. Sir William will, therefore, proceed without delay; and though he has at present much** else to do, hopes, if health permit him, to have his proof written during the present week, and printed in the course of next. There is however a strike at present among the Edinburgh printers. What effect†† this may have in regarding the publication Sir William Hamilton is unable to say.”]

My reader must take notice, that I and my accuser are at issue on some important points of social ethics. He “could not but write,” he was “constrained” to write, under “the conviction of the truth” of his charge. In the day of swords, it was one of the objects of public policy to prevent people from sticking them into each other's bodies upon trivial grounds. We now wear pens: and it is just as great a point to hinder ourselves from sticking them into each other's characters without serious and well-considered reasons. To this end, I have always considered it as one of the first and most special rules, that conviction of the truth of a charge is no sufficient reason for its promulgation. I assert that no one is justified in accusing another *until he has his proof ready*: and that, in the interval, if indeed it be right that there should be any interval, between the charge and the attempt at substantiation, all the leisure and energies of the accuser are the property of the accused. Such being my mind, I cannot now accept Sir William Hamilton, whose theory and practice are so much at variance with mine, either as an opponent in private controversy, or a judge of the result of mutual explanations. More than this, there has been throughout his correspondence, too much of the feeling that he could hold certain points as assumed, and upon those assumptions frame his own conditions, without any reference to whether I granted them or not: and too little of the feeling that when an accusation is

* I was no party to that correspondence, as was avouched by the letters themselves. It was terminated before the 10th of April, with a definite announcement, on the part of my friend Dr. ———, that he saw the matter must become public. The distinction between termination *in fact*, and termination *in effect*, is above my comprehension.

† This regretful announcement is curious, as addressed to me. What have I been pressing for all along?

‡ This will not do. *Breach of confidence* and *false dealing* must be maintained, or Sir William Hamilton abandons his charge.

** So have I.

†† It need have none at all. Why not print at Glasgow?

made, the bringer of it puts himself into the balance as well as the party whom he accuses. He is not, I am well aware from his writings, apt to think much of the mathematician out of mathematics: but he must bear with me while I quote, without meaning to apply its point offensively, something for his instruction from the mouth of a mathematician. The late Professor Vince was once arguing at Cambridge against duelling, and some one said, "Well, but, Professor, what could you do if any one called you a liar." "Sir," said the fine old fellow, in his peculiar brogue, "I should tell him to pruv it: and if he did pruv it, I should be ashamed of myself: and if he didn't pruv it, *he* ought to be ashamed of himself." I will not permit myself to suppose that Sir William Hamilton is either ignorant of, or unconsenting to, the infamy which would attach to any one who had deserved the terms he used for the conduct he described. But I doubt if he be aware of the serious character of his own position, if he persist in declining to attempt the establishment of what he has said more than once he believed in his right to assert, and what, up to this moment, he has only suspended the assertion of, with no other avowed purpose except that of inducing me to put myself on my justification for his private satisfaction, instead of proceeding to a public discussion. [And now my doubt is changed into certainty. For in the letter introduced in [], Sir William Hamilton grants me permission to see, if I please, the evidence on which I am accused before I publish my answer. Not because it is my right; not because it is due to his own character; but because he feels himself so strong that he can afford it. It is rather a better presumption for my case, that I feel I can afford to do without it. I will not wait for his tardy publication: but, if it ever do appear, I will try the "strength of his position" at once, and the reader shall soon see how I set about it.]

I know how it would be if I allowed the matter to end thus. Sir William Hamilton has by this time no doubt put it out of his own power to silence whispers, those whispers which sometimes circulate for a quarter of a century, and are then published in recollections and memoirs, perhaps at a time when neither party can correct them. Of course *I* have widely announced the purport of the letter of March 13: I receive no confidences of that kind. Accordingly I am now determined to prevent all risk as to this matter: and further, to show any one who, having the character which will give weight to his words, may be disposed to attack mine, that he will have to look to his own retreat; that his bridge shall neither be of gold, nor of my providing. I will now proceed to the extracts already promised.

To make the extracts from my paper intelligible, I premise that I consider every proposition as limited, if so mentioned, to a given range of objects of thought; the whole collection of which I call its *universe*. If no limitation be intended, then the universe of the proposition is *the* universe. *Contrary* names are those one of which must belong to every object of thought, but never both: and they are denoted by large and small letters. Thus *x* means not-*X*, and everything is either *X* or *x*. And *X* and *Y* being two names, and with reference to this order, the following notation is used:—

(A) or <i>X</i>	<i>Y</i> means 'Every <i>X</i> is <i>Y</i> '	(O) or <i>X</i> : <i>Y</i>	means 'Some <i>X</i> s are not <i>Y</i> s'
(E) or <i>X</i> . <i>Y</i>	'No <i>X</i> is <i>Y</i> '	(I) or <i>XY</i>	'Some <i>X</i> s are <i>Y</i> s'
(a) or <i>x</i>	<i>y</i>	(o) or <i>x</i> : <i>y</i>	'Some <i>x</i> s are not <i>y</i> s'
(e) or <i>x</i> . <i>y</i>	'No <i>x</i> is <i>y</i> '	(i) or <i>xy</i>	'Some <i>x</i> s are <i>y</i> s'

When *Z* is introduced instead of *X*, the same vowels are used.

Extract I. From the paper as sent to Cambridge before I had any communication whatsoever from Sir William Hamilton (without any corrections).

SECTION III. On the quantity of propositions.

"The logical use of the word *some*, as merely 'more than none,' needs no further explanation. Exact knowledge of the extent of a proposition would consist in knowing, for instance in 'some Xs are not Ys,' both what proportion of the Xs are spoken of, and what proportion exists between the whole number of Xs and of Ys. The want of this information compels us to divide the exponents of our proportions into 0, more than 0 not necessarily 1, and 1. An algebraist learns to consider the distinction between 0 and quantity as identical, for many purposes, with that between one quantity and another: the logician must (all writers imply) keep the distinction between 0 and a , however small a may be, as sacred as that between 0 and $1-a$: there being but the same form for the two cases. We shall now see that this matter has not been fully examined.

"Inference must consist in bringing each two things which are to be compared into comparison with a third. Many comparisons may be made at once, but there must be this process in every one. When the comparison is that of identity, of *is* or *is not*, it can only be in its ultimate or individual case, one of the two following:—'This X is a Y, this Z is the very same Y, therefore this X is this Z; or else 'This X is a Y, this Z is not the very same Y, therefore this X is not this Z.' And collectively, it must be either 'Each of these Xs is a Y; each of these Ys is a Z; therefore each of these Xs is a Z;' or else 'Each of these Xs is a Y, no one of these Ys is a Z, therefore no one of these Xs is a Z.'

"All that is essential then to a syllogism is that its premises shall mention a number of Ys, of each of which they shall affirm either that it is both X and Z, or that it is one and is not the other. The premises may mention more: but it is enough that this much can be picked out; and it is in this last process that inference consists.

"Aristotle noticed but one way of being sure that the same Ys are spoken of in both premises: namely, by speaking of all of them in one at least. But this is only a case of the rule: for all that is necessary is *that more Ys in number than there exist separate Ys shall be spoken of in both premises together*. Having to make $m+n$ greater than unity, when neither m nor n is so, he admitted only that case in which one of the two m or n , is unity and the other is anything except 0. Here then are two syllogisms which ought to have appeared, but do not,

Most of the Ys are Xs
Most of the Ys are Zs

Most of the Ys are Xs
Most of the Ys are not Zs

∴ Some Xs are Zs

∴ Some of the Xs are not Zs.

And instead of most, or $\frac{1}{2}+a$, of the Ys, may be substituted any two fractions which have a sum greater than unity. If these fractions be m and n , then the middle term is *at least* the fraction $m+n-1$ of the Ys. It is not really even necessary that all the Ys should enter in one premise or the other: for more than the fraction $m+n-1$ of the whole may be repeated twice.

"And in truth it is this mode of syllogising that we are frequently obliged to have recourse to; perhaps more often than not in our universal syllogisms. 'All men are capable of some instruction; all who are capable of any instruction can learn to distinguish their right and left hands by name; therefore all men can learn to do so.' Let the word *all* in these two cases mean only *all but one*, and the books on logic tell us with one voice that the syllogism has particular premises, and *no conclusion can be drawn*. But in fact idiots are capable of no instruction, many are deaf and dumb, some are without hands; and yet a conclusion is admissible. Here m and n are each very near to unity, and $m+n-1$ is therefore near to unity. Some will say that this is a probable conclusion: that in the case of any one person it means there is the chance m that he can receive instruction, and n that one so gifted can be made to name his right and left hand: therefore $m \times n$ (very near unity) is the chance that this man can learn so much.

"But I cannot see how in this instance the probability is anything but another sort of inference from the demonstrable conclusion of the syllogism, which must exist under the premises given. Besides which, even if we admit the syllogism as only probable with regard to any one man, it is absolute and demonstrative in regard to the proposition with which it concludes.

"But this is not the only case in which the middle term need not enter universally: this however is matter for the next Section. I now go on to another point."

Extract II.

"I now take the two cases in which particular premises may give a conclusion: namely

I_{II} $XY+XY=XZ$ $XY+Y:Z=X:Z$ O_{Io}

on the suppositions that the Ys mentioned in both premises are in number more than all the

Ys. If Y_1 and Y_2 stand for the fractions of the whole number of Ys mentioned or implied in the two premises, and y_1 and y_2 for the fractions of the ys implied or mentioned, we shall by a repetition of the process on $YX + YZ = XZ$ (the other being obtained in the course of the process) arrive at the following results or their counterparts: remembering that $Y_1 + Y_2$ is greater or less than 1, according as $y_1 + y_2$ is less or greater.

Designation.	Syllogism.	Condition of its existence.
I_{II}	$YX + YZ = XZ$	$Y_1 + Y_2$ greater than 1
O_{Jo}	$YX + Y : Z = XZ$
i_{oo}	$Y : X + Y : Z = xz$
O_{oi}	$X : Y + yz = X : Z$	$Y_1 + Y_2$ less than 1
i_{ii}	$yz + yz = xz$
O_{oi}	$X : Y + yz = X : Z$
I_{oo}	$X : Y + Z : Y = X : Z$

Communication received on the 4th or 5th of November from Sir William Hamilton, being the pretext for his charge that I have, with injurious breach of confidence towards himself, and false dealing towards the public, appropriated his "Fundamental Doctrine of Syllogism" privately communicated to me.

"16 Great King Street,
November 2nd, 1846.

"DEAR SIR,—I have been longer than I anticipated in answering your last letter. I now send you a copy of the requisites for the prize Essay, which I gave out to my students at the close of last session. It will show you the nature of my doctrine of syllogism, in one of its halves. The other, which is not there touched on, regards the two wholes, or quantities in which a syllogism is cast. I had intended sending you a copy of a more articulate statement which I meant, at any rate, to have drawn up; but I have not as yet been able to write this. I will send it when it is done. From what you state of your system having 'little in common with the old one,' and from the contents of your First Notions, we shall not, I find, at all interfere, for my doctrine is simply that of Aristotle, fully developed.

It will give me great pleasure if I can be of any use, in your investigations concerning the history of Logical doctrines. I have paid great attention to this subject, on which I found, that I could obtain little or no information from the professed historians of Logic; and my collection of Logical books is probably the most complete in this country. But, as I mentioned to you in my former letter, it is only in subordinate matters that in abstract Logic there has been any progress.

"I remain, dear Sir, very truly yours,

"W. HAMILTON."

Essay on the new Analytic of Logical Forms.

Without wishing to prescribe any definite order, it is required that there should be stated in the Essay,—

- 1°. What Logic postulates as a condition of its applicability.
- 2°. The reasons why common language makes an *ellipsis* of the expressed quantity—frequently of the *subject*, and more frequently of the *predicate*, though both have always their quantities in thought.
- 3°. *Conversion of propositions*—on the common doctrine.
- 4°. Defects of this.
- 5°. *Figure and Mood* of Categorical syllogism, and Reduction,—on common doctrine (General statement).
- 6°. Defects of this (General statement).
- 7°. The one *supreme Canon* of Categorical Syllogisms.
- 8°. The evolution, from this canon, of all the *species* of Syllogism.
- 9°. The evolution, from this canon, of all the *general laws* of categorical Syllogisms.
- 10°. The error of the *special laws* for the several Figures of Categorical Syllogism.
- 11°. *How many Figures* are there.
- 12°. What are the *Canons* of the several Figures.
- 13°. *How many moods* are there in all the Figures: showing in concrete examples, through all the Moods, the *unessential* variation which Figure makes in a syllogism.
(14° was missing, or those which follow 13° were wrong numbered.)
- 15°. What relation do the Figures hold to *extension* and *comprehension*.
- 16°. Why have the *second* and *third* Figures *no determinate major* and *minor* premises and *two* indifferent conclusions; while the *first* Figure has a *determinate major* and *minor* premise, and a single *proximate conclusion*.

17°. What relation do the Figures hold to *Deduction* and *Induction*.

N.B. This Essay open for competition to all students of the class of Logic and Metaphysics during the last or during the ensuing session.

April 15th, 1846.

Extract from the Addition to my Paper, taken, as can be shown, from the papers which I gave the means of identifying in January last, and which papers (though I hold it immaterial) I assert to have been written before I received any logical communication from Sir William Hamilton. (To be compared with the extracts given above.)

"Since this paper was written, I found that the whole theory of the syllogism might be deduced from the consideration of propositions in a form in which *definite quantity* of assertion is given both to the subject and the predicate of a proposition. I had committed this view to paper, when I learned from Sir William Hamilton of Edinburgh, that he had for some time past publicly taught a theory of the syllogism differing in detail and extent from that of Aristotle. From the prospectus of an intended work on logic, which Sir William Hamilton has recently issued, at the end of his edition of Reid, as well as from information conveyed to me by himself in general terms, I should suppose it will be found that I have been more or less anticipated in the view just alluded to. To what extent this has been the case, I cannot now ascertain; but the book of which the prospectus just named is an announcement, will settle that question. From the extraordinary extent of its author's learning in the history of philosophy, and the acuteness of his written articles on the subject, all who are interested in logic will look for its appearance with more than common interest.

"The footing upon which we should be glad to put propositions, if our knowledge were minute enough, is the following. We should state how many individuals there are under the names which are the subject and predicate, and of how many of each we mean to speak. Thus, instead of 'Some Xs are Ys,' it would be, 'Every one of a specified Xs is one or other of *b* specified Ys.' And the negative form would be as in 'No one of a specified Xs is any one of *b* specified Ys.' If propositions be stated in this way, the conditions of inference are as follows. Let the *effective number* of a proposition be the number of mentioned cases of the *subject*: if it be an affirmative proposition, or of the *middle term*, if it be a negative proposition. Thus, in 'Each one of 50 Xs is one or other of 70 Ys,' is a proposition, the effective number of which is always 50. But 'No one of 50 Xs is any one of 70 Ys' is a proposition, the effective number of which is 50 or 70, according as X or Y is the middle term of the syllogism in which it is to be used. Then two propositions, each of two terms, and having one term in common, admit an inference when 1. They are not both negative. 2. The sum of the effective numbers of the two premises is greater than the whole number of existing cases of the middle term. And the excess of that sum above the number of cases of the middle term is the number of the cases in the affirmative premiss which are the subjects of inference. Thus, if there be 100 Ys, and we can say that each of 50 Xs is one or other of 80 Ys, and that no one of 20 Zs is any one of 60 Ys;—the effective numbers are 50 and 60. And 50 + 60 exceeding 100 by 10, there are 10 Xs, of which we may affirm that no one of them is any one of 20 Zs mentioned.

"The following brief summary will enable the reader to observe the complete deduction of all the Aristotelian forms, and the various modes of inference from *specific particulars*, of which a short account has already been given.

"Let Q be the whole number of Xs; and *t* the number specified in the premiss. Let *c* be the whole number of Zs; and *w* the number specified in the premiss. Let *b* be the whole number of Ys; and *u* and *v* the numbers specified in the premises of *x* and *z*. Let $X_t Y_u$ denote that each of *t* Xs is affirmed to be one out of *u* Ys: and $X_t : Y_u$ that each of *t* Xs is denied to be any one out of *u* Ys. Let $X_{m,n}$ signify *m* Xs taken out of a larger specified number *n*; and so on. Then the five possible syllogisms, on the condition that no contraries are to enter either premises or conclusion, are as follows:—

1. $X_t Y_u + Z_w Y_v = X_{t+w-b, t} \quad Z_w = Z_{t+w-b, w} X_t$
2. $X_t Y_u + Y_v Z_w = X_{t+v-b, t} \quad Z_w = Z_{t+v-b, w} X_t$
3. $Y_u X_t + Y_v Z_w = X_{u+v-b, t} \quad Z_w = Z_{u+v-b, w} X_t$
4. $X_t Y_u + Z_w : Y_v = X_{t+v-b, t} : Z_w$
5. $Y_u X_t + Z_w : Y_v = X_{u+v-b, t} : Z_w$

"The condition of inference expresses itself; in the $X_{m, t}$ of the conclusion, *m* must neither be 0 nor negative. The first case gives no Aristotelian syllogism; the middle term never

entering universally (of necessity) into any of its forms, under any degree of specification which the usual modes of speaking allow. The other cases divide the old syllogisms among themselves in the following manner: they are written so as to show that there is sometimes a little difference of amount of specification between the results of different figures, which changes in the reduction from one figure to another. The Roman numerals mark the figures.

2.	$t = a, v = b$	$Y)Z_w + X)Y_u = X)Z_{u,w}$	<i>Barbara I.</i>
	$t = a, v = b$	$X)Y_u + Y)Z_w = Z_{a,w}X$	<i>Bramantip IV.</i>
	$t < a, v = b$	$Y)Z_w + X_tY_u = X_tZ_{u,w}$	<i>Darii I.</i>
	$t < a, v = b$	$X_tY_u + Y)Z_w = Z_{t,w}X_t$	<i>Dimaris IV.</i>
3.	$u = b, v = b$	$Y)X_t + Y)Z_w = Z_{b,w}X_{b,t}$	<i>Darapti III.</i>
	$u < b, v = b$	$Y_uX_t + Y)Z_w = Z_{u,w}X_{u,t}$	<i>Disamis III.</i>
	$u = b, v < b$	$Y)X_t + Y_v)Z_w = Z_{v,w}X_{v,t}$	<i>Datisi III.</i>
4.	$t = a, v = b, w = c$	$Y.Z + X)Y_u = X.Z$	<i>Celarent I.</i>
	$t = a, v = b, w = c$	$Z.Y + X)Y_u = X.Z$	<i>Cesare II.</i>
	$t = a, v = b, w = c$	$X)Y_u + Z.Y = Z.X$	<i>Camestres II.</i>
	$t = a, v = b, w = c$	$X)Y_u + Y.Z = Z.X$	<i>Camenes IV.</i>
	$v = b, w = c$	$Y.Z + X_tY_u = X_t.Z$	<i>Ferio I.</i>
	$v = b, w = c$	$Z.Y + X_tY_u = X_t.Z$	<i>Festino II.</i>
	$t = a, v = b,$	$X)Y_u + Z_w.Y = Z_w.X$	<i>Baroko II.</i>
5.	$u = b, v = b, w = c$	$Y.Z + Y)X_t = X_{b,t}.Z$	<i>Felapton III.</i>
	$u = b, v = b, w = c$	$Z.Y + Y)X_t = X_{b,t}.Z$	<i>Fesapo IV.</i>
	$v = b, w = c$	$Y.Z + Y_uX_t = X_{u,t}.Z$	<i>Feriso III.</i>
	$v = b, w = c$	$Z.Y + Y_uX_t = X_{u,t}.Z$	<i>Fresison IV.†</i>
	$u = b, w = c$	$Y_v.Z + Y)X_t = X_{v,t}.Z$	<i>Bokardo III.</i>

I can easily imagine the amusement which I shall afford, to some of those who can readily take in the preceding, by seriously setting myself to answer an imputation founded on such a pretext. I have no objection to help any one to a laugh: if I had, I should beg it might be at the expense of my accuser; as it is, I am quite willing to go halves. But, I repeat, the learning and moral worth of my accuser are the amber in which this straw would be preserved, if I did not take steps to prevent it: and, for once at least, whatever it is worth his while to assert, it cannot be totally, and beyond doubt, safe for me to neglect.

There are two questions;—First, did Sir William Hamilton communicate anything? Secondly, allowing hypothetically that he did, was it anything more than I had proved my independent right to when I sent my paper to Cambridge? He says (November 2) that the requisites will “show the nature of his doctrine of syllogism, in one of its halves,” and again (March 13) that he communicated “the fundamental doctrine.” And (April 13) he speaks of a doctrine founded on the *expressed quantity* of the predicate, as the thing in question. It is therefore the *second* requisite on which this part of the case turns. Now no predecessor of mine that I know of ever *expressed* or *understood* any quantity in a logical assertion except *none*, the indefinite *some* of the particular proposition, and *all*. Sir William never hints at *defining* the quantity of the terms in *particular propositions*, on which the whole of my second system turns. And it is for him to prove, both (as I shall presently remark) that he and I mean the same thing by *quantity*, and also that he has produced to his class a syllogism of *particular* premises with *definite* quantities. When

the *Requisites* above cited first reached me, and I saw mention of the quantities which both subject and predicate *always have* in thought, all I could do was to wait for the *more articulate statement* which was promised me. It remained to see whether Sir William Hamilton was really speaking of what they *always have* in thought, which the common system represents, or of what they *have not but always would have if our knowledge were exact enough*, which is what my extension of it supposes.

The reader who is not used to logic will be rather surprised to learn that a person kept close to Aristotle's forms could not prove that in a company of men, most of whom have coats and most of them waistcoats, it must be that some of them have both coats *and* waistcoats. The inference is most demonstrably legitimate: but all the schoolmen that ever lived could not put it into the form of an Aristotelian syllogism. If Sir William Hamilton can prove that he ever, before his communications with me, actually exhibited such an inference to his class, or another of a like kind, then, and not till then, will he show that he so far preceded me. I will presently put the burden of proof on him more distinctly.

Next, could I, after writing what is in my paper, want anything necessary for the addition? I will here observe, that my system in no degree depends upon giving definite quantity to the predicate, as *predicate*. It consists entirely in the quantification of the *middle term*, be it subject or predicate. In fact, perfectly definite quantification destroys the necessity of distinguishing subject and predicate. To say that some 20 Xs out of 50, are all to be found among 70 Ys, or that 20 out of 50 Xs are 20 out of 70 Ys, is precisely the same thing as saying that 20 out of 70 Ys are 20 out of 50 Xs.

Now the only distinction between the original paper and the addition is, that in the former I have been thinking of the cases of inference which Aristotle could not include, and in the latter of those which he did include: both on one principle, explicitly stated in both. In the former I have only quantified the middle term, *be it subject or predicate*, in the latter I have noted the manner in which that same canon derived from the middle term is expressed when there is the quantification of *all* terms. In the former I have expressed the quantity of my conclusion, there called *the middle term*, being as much as is really *middle*, by $m+n-1$, and Y_1+Y_2-1 : in the latter by $v+w-b$, &c.; the resulting number instead of the resulting fraction of the whole. I am really so much at a loss to imagine what I have done in the Addition with any tool not used in the Memoir, that I must wait till Sir William Hamilton points it out, which I hope he will do very explicitly. He must, for his own character's sake, attempt to join my Addition on to the Memoir by a hook of *breach of confidence* and an eye of *false dealing*, manufactured out of the materials of his letter of November 2, and the *Requisites* which he sent with them. I hope he will not draw back, but set manfully to work, and try it: I am curious as well as interested.

In his letter of March 13, Sir William refers again and again to my having acknowledged his priority: this I must briefly set right. At every period I have given him all that I thought it likely he had. Accordingly, in my correspondence, when I knew nothing but his *results* as stated in the prospectus, I treated it as certain that we must have something in common, in which case he, as having published it in lectures, must have had undoubted priority. But when, after reading Mr. ———'s notes, my impression was much weakened, I used the less confident language quoted above in the extract from my *Addition*. And now, my belief in our having anything in common being reduced to thinking it very improbable, by further search through those notes,

I tell him that I will admit such priority as he shall prove, and no more. Nor will I admit his own simple assertion: for though I have full reliance on his veracity, yet he is as liable as myself to the well-known mistake of investigators, upon whose previous views the approach of those of others frequently throws such a new and sudden light, that they think they always must have seen that which they do then see. In my case I contend that *others* will see first, that he never did open his views; secondly, that if he did, any one (not myself merely) can see that I had nothing to gain by it. And to others I submit those points.

I now come, in the last place, to what would have been the previous question, if this statement had been purely literary. It is this, is or is not Sir William Hamilton under a mistake in supposing that he ever used the term *quantification* in the sense in which I use it? On this point, I will, as I said, bring forward enough to throw the onus of proof upon him.

First, even in the Prospectus, *definite* quantity is divided into *universal* and *singular*; *all*, and *one*. No hint of any proposition in which *half*, or *more than half*, or *two-thirds*, &c. are mentioned. In fact, whereas the old logicians made the *singular* proposition count as *universal*, Sir William makes a distinction between *universal* (all, when there are more than one,) and *singular*, (all, when there is only one,) and calls them both *definite*, and the particular proposition *indefinite*, thereby at once distinguishing his system from mine, the very essence of which is the consideration of *definite particulars*. Had I taken notice of this when I first saw the Prospectus, I should not have supposed it likely we had anything in common. Sir William Hamilton cannot even maintain his *claim* against me, much less his *charge*, without directly contradicting the very Prospectus which he published "formally to establish" his "right of authorship."

The passage from the Prospectus is as follows :

"That the *preindesignate terms* of a proposition, whether subject or predicate, are never, on that account, thought as *indefinite* (or indeterminate) in quantity. The only indefinite, is *particular*, as opposed to *definite*, quantity; and this last, as it is either of an extensive *maximum* undivided, or of an extensive *minimum* indivisible, constitutes quantity *universal* (general), and quantity *singular* (individual). In fact, *definite* and *indefinite* are the only quantities of which we ought to hear in Logic; for it is only as indefinite that particular, it is only as definite that individual and general, quantities have any (and the same) logical avail."

When Sir William Hamilton mentioned to me that he had requested Mr. _____ to give me information on his system, he described that gentleman as having obtained one of his highest class honours, and therefore fully competent to afford me information. And further, that various topics (page 4), among which was quantification of the predicate, were discussed in the class, partly given out as exercises, and "in fact mere common-place." Mr. _____ (who was my own pupil, and to whose intelligence I add my testimony) lent me his notes, full and clear, and with a table of contents. Surely then, there must be something in these notes to show that Sir William Hamilton had preceded me. *All I can say is I can find nothing*. I can find no notion of *quantity* except the old one of *plurality of attributes* and of *genus* and *difference*. I can find no notion whatever of *numerically definite* quantity. I cannot find one single inference that is not obtained in the old system. I cannot find one single syllogism in which the middle term is not universal in one premiss. Mind, I say this simply to put the proof upon Sir William Hamilton. I do not deny that such things may be there: I cannot find them. I do not deny (and, when I see it, shall admit) that *other* notes may supply what is not in these: all I know is that *these* notes

are Sir William's chosen evidence, and I cannot find anything in them which seems to me at all connected with the points on which I ventured to propose a departure from the established system. There is much which is new to me, and much which I shall study with great interest when Sir William Hamilton's treatise appears: in particular, the manner in which life is given to the (as I once thought, it) useless distinction of figures. But any consideration of definite particulars, any inference which is not an Aristotelian syllogism, is wholly wanting, so far as I can see after a very attentive examination.

But if there really be anything in which Sir William Hamilton has preceded me, I shall be, of all men except himself, most interested in his having his full rights. And I make him this offer, and will take his acceptance of it as reparation in full for his suspicions and assertions. With the consent of the gentleman to whom these notes belong, which I am sure will not be refused to our joint application, I will forward to him a copy of their table of contents, having more than a hundred and fifty headings. From these Sir William Hamilton shall select those which are, in his opinion, sure to contain proof of his priority on any point which I have investigated. Of these I will have copies made and sent to him: and will print in the work on Logic which I am preparing (and in some one part of it) the parts which he shall select as fit to prove (or to show that he could prove, let him call it as he likes) his case, or the germs of his case (as he pleases, again). Provided always, that the matter shall not run beyond some eight or a dozen octavo pages of small print. And I on my part propose that I shall be allowed to print, to one-half the amount selected by Sir William Hamilton, of additional extract: but if this be refused I will not insist on it. With this I will put a heading fully descriptive of the reason and meaning of the insertion, and such distinct reference and account at the beginning of the preface as shall be sure to call the reader's attention to it. So that my book shall establish the claim, if it can be established from the notes of one of the best students. If this offer be not accepted, an account of it will take the place of any other result. If Sir William Hamilton, or any one else, can propose anything to make this offer fairer, I shall probably not be found indisposed to accept the addition. And though, I will frankly say, my present conviction is that the acceptance of the offer would alone cause my work to knock Sir William Hamilton's assertions to atoms, yet I will pledge myself, in any case, to abide by it.

And now I have done with the matter, at least until Sir William Hamilton shall have had his turn. And I trust this will speedily come about; for lengthened controversies are nuisances to the parties engaged, and would be to others, if there did not exist that plenary power of not reading which makes apologies for publication superfluous.

A. DE MORGAN.

University College, London,
April 27, 1847.

For a continuation of this controversy
see the *Athenaeum* Nos 1022 and 1023,
18

A LETTER

TO

AUGUSTUS DE MORGAN, ESQ.

OF TRINITY COLLEGE, CAMBRIDGE,

PROFESSOR OF MATHEMATICS IN UNIVERSITY COLLEGE, LONDON,

ON HIS CLAIM TO AN INDEPENDENT RE-DISCOVERY
OF A NEW PRINCIPLE IN THE THEORY OF SYLLOGISM.

FROM

SIR WILLIAM HAMILTON, BART.

SUBJOINED,

THE WHOLE PREVIOUS CORRESPONDENCE,

AND A POSTSCRIPT IN ANSWER TO

PROFESSOR DE MORGAN'S

"STATEMENT."

LONDON AND EDINBURGH:

LONGMAN, BROWN, GREEN, AND LONGMANS;

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MDCCCXLVII.

141

"Ὁ βούλεται, τοῦδ' ἕκαστος καὶ οἶται."

DEMOSTHENES, GALEN, CHRYSOSTOM, ETC.

"Quod volunt, id credunt, homines."

CÆSAR, QUINTILIAN, HEGESIPPUS, ETC.

"The Wish is father to the Thought."

BEN JONSON.

"Ἦαντοῖς, ὀνειροπολοῦσι."

GREEK PROVERBIAL.

"Fingunt, creduntque."

TACITUS, CHRYSOLOGUS, BACON, ETC.

"As their own dreams at length deceive,
Which, oft repeating, men believe ;
Just so *with you*, my friend, it fares,
Who deal in philosophic wares :
Your happy whimsey you pursue,
Till you at length believe it true ;
Caught by your own delusive art,
You fancy first, and then assert."

PRIOR.

TO PROFESSOR DE MORGAN.

Edinburgh; 27 April, 1847.

SIR,

As there is no longer any prospect, that your claim of having originally thought out the new principle of Syllogism, contained in the thorough-going quantification of the predicate, prior to any communication of it to you by me, may be withdrawn; and our difference terminated, by an admission, on your part, of a possible error of memory, in the confusion of dates: I have no alternative, without compromising truth, surrendering my rights, and "shrinking" from your "challenge," but publicly to prove that such a claim is groundless;—groundless, at once, on all general presumption, and on all special evidence.

In performing this disagreeable duty,—disagreeable it is in every way,—I am well aware of the careless incompetence of the many for such discussions; and therefore to insure that *all* the materials for a just decision should be laid before the *same* competent few, I proposed, that the whole relevant correspondence, with our several statements, interchanged and revised, should be jointly published. This proposal, I regret, you "altogether declined." I trust, however, that in any attempt you may make at vindication, you will not be content with gratuitous assertion, but give, at least, the series of letters, on which alone your averments can pretend to any weight.—I now proceed to the refutation of your claim; and on grounds, which may be misrepresented or ignored, but cannot, I think, be stated and repelled.

But what is the claim, which you maintain, and which I dispute?—It regards the principle of Syllogistic, afforded by the quantification—the expressed quantity—of the predicate; (that of the subject, as old, and common-place, and, to our point, irrelevant, should be omitted.) Now, it is not disputed, that the doctrine, founded on this principle, has, for many years, been publicly taught, by me, in the University of Edinburgh. You, therefore, limiting your claim to a *personal originality*, maintain:—*that, AFTER having finished a paper, (dated 3 Oct. ult.) for the Cambridge Philosophical Society, "On the Structure of the Syllogism," in conformity with the old exclusive theory of quantity; you thought and wrote out a second view of Syllogism, founded on the new and non-exclusive principle of a quantified predicate, and this BEFORE receiving my answer (dated 7 Oct.) to your first letter to me (dated 30 December, erroneously for September) which letter was thus written previously to your alleged discovery [37, 1, 2.]* *—On the contrary, I maintain:—*that this assertion is gratuitous,—opposed to all presumption,—probability,—and proof; that your second view is borrowed from me, and was not apprehended by you, previously to last November, nor until after the document containing it had, at least, been for nearly a fortnight in your hands.* [VI. 14, and letters *passim*.]

These are the counter statements: on what grounds can they be established or disproved? On two: on *general presumption*; and on *special inference*.

I. *General presumption*.—Necessity, besides reason, has established, in the scientific world, the rule,—*That when a discovery has once been publicly made known, all pretence of re-discovery (except under peculiar circumstances) is ignored or scouted as ridiculous.* Were this not recognised as "the standing

* The Numerals within square brackets refer to the Correspondence; the *Roman* to the several documents; the *Arabic* to important passages.

order," against presumption and delusion ; for a single Dennis, shouting, upon one occasion,—“ By Jove !—my thunder ; ” at each step in progress, we should have a host of claimants, clamouring, and some sincerely,—“ By Mercury ! my own invention ; ” and all in choral malediction, with the old grammarian, of “ these,” (as Swift expresses it,)—“ these damned forestallers,”—“ Pereant, qui ante nos, nostra dixerunt ! ”—If such appropriation, indeed, were not put down, we should have, in another sense than Solomon’s, “ nothing new under the sun.” The sun, in fact, would shine upon “ nothing new,”—upon nothing fresh,—upon nothing which did not, on being drawn into the light, prove fly-blown, contaminated, and putrescent.—Now my peculiar views in Logic had, for many years, been academically published ; they might thus, immediately or mediately, have been known to thousands ; and of these there were, surely, many from whom a London logician could easily obtain his information. I was thus, I thought, secure. Nor am I aware, that a single circumstance can be alleged, in reference to you, why the rule should be restricted in its application. On the contrary, you, of all men, after your third communication, were the last whose interference I should have anticipated ; and am now, under the second head, to show, that, independently of any general presumption, there is even a redundancy of special evidence to make your individual claim absurd.

II. *Special inference.*—Under this head, I may canvass your claim, on *two* suppositions. I may either, *take it on your own statement, and show;—that, since you here represent yourself, as acting so dishonourable, so foolish, and so unnatural a part, your unsupported assertion of a claim, in itself the most improbable, is, on your own authority, good for nothing ; or, if entitled to hold you for a man of common honour, common sense, and common feeling, that I may, and in your own favour, redargue your claim and statements, as the result of a mistake.*

I confess, that, having no knowledge of your character, the former supposition was, as was natural, the first suggested. For when I found you (*inter alia*) asserting, that you had *previously held*, what you had kept *previously concealed* ;—*appropriating*, the one day, what you *declared incomprehensible*, the other ;—and citing as *your witness, me* the only *party aggrieved*, and *you* the only *party aggrieving* :—I confess, that, for a time, I regarded your pretension, as an attempt at plagiarism, cool as it was contemptible.

From this view, feeling, information, reflection turned me ; and I now, Sir, tender you my sincere apology, for admitting, though founded on *your own* statements, an opinion so derogatory of one, otherwise so well entitled to respect.

In itself, this view was, to me, painful and revolting.—The character, too, which you bear among your friends, I found to be wholly incompatible with a supposition so odious. You are represented as an active and able man, profound in Mathematics, curious in Logic, wholly incapable of intentional deceit, but not incapable of chronological mistakes. Your habitual confusion of times is, indeed, remarkable, even from our correspondence. Your dates are there, not unfrequently of the wrong month, and not always, even of the right year. With much acuteness, your works show you deficient in architectonic power, the concomitant of lucid thinking ; and, that you are not guiltless of intellectual rashness is sufficiently manifest, from your pretension to advance Logic, without having even mastered its principles.—I have, however, here to reproach myself, more especially with *two* omissions.—In the *first* place, though a Professor of Logic, and often sedulously

cautioning my pupils against the sophistry of the affections, I yet, for a time, neglected to interpret your statements, under the possible, the probable influence of self-love; that "universal passion," the root in fact of every other, whose agency on our opinions has always been paramount and proverbial. The hemistich,—"*credunt quod credere cordi est*;" its converse,—"*tarde, quæ credita lædunt, Credimus*?" and the verse—"Credimus? *an qui amant, ipsi sibi somnia fingunt*;"—these are but three partial phases of an adage, which has been said and sung in all languages and under an endless multitude of forms. I was wrong, therefore, in not, at once, extending to you the benefit of this endemic self-illusion (see p. 2),—wrong in presuming you to be a deceiver, and not rather a deceived; more especially, when even your intellectual rashness showed that the principle of self-love wrought so strong within you. On this ground all is easily explained:—You wished to be an inventor; you dreamt your wish; you believed your dream. "*Quicquid optant*," says LAMPRIDIUS, "*verum esse asseverant*."—In the second place, though I had myself written, and collected the consentient testimony of others, on the evil effects of a too exclusive study of mathematics, as enervating the intellect;* still I did not, at first, accord to you the indulgence, to which mathematicians are, in equity, entitled, when, leaving the level railroad of their own, they venture to commit themselves to the pathless plains, to the hills and valleys, the rocks and quagmires, of the other sciences. I was wrong in not considering, that a blind credulity (or a blind scepticism) is, out of demonstration, the sin which most easily besets a mathematician. "I have never met," says MR DUGALD STEWART, who had himself been a Professor of Mathematics, "with a mere mathematician, who was not credulous to a fault;" and WARBURTON,—"*The oldest mathematician in England is the worst reasoner in it.*" So all competent anthropologists. I do not, indeed, consider you as a mere mathematician; but it would be unfair to omit your principal pursuit among the causes which may account for your illusion.

Having now, therefore, renounced your own supposition, and apologised for having held it; I proceed to refute your claim, upon the other. This I prefer, as being, at once, more charitable, and not less effective. More fully stated it is as follows:†

In the answer (7th Oct.), to your first application, you were expressly informed, that my doctrine "afforded, what I thought, a full extension, and, thereby, a complete simplification of the syllogistic theory [3];" subsequently, (2d Nov.) that the Requirements then sent "showed the nature of that doctrine in one of its halves [11];" which doctrine was "simply that of Aristotle, fully developed [12]." Further, you could not be ignorant, of what even the tyro in Logic is aware,—that the existence in thought, of a quantity to be annexed to the predicate, in its technical enunciation, is a condemned paradox, a logical neologism. The second proposition of the Requirements, which avows such heresy, could not therefore but strike you; nor could the inculcation of this new principle, and at their very outset, fail of revealing it to you, as the basis of my new doctrine. But, even to one wholly ignorant

* Edin. Rev. No. CXXXVI. An article, extracted and translated into German, French, and Italian; and still unanswered, for Professor Chevalier's pamphlet leaves it quite untouched.

† The reader is requested, before passing to the sequel of this Letter, to peruse the appended Correspondence;—this Correspondence containing the special evidence, on which Mr De Morgan's claim of having re-discovered the principle in question, is, by himself, disproved.

of Logic, this proposition would clearly indicate the fact,—that the supplying the ellipsis, prevalent in common language, of the quantity of the predicate (though what that is he certainly might not know,) made part and parcel of my logical theory. The words are of the plainest:—"State.—The reasons, why common language makes an *ellipsis* of the *expressed quantity*, frequently of the *subject*, and more frequently of the *predicate*, though both have always their quantities in thought." Limiting ourselves to this one proposition of the Requirements:—In saying that you "*did not comprehend these at all* [34]," you could not mean, that, as a logician, you did not understand from this article, and apart even from my letters, that the half system in question, was founded on the technical quantification of the predicate; far less, that as an ordinary individual, you could remain ignorant of such quantification, as a fact, being involved in the doctrine of which the Requirements were exponent. I must, therefore, interpret your language to mean—that, *at first*, and speaking *very loosely*, you did not comprehend the Requirements at all. But it may, indeed it must, in lenience, be supposed, that a subsequent study of the document obtained for you some illumination; that the principle of a quantified predicate gradually dawned on your understanding; that your cogitations were committed to writing; that you rapidly forgot the date and the external source of your new light; so that when the Prospectus arrived some two months thereafter, you conscientiously imagined ("finxisti, simulque credidisti,") that, in reading my words, you were reading your own doctrine. On this hypothesis, your good faith is preserved; and besides the illusion, we have little more than to suppose,—what is no marvel, —an error of recollection, in regard to the time and circumstances under which the new theory was first revealed to you. This is the more charitable opinion, to which, as I said, I am now inclined. But I can adopt it only in showing, for you, though against yourself, that the statement you have publicly hazarded, in regard to the date of your second view of syllogism, and to your personal originality therein, is wholly untenable. Untenable:—because, it would imply, *in the face of your character as a man, that you have acted contrary*;—i. *to all fair and candid and honourable principle*;—ii. *to the dictates of the most ordinary prudence*;—iii. *to the strongest tendencies of our common nature*;—and—iv. *in opposition to your character as an author, that you maintain an assertion belied by the whole doctrine of probabilities*.

Under these four heads I now, therefore, proceed to discuss your claim.

i. *Your claim and relative statements are erroneous, because incompatible with your character, as an honourable man.*—For these would implicate you—1°, *in a suppression of the truth*;—2°, *in a suggestion of the false*; 3°, *in an assertion of the false*.

1°, *Suppressio veri*.—An honourable man would not seek, would not receive, confidential information, without, on his part, candidly declaring to his informant whatever might affect the grant of information at all, or the quantity and quality of its contents; the informant is, therefore, entitled to believe, that whatever it is incumbent on the receiver to state, but is not stated, does not exist.—Now you gladly obtained information from me—I wanted none from you. Even from my first answer, you were informed of my possession of a new doctrine of syllogism; and without suspecting any want of common candour upon your part, I then frankly offered to your curiosity the satisfaction it was in my power to give.—But what, on your own shewing, was the proceeding towards me? Your doctrine, as contained in the First Notions, was published and patent; any mere expansion of that required, therefore, no dis-

closure. But if your statements be correct, (which they surely cannot be,) you "thankfully accepted" the offer of information, in regard to my peculiar doctrine [9, 17], whilst yourself adopting a new opinion; a change touching which you not only gave me no hint, but left, may led me to believe, that you still held by that doctrine, in relation to which, you had originally applied to me.—Was that fair or honourable? [7, 8, 9, 10, 17.]

But the uncandid concealment of the true does not alone, or principally, prove your statement to be morally impossible; there is further and higher evidence of this in the

2^a, *Suggestio falsi*.—For, at the time, when, as you now state, you had *two* systems, you are found writing to me, as if you had but one [7, 8, 9].—Nay, what is even more remarkable,—on the receipt of the Requirements, you specify, and exclusively, two subaltern and undeveloped articles [V. prs. 4, 7] as those, on which you might have to claim a secondary originality; virtually, therefore, acknowledging, that you had no claim to the principal and developed proposition [V. pr. 2], on which my system is established.

But there is still a stronger confutation of your statements, than even the suggestion of the untrue; for its accuracy would infer against you no less than the

3^a, *Assertio falsi*.—And this in two ways.

a, You assert of the Requirements [V.] that you "did not comprehend them at all" [34]. Now, the truth of this assertion is repugnant to that of the statement that you had excogitated the doctrine of a "definite [expressed] quantity of the predicate," (we need say nothing of the subject,) not only before the receipt of the Requirements, but before the receipt of any communication from me [37]. But, if your new doctrine were of so early a date, you must have read it in the propositions of the Requirements, as you read it in those of the Prospectus; and you never could have averred of the former,—"I did not comprehend them at all" [34].

b, Again, among the propositions in the Prospectus [VIII.] which are comprehended and "subscribed to," as your own, there are many which as equally undeveloped, are not more comprehensible than those of the Requirements, declared, one and all, to be "not comprehended." The contradiction is here insoluble, if we suppose your new view to have originated *before the receipt of the Requirements*, as before the receipt of the Prospectus; and your veracity can only be saved, on the supposition of a mistaken recollection, in regard to the date of its origin. On this supposition, the Requirements were at first, and even naturally, not comprehended, under the *old* doctrine. This was recollected; while, perhaps not less naturally, their influence was lightly forgotten, in originating the *new*.

ii. *Your claim and statements are erroneous, because incompatible with your character as a man of (more than) common sense.*

Had you excogitated, as you state, a new theory of syllogism, your grand aim must have been, to establish your rights as a discoverer. But the concealment practised towards all the world, and especially towards the individual from whom you sought information in regard to his peculiar views of syllogism, precluded you, and by your own act, from ever establishing a claim, even to personal originality,—*in the event of your informant's doctrine proving the same as yours*. For, if in these circumstances any one proclaimed—"I also am a discoverer," he would reap only derision for his pains. No person, therefore, even of ordinary intelligence would pursue this course; and you being a man of far more than ordinary intelligence, *multo minus* could it be

pursued by you. The statements, therefore, which involve you in this predicament, must be founded on a circumstantial mistake.

There is only one hypothesis, on which your statements can, even partially, be reconciled to your common sense; but as this would wholly compromise your common honesty, it must, also, be peremptorily rejected, as morally impossible. It is:—That receiving the glimpse of a new doctrine of syllogism, you, at the same time, were made aware, that the said doctrine was mine. On the standard, therefore, of a grovelling prudence, (grovelling, I say, for absolutely speaking, "Honesty is the best policy,") concealment might be thought expedient:—1°, to obtain ulterior information;—and 2°, to avoid a premature identification, with its disagreeable consequences.—I ought, however, to apologise for even alluding to this possibility.

iii. *Your claim and relative statements are erroneous, because incompatible with your character, as a man, not destitute of the common feelings of human nature.*—This is shown in two ways.

1°, Your logical discovery, you say, was wrought and written out, after the *third*, and before the *ninth* of last October [37]; of its importance, (and the quantification of the predicate is the most important step in Logic, since the days of Aristotle,) you avow yourself aware [25]; and you hide it from the whole world until, some *three months thereafter*, on receiving, and that for the *second* time, a summary of my doctrine, you, for the *first* time, confess to me your possession of a logical discovery, and the coincidence of that discovery with mine [29].—Now is this natural? Is such concealment what one actuated by normal impulses would practise? Let us interrogate the best observers of mankind.

"It has been well," (says PLATO, and he has several passages to the point,)—"it has been well, I think, observed by HOMER,

Through mutual intercourse and mutual aid,
Great deeds are done and great discoveries made;
The wise new wisdom on the wise bestow,
[Whilst, the lone thinker's thoughts come slight and slow.]

For in company we, all of us, are more alert, in deed and word and thought. *And if a man excogitate aught by himself,—forthwith, he goes about, to find some one, to whom he may reveal it, and from whom he may obtain encouragement, aye and until his discovery be completed.*"—The same doctrine is maintained by ARISTOTLE, and illustrated by the same quotation; (to which, indeed, is to be referred the adage,—"*Unus homo, nullus homo.*")—"We rejoice," says THEMISTIUS, "in hunting truth in company, as in hunting game."—"LUCILIUS—"scire est nescire, nisi id me Scire alius scierit;"—paraphrased in the compacter though far inferior verse of PERSIUS,—"*Scire tuum nihil est, nisi te scire hoc sciat alter.*"—CICERO'S CATO testifies to the same truth:—"Non facile est invenire, qui quod sciat ipse, non tradat alteri." And SENECA:—"Si cum hac exceptione detur sapientia, ut illam inclusam teneam nec enunciem, rejiciam. Nullius boni, sine socio, jucunda possessio est."—But it would be idle to multiply confirmations of so confessed a principle. We must, therefore, in as much as you are not yourself a psychological monstrosity, abandon your hypothesis of a three months' concealment, of what other men would, from the first, have been burning to reveal.

2°, Again, on the testimony of your Letters,*—up to the moment of re-

* The Addition [38] seems contradictory of the Letters. For whilst the Letters explicitly deny all comprehension of the Requirements [34], the Addition appears to admit these as "information in general terms."

ceiving the Prospectus, you were wholly unaware that your discovery had been anticipated—anticipated by me—by me your correspondent—and by your correspondent, of all mankind. In these circumstances, disappointment would have been natural, and astonishment not to be repressed.—

“ Can such things be,
And overcome us, like a summer cloud,
Without our special wonder !”

But your Letters indicate nothing of the kind. What occurs, is only what they seem to expect. We must, therefore, from your Addition [38], correct or supplement your Letters [34] :— hold that the Requirements, *after the first*, were found not wholly incomprehensible, and that the Prospectus, when it came, in fact, was seen only to state more in detail, “ *the information*” which the Requirements had previously taught more “ *in general terms*.”—On this supposition the present anomaly is smoothed down ;—but at the expense of your claim and statements.

iv. *Your claim and statements are erroneous, because incompatible with your character as an author, these being, by the science of probabilities, shown to be absurd.*

You are an author—an authority, on the calculation of chances. I have no objection, either to the doctrine or to the doctor, and should willingly leave you to rate, if you can reach it, the improbability of a claimant being personally original, under the following circumstances :

1°, The matter of discovery—the point in question, has for thousands of years, and by millions of relative proficient, been scouted or overlooked.* The chances, against any one proficient realising the discovery, thus amounts to millions. 2°, But the odds will be doubled, if we suppose this done by two proficient, independently of each other ; and multiplied, in a rapidly rising, though indefinite, ratio, against the second, should he be,—3° a contemporary ;—4°, a fellow-countryman : 5°, and still more, in general, a scientific correspondent or acquaintance of the first.

This fivefold improbability the case involves. But, although, already, transcending all definite imagination, this is but a petty fraction of the mass of unlikelihoods which it finally accumulates.—Denoting the admitted discoverer by Z, and the claiming re-discoverer by A :

6°, A, then, is, further, no proficient—no thorough student,—in the science [32], the most learned and inventive masters of which he virtually declares himself to have surpassed.

7°, The point in question had not only been discovered, but at the time of its asserted re-discovery, having for many years been academically published, might thus, at first or second hand, then be known to thousands ; 8°, and whilst A is curious in such novelties, he has, within his sphere,

* I would not, however, be understood to say, there are none who ever held, that the predicate had always a quantity in thought, and that such quantity ought to be technically expressed. Alas for the principle if this were true ! Since excogitating the new view I have been curious in taking note of any previous indication of the doctrine ; and shall state the results in the second part of my proposed Essay towards a New Analytic. I may, however, now mention, that the opinion is found, as a paradox, even in ancient times. This is shown by the elaborate but futile refutations, in more than one of the ancient logicians ; and, whilst the true doctrine has been overlooked, these, without criticism, have been blindly transmitted, through Aristotle's authority, from logician to logician, down to the present hour. To the present hour, these refutations stand unrefuted,—the consequences of the principle undeveloped,—the principle itself in darkness.

those who could afford him ample information. Z is not only A's contemporary and countryman, and correspondent; but became his correspondent recently, yet prior to the alleged re-discovery;—9°, on matters special to the science in question, and on these alone,—10°, in consequence, too, of an application from A, requesting information in regard to their history [1.]

11°, Z confidentially communicates to A, in its principle and chief results, one portion of his new doctrine;—12°, in compliance with A's wishes [9, 17];—13°, briefly, but in distinct propositions and terms of the clearest. (V.)

14°, But although A accepted, and solicited, a confidential communication of Z's new doctrine; as he now openly professes to have possessed [23—31, 37], so he now virtually confesses to have disingenuously concealed, a relative doctrine of his own.

15°, Again; A afterwards avers, that he was unable, even to surmise the purport of the said communication, protesting, in fact, that its contents were what he "*did not comprehend at all*" [34]. 16°, At the time, however, he made no complaint, virtually treating the communication as comprehensible [14]; whilst, ultimately, he even speaks of it as "*information conveyed in general terms*" [38.] 17°, But if correct in his assertion, that he had previously thought and written out the doctrine of that communication [37, &c.,], A could not but have comprehended it, certainly in principle, probably in detail.—18°, Nay, when it is not to receive the information of Z's discovery, but to appropriate that discovery as his own, A can, without assistance, both comprehend (or profess to comprehend) and "*subscribe to,*" among others, the least developed of those propositions, which he otherwise protests that he "*did not comprehend at all*" [23, 31, and V. compared with VIII.] 19°, In his eagerness, indeed, to appropriate Z's new doctrines, A runs himself into contradiction with the very title of his system [VIII. pr. 1, 24.]

Nay more:—20°, Whilst, after a ten days' possession of the aforesaid communication, A, in writing to Z, hints at no revolution in his views, but on the contrary, apparently speaks of it as if his mind were still unchanged [14];—21°, he yet, soon thereafter, in disparagement of the one doctrine,—which had previously occupied him,—which he complacently denominated "*his system,*"—in reference to which he had applied, and hitherto alone spoken to Z,—which he had sent in abstract, to a learned Society,—and was in the declared course of giving to the world, entire;—this, his one cherished system, he suddenly, after Z's "*not comprehended*" information, supersedes, by starting a second [23—38.]—22°, This second system, "*from a perfectly different point of view*" [35] is in principle, and bating all imperfections of detail, manifestly that of the "*not comprehended*" communication [V. VIII. IX. X. XII.]; 23°, exclusively, too, of the one portion of Z's doctrine, therein alone contained;—24°, and descriptively entitled, even by analogous terms [24.]—25°, Thus, while A is professedly unable to "*comprehend at all*" a doctrine in the words of its author [34], he yet holds himself out, as able to be, nay as actually being, an author of that very doctrine itself [23—38.]

26°, By a mighty marvel,—after a studious concealment of the re-discovery, for some three months, its date is found to fall within the very week,—the one short interval of six days, during which alone A could, with any show of verisimilitude, lay claim to independence [37.] This he could, not before the *third* of a certain month, for, in that case, A's own paper to the learned Society would disprove the pretension; not after the *ninth* of the same month, for on that day (if he knew it not before) he would learn, for certain, the

fact of Z's possession of a new doctrine of Syllogism, the fact of its academical publication, and the consequent fact of the persons from whom the requisite information could be easily obtained [2].

27°, By a second marvel, in this memorable week, A of himself makes Z's discovery not only while ignorant of Z's priority, but *after writing to Z, yet before receipt of Z's answer* [37.] ;—28°, By a third marvel, some three months after, Z, the not comprehended communicator of his own novelty, was, of all mankind, “the first person to whom his having such a method was mentioned” by A [29.] ;—29°, but, not without an indirect attempt at defence, as an original excogitation by A himself [23, 26, 30, 33.]

Such, and so more than wonderful, are the correlations of this unexampled pair, whose train of mutual coincidence—whose “Pre-established Harmony”—would be too startling even for romance,—on the supposition always, that the one did not borrow from the other ; for on this alternative, all relapses into common-place. But the rising marvel in the hypothesis does not yet cease ; the supernatural must intervene. 30°, For on the one hand, it was by superhuman illumination, and no human knowledge, that A was lighted to the re-discovery of what had so long lain hid ;—31°, and, on the other, it is only on a miraculous occultation that we can explain his “not comprehending at all” the disclosure, which, to all common eyes, lay plain and patent in his hand.

But A is not only *supernatural*, he is also *unnatural*. 32°, For he manifests none of the natural, none of the irrepressible feelings of a consciously independent thinker,—no astonishment, no disappointment, and the like,—on Z's priority coming so unexpectedly to light.—33°, On the contrary, he acquiesces in it, as a matter of which he had been all along aware [IX. X.] ; and now laying claim, of course, to only secondary originality, he manifestly betrays his augury of a disallowance even of that [30.] ; nay, humiliates himself so far, as to volunteer some documentary evidence, and that, too, wholly irrelevant, in support of his allegations [30.] ; whilst hoping, as it would seem, to disarm Z as an accuser, though previously only anxious “to satisfy him” [30], he actually bespeaks, nay assumes, Z's testimony, against some other and these imaginary parties, who possibly, he professes to fear, may hereafter dispute his (A's) asserted rights. [33.]

34°, And yet A, the individual who thus betimes so cautiously prepares his documents, and marshals his witnesses, against visionary, nay impossible, opponents, rushes headlong and without defence, upon a charge of plagiarism, by concealing, from Z, what, in the circumstances, he was bound to have disclosed, and what every original and candid thinker would, from the first, have been anxious to promulgate. Nay, in explanation, or extenuation, of his self-condemning silence, and, as if his originality were not, and were not alone, in question ;—35°, A actually declares, in one place, that he was desirous to have Z, (from whom he could appropriate anything,) detail his doctrine, before that he, (from whom Z could appropriate nothing,) should divulge his own [36.] ;—36°, and in another, though with ingenuity to invent, and sense to appreciate, the discovery, professes himself “willing to avow the importance he had attached to it in his own mind,” only on becoming aware, that Z had previously done the same [25].—37°, Had there, however, been on A no moral obligation for a candid disclosure to Z ; still a three months' concealment of his discovery, from all the world, would be most unnatural and most unlikely. [29, 37, and p. 8.]

38°, And why does A not perform a certain tentative incumbent on him?—

Because, professing to be able to do it by two methods, he holds himself excused from showing his ability of doing it by any [35.]

39°, But drawing to a close :—If the tree be known by its fruits, the scientific system is here transplanted, not of native growth. In A's hands it is wholly barren. If A understood, as he professed, the result of Z's exposition, it is strange that he has turned to no better account the principle of which he maintains himself a re-discoverer. But, says PLATO, "Art is easy, Use is difficult ;" and application is difficult, to all but an inventor.

40°, In fine :—To all this, there is opposed only A's assertion [29]. But A's assertion, though honestly expressing his belief, as against evidence and on his own behalf, is of no avail ; for the shrewdest observers of mankind concur in noticing that men,—intelligent men,—honourable men, believe, in general, according to their wishes. In the words of a sainted Father of the Church :—"*Quæ volunt sapiunt, et nolunt sapere quæ vera sunt.*" (See also p. 2.)

Under this mountain of unlikelihoods, what infinitesimal, I now ask, will it require to measure the probability of A being an independent speculator,—if this be not, at once, derided, as a contingency too ridiculous—too palpably null, for computation ?

But I have done ; performed, I presume to hope, not ineffectually, the task which your illusion and my own facility combined to impose on me. If I satisfy others of the futility of your claim, I shall be content ; to convince yourself,—I say it without disrespect,—I never had an expectation. For to quote the full testimony of St CHRYSOSTOM.—"What a man wishes, that he believes ; what he believes, that he takes for granted : *those who contradict him, they are babblers.*"

I remain,

Sir,

Your most obedient Servant,

W. HAMILTON.

[The following Correspondence is given full and unaltered. The words in *italics* are such as stood underlined in the original; and any insertion is marked by an inclosure within square brackets.]

CORRESPONDENCE

BETWEEN

PROFESSOR DE MORGAN AND SIR WILLIAM HAMILTON.

I.

From PROFESSOR DE MORGAN.

SIR,—[1.] With proper apology for intruding a stranger's inquiries upon you, I beg leave to ask a question, which, I judge from your writings, no one in this country is so likely to answer.

Having been recently engaged in an examination of the Aristotelian syllogism, which has led to results which I intend to publish, [N.B.] and which certainly are not to be found mentioned in, it may be, a couple of dozen of works on formal logic which I have examined, from the invention of printing till now;—I am impeded by the absence of all history of the *syllogism*, its form and fabric, from the works which have fallen in my way on the history of philosophy. Can you inform me whether there exists such a history to any greater amount, for example, than is prefixed to the older editions of Aldrich, which amounts to very little?

If you can indicate any line of books in which what I want may be found, or any one book which deals in authorities, you will confer a great obligation upon me,—I remain, Sir, your obedient servant,
A. DE MORGAN.*

UNIVERSITY COLLEGE, LONDON, December [September] 30, 1846.

II.

From SIR WILLIAM HAMILTON.

ABERDOUR, FIFESHIRE, 7th Oct. 1846.

[2.] SIR,—I have to apologise for the delay in answering your letter of the 80th September, which, in consequence of being absent from Edinburgh, did not reach me till yesterday. In reference to your inquiry about the history of the Aristotelic theory of the syllogism, though I am aware, I believe, of all that has been written on the subject, I may safely say that there is no history which gives any information of the smallest interest in regard to its progress; there having been, in fact, no progress made in the *general* development of the syllogism since the time of Aristotle, and in regard to the few *partial* improvements, the professed historians seem altogether ignorant. I shall not, therefore, refer you to the many nominal histories where you will find nothing of importance recorded; but it will give me much pleasure to communicate to you what I have discovered touching the authors of any particular doctrines you may specify.

[3.] I have for many years taught in my class what I think affords a full extension, and thereby a complete simplification of the syllogistic theory; [N.B.] and through the notes and essays of my students this development of the doctrine has obtained considerable publicity, [N.B.] though I have not yet given it to the world through the press. [4.] Should you feel any curiosity on this

* After this letter was despatched (after indeed the paper on Syllogism for the Cambridge Philosophical Society was finished), but before the receipt of my answer—that is, between the 8d and 9th Oct. 1846—did Professor De Morgan excogitate and write out his second theory of Syllogism, according to his statement made (27th Feb. 1847), in his addition to the aforesaid paper [37.]

matter; or should you wish for a list of the professed histories of logic, I will do what I can to satisfy you on my return home, about a fortnight hence, when I shall have my books at command. It would save trouble, if you would state the historians you have already consulted. Do you read German? [5.] I have your little work on logic, published some years ago, which I read with much interest.—I remain, &c.

III.

FROM PROFESSOR DE MORGAN.

DEAR SIR,—[6.] I am much indebted to you for your letter of the 7th. I was not aware till after I had sent it of your state of health; and I am fully sensible of the additional obligation which your attention to my request under such circumstances imposes.

I was afraid, from all I could learn, that the history of the syllogism was unattainable. But the assertion of such a negative was not for me, nor for any one that I know of, who has written on the history of philosophy in Britain, except yourself. There is one friend of mine whose depth in the history of such subjects is very unusual, but who will not write;—I mean Dr —, of the — College at —. His opinion, on the inaccessibility of the information I asked for, coincides with yours.

[7.] Hoping that what I shall hear of the state of your health will warrant my acceptance of your kind offer, I shall consider how far I can introduce the history of this subject into a work which I am projecting,—a wider development of formal logic than is contained in the slight tract* which I am much gratified to find has attracted your notice.

[8.] But in the meantime, I have sent to the Cambridge Philosophical Society a paper on the syllogism,† followed by one on the probabilities of combined argument and authority.

[9.] My system,‡ as regards the syllogism, has little in common with the old one.§ [10.] As you state that you also have given in lectures your own views of the structure of the syllogism, and as I am therefore bound to save your rights, if it should happen we have anything in common, I shall thankfully accept your offer of communicating to me the heads of your system, so far as it differs from that received. Perhaps you have a pupil whom you could trust to communicate so much. And I should also ask permission, after restitution, of course, of all in which you have preceded me, to complete my paper, by giving my own account of the remainder of what is peculiar to you.—I remain, Dear Sir, your much obliged servant,

A. DE MORGAN.

UNIVERSITY COLLEGE, LONDON, October 12, 1846.

IV.

FROM SIR WILLIAM HAMILTON.

EDINBURGH, 2nd Nov. 1846.

DEAR SIR,—[11.] I have been longer than I anticipated in answering your last letter. I now send you a copy of the Requisites for the prize essay, which I gave out to my students at the close of last session. It will show you the nature of my doctrine of syllogism in one of its halves. The other, which is not there touched on, regards the two wholes or quantities in which a syllogism is cast. I had intended sending you a copy of a more articulate statement, which I meant, at any rate, to have drawn up; but I have not, as yet, been able to write this. I will send it when it is done. [12.] From what you state of your system having "little in common with the old one,"

* There is here no hint of any new view.

† This paper thus contains Mr De Morgan's *single* doctrine.

‡ "My system." Thus, *only one*, as yet.

§ No allusion is here made to any second theory, to any new view of the author.

This letter thus manifests throughout, that Mr De Morgan had, at the time, *only a single* doctrine of syllogism. To suppose that he had two, concealing the fact, nay virtually professing that he had but one, is to suppose—"quod dicitur nota." The *suppression* *veri* would in fact be equivalent to the *suppositio falsi*. *Incredulus odi.*

and from the contents of your "First Notions," we shall not, I find, at all interfere; for my doctrine is simply *that of Aristotle fully developed*.*

[13.] It will give me much pleasure if I can be of any use in your investigations concerning the history of logical doctrines. I have paid great attention to this subject, on which I found that I could obtain little or no information from the professed historians of logic; and my collection of logical books is probably the most complete in this country. But, as I mentioned in my former letter, it is only in subordinate matters that *abstract* logic has made any progress.

I remain, &c.

V.

SIR WILLIAM HAMILTON'S REQUIREMENTS.

[Propositions sent on 2d. November to Professor De Morgan, but "*not comprehended at all*" by him ([34]); being the Requirements given out at the close of the session 1846, by Sir W. Hamilton, for his summer prize.]

ESSAY ON THE NEW ANALYTIC OF LOGICAL FORMS.

Without wishing to prescribe any definite order, it is required that there should be stated in the Essays:—

1°. What logic *postulates* as a condition of its applicability?

2°. The reasons why common language makes an *ellipsis* of the *expressed quantity*, frequently of the *subject*, and more frequently of the *predicate*, though both have always their quantities in thought.† [N.B.]

* I had, therefore, no scruple in giving Mr De Morgan an unreserved summary of my doctrine; which, partially at least, I could do without trouble or delay. By sending him a copy of the Requirements for my last vacation essay. I confess to a strong dislike of having my "rights saved;" and had Mr De Morgan not led me to believe that his doctrine and mine stood wide apart. I should have thought twice before despatching my communication. But after all, though *inaccurately* described, Mr De Morgan's doctrine, whether as contained in his First Notions or in his Cambridge Paper, (exclusive of the Addition), has, except on one point, no analogy with aught of mine.

† The full *Ersepectus* of the *Essay towards the New Analytic* will be found at the end of this pamphlet.

‡ The "*Indefinite*" (the non-expressed extension) of the *subject*, though given by Aristotle as constituting a peculiar quantity of propositions, has been long abandoned by logicians. In requiring, therefore, that the quantity of the *subject* should be explicitly stated as implicitly thought, there is required nothing new. It is only in postulating the overt quantification of the *predicate* that the foundation is established for a logical doctrine essentially differing from the common. I may observe, however, that logicians, in abandoning one error of Aristotle, have themselves run into an opposite. The want of a pre-designation in language, may, and often does, indicate the indetermination of the mind as to whether a term should be taken in the whole, or in part of its extent. It thus denotes—*some, at least*; and this applies on the old doctrine to the subject, on the new to the subject and predicate.

§ Against the following (omitted) clause, which is not subscribed, as probably not understood, Mr De Morgan places a "7"

¶ In underlining this, Mr De Morgan does not notice that my use of *definite* and *indefinite* is different from that of the logicians, and of Mr De Morgan himself. Did he really adopt my sense of these terms, and retain his own definition of his second system, he would have *no particular* (or, as I call it, *indefinite*) propositions at all. See [24.]

VIII.

(Out of order.)

Extracts from SIR WILLIAM HAMILTON'S PROSPECTUS.

[Propositions sent to Professor De Morgan, on 28th December, and not *only comprehended*, but, as expressing his new doctrine, "*subscribed to*" by him, ([23, 31]); from Sir W. Hamilton's published Prospectus† of his]

ESSAY TOWARDS A NEW ANALYTIC OF LOGICAL FORMS.

That we can only rationally deal with what we already understand, determines the simple logical postulate,—*To state explicitly what is thought implicitly*. From the consistent application of this postulate, on which Logic ever insists, but which Logicians have never fairly obeyed, it follows:—

That, logically, we ought to take in to account the *quantity*, always understood in thought, but usually, and for manifest reasons, elided in its expression, not only of the *subject*, but also of the *predicate*, of a judgment. This being done, and the necessity of doing it, will be proved against Aristotle and his repeaters, we obtain, *inter alia*, the ensuing results:—

1°. ... The only indefinite, is *particular*, as opposed to *definite quantity*. [...] In fact, *definite* and *indefinite* are the only quantities of which we ought to hear in Logic; for it is only as indefinite that particular, it is only as definite that individual and general quantities, have any (and the same) logical avail.§

3°. Conversion of propositions on the common doctrine.

4°. Defects of this.

5°. *Figure and Mood of Categorical Syllogism and Reduction*,—on common doctrine. (General Statement.)

6°. Defects of this. (General Statement.)

7°. The one *Supreme Canon* of Categorical Syllogisms.

8°. The evolution from this canon of all the *Species* of Syllogism.

9°. The evolution from this canon of all the *General Laws* of Categorical Syllogisms.

10°. The error of the *Special Laws* for the several *Figures* of Categorical Syllogisms.

11°. *How many Figures* are there?

12°. What are the *Canons* of the several *Figures*.

13°. *How many Moods* are there in all the *Figures*; showing in concrete examples, through all the *Moods*, the *unessential* variation which *Figure* makes in a Syllogism?

15°. [Mis-numbered.] What relation do the *Figures* hold to *Extension* and *Comprehension*?

16°. Why have the *second* and *third* figures *no determinate major* and *minor premises*, and *two indifferent conclusions*; while the *first* Figure has a *determinate major* and *minor premise*, and a *single proximate conclusion*?

17°. What relation do the *Figures* hold to *Deduction* and *Induction*?

15th April, 1846.

2°. The revocation of the *two Terms* of a *Proposition* to their *true relation*; a proposition being always an *equation* of its subject and its predicate.

3°. The consequent reduction of the *conversion* of *Propositions* from three species to one—that of *Simple Conversion*.

8°. A manifestation that *Figure* is an *unessential variation* in syllogistic form; and the consequent *absurdity* of *Reducing* the syllogisms of the other figures to the first.

4°. The reduction of all the *General Laws* of *Categorical Syllogisms* to a *Single Canon*.*

5°. The evolution from that *one canon* of all the *Species* and *varieties* of *Syllogism*.

4°. The reduction of all the *General Laws* of *Categorical Syllogisms* to a *Single Canon*.*

6°. The *abrogation* of all the *Special Laws* of *Syllogism*.

12°. Their (moods to wit) numerical *equality* under all the figures.†

8°. A manifestation that *Figure* is an *unessential variation* in syllogistic form; and the consequent *absurdity* of *reducing* the syllogisms of the other figures to the first.‡

VI.

From PROFESSOR DE MORGAN.

DEAR SIR.—[14] I have to thank you for your note of the 2d inst., and first to say, that from the abstract of the proposed thesis, I am not at all clear that I shall *not* have to claim only

* To his subscription of this proposition, Mr De Morgan appends the interrogation:—"Arithmetical?" I answer—No; logical, of course.

† His subscription of this proposition, Mr De Morgan qualifies by the following words, which show how little he saw of the most proximate consequences of his adopted theory:—"If I understand this rightly, I may underline it, I think. A. De M."

‡ Besides the propositions here extracted, Mr De Morgan underlines of the prospectus, on page 1, paragraph second, the words—"to complete and simplify the old;" on page 2, paragraph first, the words—"the system which, ——— supply;" on page 3, paragraph twelfth, the words—"in the *third* ——— simplicity;" paragraph fourteenth, the words—"thus, on ——— vanished."

secondary originality on several points.* When I see "defects of the common doctrine of conversion," and a "supreme canon" of categorical syllogism, I must wait for further information. [N.B.]†

[15.] When I said I had little in common with Aristotle, of course I did not mean that I could avoid, or evade, or refuse to comprehend all his system.‡ But I found his general principles not general enough, and his exclusions (to me) arbitrary.

I think I may yet be able to flatter myself that I have followed you in some points unknowingly.

[16.] I should be much obliged by any suggestion of yours on the following language:||

I want to classify the four forms A E I O more definitely. For instance, first I employ the following notation, which has been of singular service:—

Every A is B	A) B
No A is B	A. B
Some As are Bs	A B
Some As are not Bs	A : B

I took these because A . B and A B are mathematical symbols, which an algebraist is used to convert into B . A and B A; and A) B and A : B are also mathematical symbols, which he is equally used not to convert. [N.B.]

Now A) B, for instance, being consistent with B) A and also with B : A; we have a distinction here, which I want to signify.

Suppose A) B and B) A; then, of course, call A identical with B, or an *identical* of B.

But if A) B and B : A; call A a *sub-identical* of B, and B a *super-identical* of A.

Again, let *a* mean everything but A, or not-A, either in the whole universe, or in a *universe* made to be the universe of the proposition by express definition.

Let *b* be the same with respect to B. Then A and a I call *contraries*.

When A . B and a . b, then A and B are *contraries*.

But when A . B and a b, so that A and B exclude each other, but do not fill up the universe between, I call A and B *sub-contraries*. I hope logicians will surrender this term, for surely I and O were only called *sub-contraries*, because they come under, or fall within, A and E; come under them in the old scheme, $A \times^E O$ or fall short of them in extent of assertion;

But when A B and a . b, so that A and B fill up the universe and overlap besides, I call them *super-contraries* of each other.

Finally, when A B, A : B, B : A and a b, I call this for the present a *complex particular* proposition, for want of a better name.

Now, in these seven forms all assertions are contained.

I introduce these contraries into the analysis of the simple proposition. They give eight distinct modes of predicating by means of A and B, name[ly]

1. A) B	=	A . b	=	b) A	} Contradictories.
2. A : B	=	A b	=	b : a	
3. B) A	=	B . a	=	a) b	} do.
4. B : A	=	B a	=	a : b	
5. A . B	=	A) b	=	B) a	} do.
6. A B	=	A : b	=	B : a	
7. a . b	=	a) B	=	b) A	} do New Forms.
8. a b	=	a : B	=	b : A	

* "Secondary originality on SEVERAL POINTS." Therefore not on THE PRINCIPLE; as shown, indeed, articulately by the specification which follows of two subordinate propositions, (4, 7) to the exclusion of the one fundamental. (2.) Be it observed, that, absolutely "A NEW ANALYTIC of Logical Forms," was proposed; and this supposed a NEW PRINCIPLE.

† See in the Requirements (V), Propositions 4 and 7. It is strange that, in the circumstances, Mr De Morgan did not here state how he remedied the defects of the common doctrine of Conversion, which I subsequently borrowed from me; and what was his Supreme Canon of categorical syllogisms, which he has never yet shown? Why did he wait? If original, now only could he show his originality. I could take nothing of his; and he was bound, morally, to act candidly by me, and,—in prudence, not to create against himself the presumption of taking anything of mine. * But, as before noticed, the reference—the exclusive reference—to these two subordinate and inexplicit propositions of the "New Analytic," as those alone in which he might "claim secondary originality," proves that AT THIS TIME (though for ten days in possession of the Requirements) he had no theory of syllogism, founded on the paradox of a quantified predicate, as explicitly stated in proposition 2.

‡ Could I, or any one, so have understood it? And on my understanding, I regulated, of course, my communication. It is almost superfluous to say, that this only refers to the view sent to the Cambridge Philosophical Society. [8, 9.]

|| The following details further concur in showing that, AT THIS TIME, the thorough-going quantification of the predicate was no principle of Mr De Morgan; nor a universal and simple conversion.

B

[17.] I shall await with much interest any thing you may send on your views. [17*] I have asked several persons whether they have heard of your mode of teaching the syllogism, and find them all answer *no*, except to the fact of having heard of some variations from the old mode, which they are curious to learn.* Surely a syllabus of results would be a great service to those who pursue this subject, if you are not inclined to do more.—I remain, Dear Sir, yours very truly,
A. DE MORGAN.

7 CAMDEN STREET AND TOWN, Oct. [November] 14, 1846.

VII.

From SIR WILLIAM HAMILTON.

EDINBURGH, 28th Dec. 1846.

MY DEAR SIR,—[18.] I send you what I promised, a more articulate statement of my syllogistic theory. I should have made it, however, less summary, had the prize essays (of which I sent you the requisites) been given in; but the day of delivery is not till the 1st of January.—[19.] I have appended this statement to my edition of Reid's Works which has just appeared, and should have sent it to you sooner, but waited for an answer to my application to Mr ———, which ought to have come some ten days earlier. That gentleman was a pupil of mine six years ago, and obtained one of the highest honours of the class; he was, therefore, fully competent to afford you information, which I begged him to do, in regard to my logical doctrines, as they were taught so far back. I knew him to be a graduate of your college, and he tells me that he was for three years a pupil of your own. If you are still interested in the matter, you can, therefore, obtain from him, as an acquaintance, what information you wish, more agreeably than from a stranger. When he attended me, besides the twofold wholes in which the syllogism proceeds, the quantification of the predicate, and the effect of that on the doctrine of conversion, on the doctrine of syllogistic moods, on the special syllogistic rules, &c., were topics discussed, and partly given out for exercises. They were, in fact, then, mere common-place.

[20.] From your last letter it appears that you have much more in common with Aristotle than, from the one previous, I was led to expect. [N.B.]

[21.] I regret that you did not state on the two points in the requisites of which you speak—what your opinion actually is touching “the defects of the common doctrine of Conversion,” and “a Supreme Canon of categorical syllogisms.” This statement would at once have shown that your doctrine and mine were not coincident; for mine on these points is always dependent on the quantification of the predicate terms, and on this essential doctrine you do not surmise that we may be at one. [N.B.] From the printed summary now sent, you will see that I cure the defects of the common doctrine of conversion, simply by abolishing two of its species,—that by Accident, and that by Contraposition. I am curious to learn what is your Supreme Canon of categoricals. [N.B.]

As to your question touching the classification of A, E, I, O, I am sorry that I can afford you no assistance; in fact, I am afraid that I have not mastered it [your notation] sufficiently to speak of it with competent understanding; I could, indeed, give you my own modification of these symbols, but as that is very simple, and rests immediately on my fundamental doctrine, I do not think it would be of any use. It is, however, at your service.

I shall have the pleasure of sending you the fragment, now issued, of my Supplement to Reid. You will find in the first Dissertation a speculation in regard to the introduction of the term *Axiom* into mathematics. I should be glad to know what you think of its probability.—I remain, &c.

VIII.

[The Prospectus of the “Essay towards a New Analytic of Logical Forms” will be found appended to this pamphlet. Extracts from it are also given in contrast to the Requirements, [v.]

IX.

From PROFESSOR DE MORGAN.

Dec. 31, 1846.

MY DEAR SIR WILLIAM,—[23.] I received your obliging communication *this morning*, and am

* It is not wonderful that “several persons asked” should know little; it is only wonderful that they should have known so much more than Mr De Morgan, so much interested in logical matters; indeed, any thing at all of my lectures or of me. Could a pupil not have been interrogated? And of the competency of pupils in this respect, Mr De Morgan and myself had both previously spoken. [10, 3.]

now fully satisfied that I have, in one* of my views of syllogism, arrived at your views in substance, or something so like them,† that I could subscribe in my own sense to a great part of your paper. [See 31.]

[24.] I should call my view that of annexing definite‡ quantity to the subject and predicate of propositions, and drawing out a definitely quantitative conclusion.¶ [N.B.]

[25.] The importance you attach to it makes me willing to avow the importance I had attached to it in my own mind; § [26.] and to put all question about invention on a clear footing, I will state as follows:—¶

[27.] 1. The method to which I allude is not that now before the Cambridge Philosophical Society, but one thought of since.

[28.] 2. You are of course the first publisher of it, having given it in your lectures,—presuming mine and yours to be the same.

[29.] 3. You are the first person [N.B.] to whom I have mentioned my having such a method,** except puzzling†† a few friends with a challenge to reduce

{ Most Xs are As
Most Xs are Bs
Some As are Bs

to an Aristotelian syllogism. This, of course, is rather an approach to an indefinite form.‡‡

[30.] 4. To put myself in a position, should it ever be necessary, to satisfy you [N.B.] |||| as to what I am now talking of, I state that I mean what is contained in certain papers which I shall be able to produce; to wit, fourteen leaves of glazed draft paper, written on one side, numbered 1 to 14, headed, "On syllogisms of definite quantity," and severally beginning with the following:—

1. In this and the ...
2. One of the Ys
3. Case, and which
4. means that some
5. Followed by inference
6. Form of the syllogism (Table)
7. $v = b, w = c$ (Table)
8. u, it is the
9. This is truly
10. The *a fortiori* argument
11. Might have been adduced
12. b less than b
13. In order however
14. Question that can arise

[31.] 5. This chapter and its results I might express in your words, wherever they are underlined in the prospectus §§ which I return, hoping you will send another. [See 23.]

[32.] To mood and figure I have attended but little; what I get on those points will be from your hint, or from your book.¶¶

* This, the reader will remark, is the first confession that Professor De Morgan had more than one view of the Syllogism, and this after he had been, for the second time, informed of the purport of my doctrine. See [35, 36, 37, 38.]

† How can there be a doubt of an identity in principle? or indeed (as appropriated from me) of an identity in detail? [28.]

‡ See p. 15, note §, on pr. 2, 1°, of the Prospectus.

¶ This last is superfluous. As are the propositions (major and minor), so must be the conclusion of a Syllogism.

¶¶ This looks too like an attempt by Mr De Morgan to explain his previous unaccountable silence, as to having any such method, on the supposition that he had it. It is a difficulty for the milder hypothesis.

¶¶ The following details, especially that in [30], might be viewed as indicating a consciousness that I would not be "satisfied" with this unexpected appropriation of my doctrine. This also is for me a difficulty.

** Mr De Morgan, on his own statement, excoigates a doctrine, which affords a new and momentous development to the Aristotelian logic;—he attaches to it the high importance it deserves, [25.];—but he whispers it to no human ear, until three months are come and gone, when (after a studious concealment) he first reveals it as a re-discovery to the discoverer himself, on receiving, for the second time, a communication of the discoverer's doctrine. Is there not, therefore, a chronological confusion?

†† This "puzzle" has no reference to the new doctrine more than to the old; and it could not have puzzled a proficient, to give, at least, the vulgar solution.

‡‡ What Mr De Morgan means by this, I cannot divine.

|||| How will this statement ever "satisfy me," if these papers, supposing them to enounce my theory, are not proved to have been written anterior even to the receipt of my Requirements?

§§ This adoption of my Prospectus makes Mr De Morgan's total non-comprehension of my Requirements the more unintelligible, [34.] The contrast is given under [V.]

¶¶ And yet, though thus confessedly to seek in the very alphabet of the science, Mr De Morgan would be a logical inventor! What is here acknowledged in terms is sufficiently manifested from mistakes.

As I wish to forward this as speedily as possible, I will add nothing but my thanks for your attention,—I remain, Dear Sir, yours truly,

A. DE MORGAN.

X.

From PROFESSOR DE MORGAN.

MY DEAR SIR WILLIAM,—[33.] When I received your note, inclosing the abstract of your system of syllogism, I immediately (that is, as soon as evening came, which is my time for such things) wrote to you my note of the 31st ult.; because I wished, if need were, to establish against others [N.B.] (not against you, for you are my witness)* [N.B.] that I had independently [!] taken up the notion of 'quantification' of the terms of a syllogism.

[34.] I did not comprehend your instructions for the prize essay at all. [N.B.] It was your intention, I have no doubt, that they should suggest nothing even to a careless student at your lectures—still less to those who had not been in your class. [35.] And I did not mention my general canon of syllogism in my former notes, because in fact I have two, [N.B.] in two perfectly different points of view. [N.B.] [36.] And I wished to divine, if I could, from unsuggested communication from you, [N.B.] with which of the two systems, if with either, yours was connected.†

I will now briefly state the heads of both systems.

[I. NEW SYSTEM.] Aristotle either has, or has been interpreted to have two exclusions. I cannot see that he expressly rejects all he does not use; but his followers have made him appear to do so.

First, he allows, or his followers allow, no inference from anything but the *dictum de omni et nullo*. [?]

Secondly, he rejects, or half rejects, the use of contrary terms. *Not-man*, he says, is not the name of anything: afterwards he admits it as an *aoist* name.

On looking at a proposition so called, I find it is in truth several propositions; that is, there are several distinct assertions, each of which is a contradiction of the proposition.

The proposition, for instance,

Every A is B
asserts that

As and Bs are compared;

That coincidences are obtained;

That every A has been compared with one or more Bs;

That the greatest amount of coincidence is attained.

No A is B

asserts that

As and Bs are compared;

Exclusions are obtained;

That every A has been compared with every B;

That the greatest amount of exclusion is attained.

* In the letter of the preceding day it is expressly said, that the statement was intended to "satisfy me." [30, 25.] In the present, we have it here no less explicitly affirmed that such statement is to establish Mr De Morgan's independence, "not against me," but "against others"—(Who may these be?)—Nay, I am even cited as "his witness," though he, of all men, is the individual whose independence my evidence would go articulately to disprove.

† Strange!—Doubly strange! How, if Mr De Morgan had excoagitated a theory of logic, founded on the quantification of the predicate, before receiving the Requirements, these Requirements should "not have been by him comprehended at all," is inconceivable; and it is equally inconceivable how, "if not comprehended," so many propositions of the Prospectus, the same, or not more articulate, should have been both comprehended and adopted. Still it was necessary,—on his hypothesis, or rather (believed) fiction,—for he cannot say with Newton, "hypotheses non fingo"—that the Requirements Mr De Morgan should "not comprehend at all." For were it admitted that "comprehended" they had been, forthwith would emerge the question—Why conceal, conceal even by *suggestio falsi*, what he must, then at least, have recognised,—the fundamental identity of his new doctrine and mine? Thus the impossibility of otherwise vindicating himself against the charge of intentional deception, constrained Mr De Morgan (on his hypothesis) to disown all understanding of the Requirements, although he thereby compromised his assertion of being previously in possession of the doctrine they contained, and moreover placed his utter non-comprehension of the prior document in irreconcilable conflict with his comprehensive approbation of the posterior. In point of fact, Mr De Morgan could neither have been so ignorant of the one set of propositions, nor so cognisant of the other, as he here imagines himself to be. Is the hypothesis itself, therefore, not unreal? "Chimera Chimera parit."

‡ This I do not affect to understand. Why the professing an ability to do a thing in two ways should stand as an excuse for not showing an ability to do it at all, is what surpasses me; as little can I perceive what satisfaction there could be in a concealment, culpable under the circumstances, and of which the result is an unanswerable presumption of plagiarism. I may add, what enhances the marvel, that neither in the body of the Cambridge paper on Mr De Morgan's old theory, nor in the Addition to it, on his new, is there any attempt made at a supreme canon of syllogism.

It would indeed be most unfortunate, in the circumstances, if Mr De Morgan were here correct;—that he had been in possession of a theory, and yet refrained from intimating that fact to me. For how could he vin-

Now suppose propositions in which the quantitative part of the preceding is made more definite. Say that

$X_t \quad Y_u$		and	$X_t : Y_u$	
	mean			
Every one of t Xs is one or other of u Ys		No one of t Xs is any one of u Ys		

Let the *effective number* of cases in a proposition be the number which it makes effective in inference. Then the effective number in a positive proposition is the number of cases of the subject.

The effective number in a negative proposition is the number of cases of the *middle term*.

And the criterion of inference being possible, is that the sum of the effective numbers of the two premises (not both negative) is greater than the *whole number of cases* of the *middle term*.

And the excess is the number of cases involved in the inference, of all which are mentioned in the conclusion-term (or terms) of the positive premiss (or premisses.)

For instance, let b be the whole number of Ys in existence: I ask whether we can infer anything from

$X_t \quad Y_u$	effective number	t
$Z_w : Y_v$...	v

Answer, if $t + v$ be greater than b, we can infer

$$X_t + v - b : Z_w$$

Or if each of t Xs be one or other of u Ys, and no one of w Zs be any one of v Ys, then if and v together are more in number than there are Ys, we may infer that no one of $t + v - b$ Xs is any one of the w Zs just spoken of.

All the modes of inference are as follows:—

Let a be the whole number of Xs

b	...	Ys
c	...	Zs

t the number of Xs mentioned in the premiss.

u, v the numbers of Ys ... premisses.

w the number of Zs ... premiss.

Criterion.						
I.	$\frac{X_t \ Y_u}{Z_w \ Y_v}$ $= X_t + v - b : Z_w$	$t + w > b$	No Aristotelian syllogism ever arises from this case.			
II.	$\frac{X_t \ Y_u}{Y_v \ Z_w}$ $= X_t + v - b : Z_w$	$t + v > b$	$t = a \quad v = b$	gives	Barbara, 1	Bramantip 4
			$t < a \quad v = b$...	Darii, 1	Dimaris 4
III.	$\frac{Y_u \ X_t}{Y_v \ Z_w}$ $= X_u + v - b : Z_w$	$u + v > b$	$u = b \quad v = b$	gives	Darapti 3	
			$u < b \quad v = b$...	Disamis 3	
			$u = b \quad v < b$...	Datissi 3	
IV.	$\frac{X_t \ Y_u}{Z_w : Y_v}$ $[=] X_t + v - b : Z_w$	$t + v > b$	$t = a \quad v = b \quad w = c$	gives	Celarent 1	Cesare 2
				...	Camestres 2	Camenes 4
			$v = b \quad w = c$...	Ferio 1	Festino 2
			$t = a \quad v = b$...	Baroko 2	

dicare, on honourable grounds, such concealment,—aware as he must have been that an acknowledgement of this reticence on his part, would at once have arrested all confidence and information on mine? I here throw out of account the harsher supposition, that his second theory was, and as he knew, correspondent to mine; that its declaration would therefore, as he foresaw, have led, not only to a cessation of my communications, but to questions which it might have been equally difficult to answer, and awkward to leave unanswered. Is the ground of all this, therefore, not an illusion?

V.	$Y_a X_a$	[Criterion.] $u + v \supset b$	$u = b$	$v = b$	give	Felapton	Fesapo
	$Z_a : Y_a$		$v = b$	$w = c$...	3 Feriso	4 Fresison
						3	4
	$= X_a + v - b : Z_a$		$u = b$	$w = c$...	8 Bokardo	

[II. OLD SYSTEM.] [36.*] Now, as to my other system, (now before the Cambr. Phil. Soc.) which has no more reference to quantity than the ordinary ones, [N.B.] though, of course, it might be joined with the preceding.

By *contrary* terms, I mean those under one or other of which everything in the *universe* falls. But by the *universe* I do not mean the whole *universe* of thought, but any given part of it which may, for the time being, by express hypothesis, be made the medium of the argument. Thus, in the *universe* man, Briton and alien are contraries, Briton and Frenchman are not. Whatever the universe may be, A and not-A are contraries.

Let contraries be denoted by large letters, and corresponding small ones. Thus, every not-A is a, and everything is either A or a, B or b, &c.

Let	A) B	signify	Every A is B
	A . B	...	No A is B
	A B	...	Some As are Bs
	A : B	...	Some As are not Bs

There are eight modes of predicating with respect to the division of the universe into As and as, and Bs and bs. That is, eight *distinct* modes, for there are twenty-four in all, reducible to eight. The eight distinct modes are most symmetrically expressed thus:—

	A) B	a) b	=	B) A
	A . B	a . b		
	A B	a b		
	A : B	a : b	=	B : A

And we have

$\begin{cases} A) B \\ A : B \end{cases}$	$=$	$\begin{cases} A . b \\ A b \end{cases}$	$=$	$\begin{cases} b) a \\ b : a \end{cases}$	$\begin{cases} B) A \\ B : A \end{cases}$	$=$	$\begin{cases} B . a \\ B a \end{cases}$	$=$	$\begin{cases} a) b \\ a : b \end{cases}$
$\begin{cases} A . B \\ A B \end{cases}$	$=$	$\begin{cases} A) b \\ A : b \end{cases}$	$=$	$\begin{cases} B) a \\ B : a \end{cases}$	$\begin{cases} a . b \\ a b \end{cases}$	$=$	$\begin{cases} a) B \\ a : B \end{cases}$	$=$	$\begin{cases} b) A \\ b : A \end{cases}$

I consider the convertible propositions $A . B = B . A$ and $A B = B A$ as absolutely identical.

My new forms $a . b$ and $a b$ are as follows:—

First $a b$, or some things which are not As, are also not Bs, merely says that A and B are not contraries. But the other side of the contradiction $a . b$, or there is nothing which is neither A nor B, does not say that A and B are contraries. That they divide the universe between them is true, but they may overlap and hold a part in common. That A and B are contraries requires $A . B$ and $a . b$ both.

Next, I affirm that with respect to this consideration of division into A and a, &c., the distinction between the positive and negative proposition, and between the universal and particular, is an accident of language.

Suppose we have the terms, A and B, and can say Every A is B. A Frenchman has not the term B it may happen, but only its contrary, b: he can only say No A is b. An Italian may not have the term A; to him the As may not be distinguished by a name from other Cs; he can only say Some Cs are Bs, unless the proposition itself makes him an inventor of a specific term for those Cs which are Bs. A Spaniard may be in both predicaments, and he must say Some Cs are not bs.

I affirm it, then, to be a universal canon of syllogism, that every legitimate inference, which has no more "quantification" (thank you for the word) than is effected in the formation of A E I O, as usually received—must be reducible to a syllogism with universal affirmative premises, either by substitution of a specific name belonging only to a particular some of a more general name, or by change of a term into its contrary, or both. Now, the only syllogisms with universal affirmative premises are

X) Y	+	Y) Z	=	X) Z
Y) X	+	Y) Z	=	X Z

I have some algebraical objection to this use of $+$, and shall try to look out for some mathematical symbol with which the process has more analogy; but let it stand for the present. By applying the above canon, and never admitting a syllogism in which one premiss is stronger than is necessary to the conclusion (in the manner done in my little tract), merely for condensation, I get

1. Six syllogisms which are Aristotelian, and from which, by strengthening a premiss, or by conversion, any Aristotelian syllogism may be produced.
2. Six syllogisms involving contraries, of X , of Y , or of Z .
3. Two of a peculiar kind.

1. Usual

$X) Y$	$+$	$Y) Z$	$=$	$X) Z$	} Universal
$X) Y$	$+$	$Y . Z$	$=$	$X . Z$	
$X Y$	$+$	$Y) Z$	$=$	$X Z$	} Particular
$X Y$	$+$	$Y . Z$	$=$	$X : Z$	
$X : Y$	$+$	$Z) Y$	$=$	$X : Z$	
$Y : X$	$+$	$Y) Z$	$=$	$Z : X$	

2. Unusual

$X . Y$	$+$	$z . y$	$=$	$X) Z$	} Universal
$Y) X$	$+$	$y . z$	$=$	$x . z$	
$x . y$	$+$	$z y$	$=$	$X : Z$	} Particular
$X : Y$	$+$	$z . y$	$=$	$X Z$	
$x y$	$+$	$Z) Y$	$=$	$x z$	
$Y : X$	$+$	$Z . Y$	$=$	$x z$	

3 Unusual

$X) Y$	$+$	$Z) Y$	$=$	$x z$
$x . y$	$+$	$z . y$	$=$	$X Z$

Now, if any one should say that there are no *new* syllogisms here, none but what can be reduced to Aristotelian forms, I am perfectly prepared to agree with him. But then it is to be remembered that there are no new syllogisms in the second, third, and fourth figures. It is enough for me to show that *legitimate inferences have been neglected* for the want of these forms.

This is an old trap for a beginner,

A man eats cheese,
A mouse eats cheese,
Therefore

The beginner who falls into the trap says, "a man is a mouse," and his teacher shows him, as he thinks, that *no* inference can be drawn. But there is an inference, namely, that there are things which are neither men nor mice, namely, all which do not eat cheese. This, of course, is a case of $X) Y + Z) Y = x z$.

There are a great many details with which I need not trouble you. As, for instance, every proposition which speaks ^{universally} of any name, speaks ^{particularly} of its contrary. Every proposition is then both particular and universal.

Again, every proposition is either convertible in terms, or convertible in contrary terms

$$\begin{array}{lcl} A . B & = & B . A \\ A) B & = & b) a \end{array} \quad \begin{array}{lcl} A B & = & B A \\ A : B & = & b : a \end{array}$$

The first thing which led me to this view was the general practical ignorance which I observed to prevail of the truth of this identity

Every A is B is the same as Every not- B is not- A .

Euclid, for instance, having proved the second, then proves the first: in

Every non-central point } is { not a point from which three equal
(of a circle) } straight lines can be drawn.

And then proves that

Every point from which three equal straight lines can be drawn } is { the centre.

The editors of Euclid (a class of persons whose logic I have never admired) have never noticed this, nor any other of the cases in which Euclid has shown his ignorance of the distinction between a new combination and a logical alteration of an old result.

I shall be curious to see what you say on the word *axiom*. You are, no doubt, aware that it is not a word of Euclid, and that Archimedes only uses it in the sense of prefixed *definition*.—I remain, Dear Sir, yours very truly,

A. DE MORGAN.

7 CAMDEN STREET AND TOWN, January 1, 1847.

XI.

From PROFESSOR DE MORGAN.

MY DEAR SIR WILLIAM.—On this evening I have received both your Notes on Reid and Mr —'s notes on your lectures, all which I shall have much interest in.

I searched first for the word *axiom*, which I have read attentively. In every thing you state of the mathematicians, you are right, to my certain knowledge. I had always the impression that it must have been from Proclus that the word took its mathematical sense; but whether earlier Platonists had used it, I could not tell. I shall have to read the *Syntaxis* shortly, and shall find out whether Ptolemy uses it—I never found it except, as you mention, in Archimedes.

Perfidious is not an improper word for the editors of Euclid; but the earlier ones have this excuse, that there was a general impression that *Euclid* only gave enunciations, and that *Theon* was the demonstrator. They thought they had as good a right to demonstrate their own way as Theon. Robert Simson is my utter aversion as an editor. If you will look into an article, *Euclid*, which I wrote in Dr Smith's Biograph. Dict., you will see, if you do not know it, what the Theon mania was; also a great deal of Euclid-bibliography, a little more correct than Fabricius, &c. gave it, or Scheibel.—I remain, yours, very truly,

A. DE MORGAN.

7 CAMDEN STREET AND TOWN, January 13, 1846, [1847.]

XII.

Extract from PROFESSOR DE MORGAN'S ADDITION.

[Dated 27th February 1847, to his paper in the Transactions of the Philosophical Society, Cambridge, "On the Structure of the Syllogism," dated 3d October 1846. This was received by Sir William Hamilton on 30th March, (*which the reader is requested to bear in mind*); but it has been thought better to arrange it by its date of writing. The Addition thus begins:—]

[37.] Since this paper was written, [3d Oct.], I found that the whole theory of the syllogism might be deduced from the consideration of propositions in a form in which *definite quantity* of assertion is given both to the subject and the predicate of a proposition. I had committed this view to paper, when I learned from Sir William Hamilton of Edinburgh, [by 9th Oct.], that he had for some time past publicly taught a theory of the syllogism differing in detail and extent from that of Aristotle.* [38.] From the prospectus of an intended work on logic, which Sir William Hamilton has recently issued, at the end of his edition of Reid, as well as from information conveyed to me by himself in general terms,† I should suppose it will be found that I have been more or less anticipated in the view just alluded to. To what extent this has been the case, I cannot now ascertain: but the book of which the prospectus just named is an announcement, will settle that question. From the extraordinary extent of its author's learning in the history of philosophy, and the acuteness of his written articles on the subject, all who are interested in logic will look for its appearance with more than common interest.

* Mr De Morgan does not mention, or mis-states:—1°, That this my *first* information was given, in answer to an application on his part; and to the effect;—2°, that "I had for many years taught—3°, "what I thought afforded a *full extension*, and thereby a *complete simplification* of the syllogistic theory;"—4°, that, "through the notes and essays of my students, this development of the doctrine had obtained considerable *publicity*;"—and 5°, that his alleged re-discovery of my view, and its reduction to writing, fell in the few latter days that intervened between his *first* writing to me and the receipt of my answer, more definitely between the 3d and 9th of October.

† This "information in general terms," must refer to the Requirements; and these Mr De Morgan, not a month before, avers that he "did not comprehend at all." [34.]

XIII.

From SIR WILLIAM HAMILTON.

Sir William Hamilton has been confined to bed for about two months by inflammatory illness, and has been unable to consider Professor De Morgan's last letters till to-day. He finds it necessary, in answering them, to refer to his letter of 2d November, *that inclosing* the Requirements for the prize essay, of which he has no duplicate; he will therefore be obliged to Mr De Morgan for a sight of it.

EDINBURGH, *March* 1846.

XIV.

From PROFESSOR DE MORGAN.

MY DEAR SIR WILLIAM—I am sorry to hear of your illness.

I return the heads of the essay which accompanied your note of November 2. As it is your only copy, I have taken a copy, and therefore need not trouble you to return it.

What I see most distinctly from Mr ———'s notes is the truth of 15^e in the list.

I cannot make out *from these notes* that we clash; but this I cannot decide on, and hold you much better able to settle.—I remain, faithfully yours,

A. DE MORGAN.

7 CAMDEN STREET AND TOWN, *March* 5, 1847.

XV.

From SIR WILLIAM HAMILTON.

Sir William Hamilton presents his compliments to Mr De Morgan, whose note, with its inclosure, he received to-day. He is sorry to be so troublesome, but Mr De Morgan has sent the wrong paper. The one Sir William requested to see was "his letter of the 2d of November, *that inclosing* the Requirements for the prize Essay." Of the Requirements themselves he has, of course, a copy. He accordingly returns them.

EDINBURGH, MONDAY, 8th *March* 1847.

XVI.

From SIR WILLIAM HAMILTON.

EDINBURGH, *March* 1847.

DEAR SIR,—Your note of the 10th, with its inclosure of my letter, I received this morning. Having again read over the whole correspondence, and being now sufficiently recovered, I proceed briefly to answer your communications of the 31st December and 1st January, ult., which reached me after I was laid up by a tedious inflammatory attack. From these (if I am not mistaken) it appears, that you claim for yourself the independent re-discovery of the fundamental doctrine of syllogism, which I privately communicated to you, and of many of its most important consequences, more fully developed in the printed prospectus. This claim, though it be only to secondary originality, I am altogether unable to admit. To me it is manifest, that, for the principle of the doctrine, you are wholly indebted to my information; and I cannot but think, that if you (though recognising always my priority), give forth that doctrine as a speculation of your own, you will be guilty—pardon the plain speaking—both of an injurious breach of confidence towards me, and of false dealing towards the public. Am I, therefore, correct in my understanding of your letters? I shall be glad to find that I am wrong.—I remain, &c.

XVII.

From PROFESSOR DE MORGAN.

MY DEAR SIR WILLIAM,—Your letter of the 13th instant, which I have read with astonishment, shews me the propriety of abstaining from further private correspondence upon the subject in question.

When my paper appears, which I expect it will do in a few days, I shall have the honour of requesting your acceptance of some copies, that you may be able to put them into the hands of those with whom you may think proper to advise.

I will not further allude to the hasty manner in which you have expressed your suspicions of an odious charge, except to state that it does not diminish the sincere respect with which I subscribe myself your most obedient servant,

A. DE MORGAN.

7 CAMDEN STREET AND TOWN, *March 16th, 1847.*

XVIII.

From PROFESSOR DE MORGAN.

MY DEAR SIR WILLIAM,—The more respect I have for you, and it is not small, the more I am bound to bring the charge you have made to a speedy, and, if necessary, a public issue.

You will before this have received some copies of my paper, containing the views which I communicated to you in writing. I trust you will soon be prepared either to retract or to maintain your assertion, that in publishing these views I am publishing what I received from you, and also your words following—"You will be guilty—pardon the plain speaking—both of an injurious breach of confidence towards me, and of false dealing towards the public."

I have no desire to hurry you unduly, and if you wish me to fix some later, but not unreasonably later, date, for the execution of the following plan, I will do it. But, failing the expression of such wish on your part, I beg to inform you that I shall wait till the 10th of next month for one of two things—your retraction—or the announcement of the specific time and manner in which you will publicly maintain the truth of the accusation which—excuse the expression—you have dared to bring against me. If I do not receive one of these two things by the date above specified, I shall then at once proceed to draw up a statement, of the publication of which, I need not say, you shall be duly apprised.

It gives me no pleasure to propose alternatives in so peremptory a manner. But there must be no mistake here: and all must see, that if you do not endeavour to maintain your words, it will not be for want of the most immediate and unequivocal challenge to do so. It gives me pain to adopt the necessary tone towards a person of your literary station and character. I am, with truth, yours faithfully,

A. DE MORGAN.

UNIVERSITY COLLEGE, LONDON, *March 25, 1847.*

P. S.—I shall transmit a duplicate of the above by the next post.

XIX.

From SIR WILLIAM HAMILTON.

EDINBURGH, *27th March, 1847.*

DEAR SIR,—I received your note of the 25th inst. this morning. Your publication, however, has not yet arrived, and as I write this late on Saturday night, it cannot now reach me before Monday, at soonest. I shall not, therefore, wait to see what, at any rate, must be perfectly irrelevant to expressions, bearing exclusive reference to your past letters. I assure you that it would afford me sincere pleasure to be enabled to retract what was written in my last, both painfully and unwillingly, but which I could not but write,—under the conviction of its truth. There are, however, only two ways in which this retraction is, for me, possible:—I must be satisfied, either that the meaning of your letters was not what I supposed them unambiguously to express, or that you have been acting under the influence of some intellectual error. It would truly gratify me—be it on the one or on the other of these alternatives—to be allowed to cancel all derogatory expressions and sentiments towards one, whom otherwise, on many accounts, I am so much inclined to respect. With your manifested feelings on the occasion I cordially sympathize; and assuredly my co-operation shall not be wanting, to lay the *whole* relevant documents fairly and fully before the public,—on the supposition always that your book prefers the same claim as your correspondence. You may accordingly depend on a statement from me of the grounds on which I felt myself constrained to advance the obnoxious allegations; and, if my health continues as it is, you will receive it before the time you specify. But as my public statement shall thus be communicated to you, of course

yours, in like manner, will be communicated to me, before their joint publication; and either, each made independent of the other, and final, or, if you be allowed to answer me, I must equally be allowed to answer you. I shall, therefore, before writing, await your determination on this point; and by that time, too, have read your publication.*—I remain, &c.

XX.

From PROFESSOR DE MORGAN.

MY DEAR SIR WILLIAM,—I very much regret that you have not received my publication, which was forwarded by a bookseller's parcel more than a week ago. I will make immediate inquiry about it, and, in the meantime, I transmit a copy by post.

With respect to the arrangement proposed by you—that the statements of each of us should undergo the other's inspection before publication, I altogether decline to accede to it. Unless you retract, I call upon you forthwith, should your health permit, publicly to attempt the proof of your accusation, as you value your own honour.

Let me be distinctly understood. When I proposed to you to retract, it was only because such an alternative is usual, and also a proper mark of consideration to the party addressed, in the manner in which I am obliged to address you. As far as I myself am concerned, I had much rather that you should attempt publicly to maintain your words; for, when such a charge has been made, I hold that the party charged is always subject to misconstruction, even after retraction, unless there be open and public statement of every circumstance.

Also let me be understood on this point. I will give you no explanations, except in print. Had you asked for them previously to expressing an opinion, you should have had them; but you preferred to make facts of your surmises, and inferences of your suspicions.—I remain, Dear Sir, your most obedient servant,

A. DE MORGAN.

UNIVERSITY COLLEGE, LONDON, *March 30, 1847.*

XXI.

From SIR WILLIAM HAMILTON.

To DR ——— †

16 GT. KING STREET, *4th April, 1847.*

MY DEAR SIR,—In sending you the letters, I may say, that on thinking the matter over, if Mr De Morgan can, on reflection, admit (what Dr ——— himself considers not unlikely‡) that he may be mistaken in regard to the date of writing down his *second* view, and that this may have been, not before first hearing from me, but after the receipt of the Requirements; in that case, I deem our approximation comparatively easy. For it is mainly on his assertions of the priority of his second view to the receipt of the Requirements that I founded my opinion of his proceedings.

* On receipt of Mr De Morgan's paper on the Syllogism, from the Transactions of the Cambridge Philosophical Society (I had supposed the publication of which he had spoken to be a book or pamphlet,) I immediately wrote to him. The purport was twofold: 1^o, to explain what, it struck me, might not be fully apprehended, in regard to my proposal of a joint publication of the correspondence; and 2^o, to show that I had previously systematised, in my own way, a point which, in principle at least, and to a certain extent, coincided with what I found in the body of Mr De Morgan's Cambridge paper. As the letter, however, when written, appeared not such as, in the circumstances, I could in propriety send to Mr De Morgan, I thought it more suitable to request my friend Dr ——— to transmit it to Mr De Morgan's friend Dr ———, leaving that gentleman to act in the matter according to his discretion. As he did not communicate the letter, beyond a partial summary of its contents, to Mr De Morgan, (retaining it, however, as a literary document) the letter forms no part of the *res gestæ*, and therefore cannot be inserted in this correspondence.

In explanation, it is here proper to state, that my friend Dr ——— had before this spoken to me, in consequence of a letter from his friend Dr ——— to whom Mr De Morgan had communicated our difference and relative documents in his possession, and who was desirous, if possible, to effect a conciliation between us. This highly respected gentleman represented Mr De Morgan as of a strictly honourable character, altogether incapable, therefore, of intentionally appropriating what did not belong to him. And as, on my part, nothing could be more gratifying, than to be enabled conscientiously to retract my charges; although no party to it, I was well pleased at the mediation.—The result is shown in the correspondence.

† This letter (see the conclusion) was transmitted to Dr ———, and by him communicated to Mr De Morgan. It forms, therefore, a document in the case.

‡ I find that *possible* should stand here for "not unlikely."

I should require nothing more than a statement in the Cambridge Philosophical Transactions, of his error upon this point, with a quotation either of the whole Requirements, or of the second Proposition alone, along with the title he gives to his doctrine, "that of annexing definite [*i. e.* expressed] quantity to the subject and predicate of propositions, and drawing out a definitely quantitative conclusion;" stating also, that I consider the second proposition of the Requirements alone, still more along with its subordinates, as sufficiently indicating to a logician the leading principle and main consequences of that part of my theory which is in question. I will have no scruple, if Mr De Morgan make this correction, in retracting *simpliciter* the hypothetical charges I advanced. If he do not, I conceive that no alternative is left me but to prove these charges publicly. If you find, through Dr ———, that Mr De Morgan is able to make the preceding admission, I will not allow the letter in which I made my allegation to stand in the way, but shall be happy to hold it *pro non scripto*. This you will, however, observe, does not of itself involve the explicit retraction of the said charges; that retraction must *follow* his correction of the date. I shall then find, however, no difficulty in making it; for if he have inadvertently confused dates, he may equally have deceived himself in regard to information and to authorship. If it save you trouble, and you think it otherwise proper, I have no objection to your transmitting this note to Dr ———, to whom, as well as to yourself, I feel greatly obliged for the conciliatory part you have both acted in this, to you, uninteresting business.—I remain, &c.

XXII.

From SIR WILLIAM HAMILTON.

EDINBURGH, 18th April, 1847.

DEAR SIR,—As I find that my letter of 13th March prevents you from favouring me with an answer to the inquiries which I would beg leave to propose, and as I am anxious that there should be no possible misunderstanding in reference to your claim of being an original excogitator of the doctrine of syllogism, founded on the expressed quantity of the predicate, I request your permission to withdraw that letter, to hold it, unconditionally, *pro non scripto*; expressing also my regret, that the allegations which it contains, though only hypothetical, were stated before you had an opportunity of explanation. This being done on my part, I trust, on yours, that you will not deny me the satisfaction of answering the questions which I am desirous to put, in the hope that the whole question may be solved by the detection of a mistake on the one side or the other.—I remain, &c.

XXIII.

From PROFESSOR DE MORGAN.

SIR,—The period mentioned in my letter of the 25th ultimo having elapsed, I now ask you whether you are prepared to give me your assurance that you will immediately proceed to the public maintenance of the charge contained in your letter of the 13th preceding; and further, I ask you, at what time I am to expect your publication?

In the event of your not giving an explicit and immediate answer to both these questions, I feel it necessary to inform you that I shall consider you as shrinking from the attempt to show cause for your assertions, and that I shall proclaim you as so shrinking in the public statement which I shall immediately draw up.*—I have the honour to be, Sir, your most obedient servant,

A. DE MORGAN.

UNIVERSITY COLLEGE, LONDON, April 12, 1847.

XXIV.

From PROFESSOR DE MORGAN.

Mr De Morgan presents his respects to Sir William Hamilton, acknowledges the receipt of his note of the 13th instant, and will answer it in the public statement which he is now preparing.

UNIVERSITY COLLEGE, LONDON, April 16, 1847.

* Mr De Morgan thus writes *after* he had read in my letter of 4th April, (XXI of the Correspondence) the passage, "if he (Mr De Morgan) do not [*state his error, &c.*] I conceive that no alternative is left me but to prove these charges publicly."

XXV.

From SIR WILLIAM HAMILTON.

Sir William Hamilton presents his compliments to Professor De Morgan, whose note of the 12th he delayed answering until he received the reply to his own of the 13th. Sir William Hamilton is surprised that Mr De Morgan could think the intimation he required even possible, so long as the conciliatory intervention of Mr De Morgan's friend, Dr ——— was not ended; and ended, in effect, it was not, until Mr De Morgan's last note terminated all hope of the difference being resolved into a mistake on the one side or the other. Sir William Hamilton has now, therefore, no alternative but to state the evidence on which he disputes Mr De Morgan's claim to an independent discovery of the new principle of syllogism; and is too confident of the strength of his position to deny his opponent the advantage of seeing the one case before publishing the other. Sir William will therefore proceed without delay; and though he has at present much else to do, hopes, if health permit him, to have his proof written during the present week, and printed in the course of next. There is, however a strike at present among the Edinburgh printers; what effect this may have in retarding the publication, Sir William Hamilton is unable to say.

EDINBURGH, 20th April, 1847.

XXVI.

From PROFESSOR DE MORGAN.

SIR,—I beg to acknowledge the receipt of your note of the 20th instant, which reached me this morning. My statement is at the printer's, and it is my intention to proceed with it without reference to your tardily expressed intention of coming forward to prove your own charge.*

I was no party to the correspondence between Dr ——— and Dr ———, unless expressing no objection to Dr ——— writing on a subject which I had mentioned to many persons, makes me one. I believe Dr ———'s letters (from which I have heard extracts read) state as much.†

I regret that you should not have been able to collect, from letters written in so determined a tone as mine, evidence enough of my being no way disposed to admit any thing short of a public discussion or a proper retraction, and, since the 10th instant, of my resolution to refuse even the latter.—I have the honour to remain, Sir, your most obedient servant,

A. DE MORGAN.

UNIVERSITY COLLEGE, LONDON, April 23, 1847.

* It is certainly strange that Mr De Morgan could *thus*, again, write, after my letter of the 4th April [XXI] had been communicated to him.

† Undoubtedly, I deemed Mr De Morgan a *consenting party* to the intervention of his friend Dr ———; and on that understanding, I had acquiesced in the negotiation. I never supposed him, more than myself, an *active party* in the matter.

POSTSCRIPT.

3d May, 1847.

SIR,

I have this day received your "Statement," for which I beg to tender you my thanks. As you there announce, that, "if my tardy publication ever appear, you will try the strength of my position," and as my Letter, which was sent last week to press, has, in fact, been retarded from the contingency above mentioned [XXV.]; I take the opportunity thus allowed me, by your rushing foremost to the printer, of proving the weakness of yours.

However unfortunate for your cause,—which, bad enough before, is now, if possible, made ten times worse,—this "Statement" renders me even a more confident believer in your good faith.

The whole plausibility to which your pamphlet may pretend, is reducible to three heads; or rather, your "Statement" itself is to be viewed as a tripod, (vulgarly, a three-legged stool,) the feet of which are:—I. *The non-production of your correspondence*;—II. *Your new and perverse confusion of one scheme of quantification with another*; and III. *Your other and miscellaneous misrepresentations*. Under these titles I shall, accordingly, consider it.

I. *The Non-production of your Correspondence.*

To some it may appear remarkable,—what, however, does not surprise me,—that, with the most unconscious simplicity, you substitute fiction for fact—assertion for proof. In mathematics, a mathematician starts from suppositions, and as Philoponus well states it, "uses his imagination for his board." But beyond the sphere of quantity this is not allowed; and he who would apply the mathematical procedure to the affairs of life, justly exposes himself and his science to derision. Now this you do. In the present instance, all that can be said for, all that can be *specially* said against, you, must rest immediately upon your own Letters. And these Letters, you seem to fear as fire. Of itself, the non-production—the suppression, in fact, so far as it was in your power,—of your correspondence, is presumption strong against you. But this presumption is enhanced to the highest, when we witness your awkward attempts at explanation; for you cannot but betray a lurking consciousness, that produced they should have been. This I do not state as a charge against your good faith, but against your sound judgment.

"I have no doubt," you say, "Sir William Hamilton would have supplied them (copies of my letters previous to March 13th), but I did not think it necessary to ask, as it is on *his* [!] communications that the charge was made." (p. 3.)—Now, in the first place, I *could not* have refused copies.—In the second, (what you omit to mention,) copies were through your friend Dr ——— ultroneously proffered.—In the third, by a singular confusion of thought, you are pleased to reverse the one possible ground of charge. It was not "*on my communications*" (though, of course, *in them*) "*that the charge was made*;" but necessarily and exclusively *on your own*. It ought to be superfluous to quote my letter of 13th March, in which I say,—"*From these (your communications,) if I am not mistaken, it appears, &c. . . . that you would be guilty, &c. . . . Am I, therefore, correct in my understanding of your Letters?*" And my understanding being admitted, *on your letters*, of course; it would then remain, for you and for me, to prove, *on them* also, that your pretension was either right or wrong.

Again; you profess to be amused at my proposal, to lay the whole relevant documents fairly before the public, our several statements being interchanged

and either mutually answered or made final, previous to joint publication, &c. "There is something racy," you say, "about the idea of private declaration and answer before going into court, circumstanced as we were. In cases of amicable literary controversy it would be an excellent plan, and would save a great deal of printing. But the notion of the prosecutor and the accused thief quietly interchanging their notes of evidence, and settling its balance before trial, would, if carried into effect, have vastly tickled the public, never indisposed to find jokes in controversies of this sort." (P. 6.)—There might, certainly, have been "something racy" in the proposal, had not "the accused thief," in the interval between the accusing document and the declined proposal, been "quietly" addressing "the prosecutor" as "My dear Sir William," in the one letter "subscribing himself, with sincere respect," and in the other, "with truth, yours faithfully." [XVII., XVIII.]—But if "racy," the proposal was, nevertheless, reasonable;—supposing always, that the end in view were *Truth*. For a *true* judgment in the case could only be expected, if the *whole* evidence and inferences on either side were submitted to the *same* judges. But, independently of truth, I was influenced to this, both by benevolent and selfish feeling. You, whom I now found it painful to regard as an intentional plagiarist, would, I was persuaded, hereby become privately convinced of the nullity of your claim, and a public controversy thus be superseded. And for myself, I care not to confess, that as I could not on principle (for "*hypotheses non fingo*,") publish, without publishing the whole proof; and as the correspondence, out of which the dispute arose, was not of my seeking: I felt it hard not only to have my time occupied with a foolish controversy, but to be put to the sole expense of printing and distributing what, at the best, would have thus only the appearance of an *ex parte* statement. But while I admit, that you had a perfect right to decline or to accept the plan of joint publication; your strange evasion of your own Letters, in not giving, upon your side, the documents which exclusively comprise the special evidence of the case, tells most unfavourably against you.

II. *Your new and perverse confusion of one scheme of Quantification with another.*

Your "Statement" is chiefly plausible from a wretched confusion of distinct things. This confusion, with which you delude yourself, and many of your readers, is of two independent schemes of logical quantification; the one, asserting an *increase* in the expressly *quantified terms*, the other, a *minuter division of the forms of quantification itself*. To disintricate this entanglement, we have simply to consider, in their contrasts, the *three* following schemes of quantification:—

The *first* scheme is that which logically—confines all expressed quantity to the *Subject*, presuming the *Predicate* to be taken—in *negative* propositions, always determinately in its *greatest* and *least* extension (universally and singularly), in *affirmative* propositions, always indeterminately in *some part* of its extension (particularly).

The *second* scheme is that which logically—extends the expression of quantity to *both* the propositional terms, and allows the *Predicate* to be of *any* quantity, in propositions of *either* quality. This not only supplies a capital defect, but affords a principle on which Logic obtains a new and general development.

The *third* scheme is that which logically—admits *more expressed quantities* than a determinately least or greatest extension (quantity singular and universal), and an indeterminately partial extension (quantity particular.) This,

though it corrects, *perhaps*, an omission, yields no principle for a general logical developement.

The first doctrine is the common or Aristotelic; the second is mine; and in the third—in so far as you have gone, and apart from the consideration of right or wrong—I do not question your originality.

Now, the second and third schemes are both opposed to the first, but in different respects; consequently the second and third may, each of them, combine with itself, *either* the whole other, *or* that part of the first to which it is not itself opposed. More is impossible.

Let the following be noted :—*Your OLD view (that in the body of the Cambridge Memoir) is a combination of the THIRD scheme of quantification with the FIRST; your NEW view (that in its Addition) is a combination of the THIRD scheme of quantification with the SECOND: and the confusion, of which you are NOW guilty, is the recent and uniform, and perverse identification, in your PRESENT "Statement," of the SECOND scheme with the THIRD.*

Before, however, proceeding to comment on your confusion of the second and third schemes, I may also relieve a confusion in the term *definite* and its reverse, *indefinite*, as applied to logical quantification.

In the *first*, common, or Aristotelic meaning, *definite*, or more precisely *predefinite* (*διοριστός, προσδιοριστός*), is equivalent to *expressed*, *overt*, or, more proximately, to *designate* and *pre-designate*; in this sense, *definite quantity* denotes *expressed*, in opposition to merely *understood*, *quantity*.

In the *second* meaning, that which I have always used, (and certain ancients, I find, were before me,) *definite* is equivalent to *determinately marked out*; a sense in which *definite quantity* is *extension undivided or indivisible, universal or singular* (this including any collected plurality of individuals) as opposed to *particular quantity*.

In the *third* meaning, which you have usurped, *definite* is equivalent to *numerically specified*; and in this sense, a *definite* is an *arithmetically articulate* quantity, as opposed to one arithmetically inarticulate.—This your meaning of the word I did not, before the appearance of your "Statement," apprehend; for of course I presumed you to use it in its first or common meaning, from which you never hint that you consciously intend to deviate. But it should be confessed, that I ought to have presumed nothing in favour of a writer who within a few lines, is found employing the term *collectively* for its converse *distributively*; and in outrage of the great logical canon, "*Lex generalis erit,—Medius concludere nescit*," silently transforming the conclusion of a syllogism from a *proposition* into a *term*! and that term—the *middle*!!—For this you may, indeed, allege in your favour a present popular English authority in Logic, who is so deep in the elements of his science, as actually to identify the terms and propositions of a reasoning.—But in this misprision of mine there is, fortunately, no harm done. For *definite*, in your sense, involves the common meaning,—and *more*.

These preliminary distinctions being taken, I now proceed to show, in what manner you come, (unwittingly of course,) to confound the judgment of your reader, by confounding the third scheme of quantification with the second, and the doctrine of your original Memoir with that propounded in its Addition, but still more articulately in your previous correspondence [VIII. IX. X].

In your "Statement" you say—"I am really so much at a loss to imagine what I have done in the Addition with any tool not used in the Memoir, that I must wait till Sir William Hamilton points it out, which I hope he will do

very explicitly" (p. 14). That I shall certainly endeavour to do, and trust that you may be no less explicit in your promised reply.

In the *first* place, your *present* assertion of the substantial identity of your first and second views, is in overt contradiction with those of your *previous* correspondence. These I shall take in the order in which they there occur [IX. X].—Now, if your two systems (I refer not to your Addition, for that affords but a scanty statement of your second doctrine,)—if your two systems were substantially identical, how could you say, in reference to the *new* principle of a quantification of the predicate, "The importance you attach to it *makes me willing to avow the importance I had attached to it in my own mind?*" [25].—How could you say,—“The method to which I allude is *not that now* before the Cambridge Philosophical Society, but *one thought of since*” [27]? On this footing, how could you think it necessary “to satisfy me,” and to offer the production of “fourteen leaves of glazed draft paper, written on one side, numbered 1 to 14, headed ‘On syllogisms of definite quantity,’ and severally beginning,” &c. &c. [30]; or deem it politic or possible (see “Statement,” p. 4) to make me “your witness” against myself [33]? And if your present assertion were correct, how could you declare,—“I did not mention my general canon of syllogism in my former notes, because in fact, I have *two, in two perfectly different points of view*; and I wished to divine, if I could, from unsuggested communication from you, *with which of the two systems, if with either, yours was connected*,” [35, 36]? Finally, how can you reconcile your present with your past assertion touching your original doctrine—“My other system (now before the Cambridge Philosophical Society) *has no more reference to quantity than the ordinary ones*,” [36*]?—But it is idle to talk to you of *reconciling*, &c. For all this you now admit,—you now maintain, to be untrue; and aver, that in making such untoward admissions, such awkward attempts at vindication, you were in an access of oblivion. “I had then,” you say! “*quite forgotten*, how nearly the papers safe at Cambridge contained the matters, which I was proving to be in my possession by thus describing them.” (p. 4).—Now there is not a little remarkable in the psychology of your case.—1°, We have here the loss of memory itself, and that of a type the most singular. You discover; and incontinently you forget, that you are a discoverer,—forget all that you have discovered.—2°, Sometimes, (in your own favour,) your recollection is null;—another *Non-mi-ricordo*!—3°, Sometimes, (against others,) your remembrance is impeccable;—a second *Themistocles*!—In fact, your memory, in your own hands, is a Lesbian rule; it is bent or straight exactly as you like it.—4°, And all this with perfectly good faith; for if you unconsciously believe as you wish, you conscientiously speak as you believe.—Be this, however, as it may, I would confide more in your treacherous memory than in your illusive judgment. For,

In the *second* place, looking to the Memoir, apart from the Addition, it contradicts your present “Statement.”

1°, The doctrine of Conversion, which you there hold (pp. 3, 12), belies the doctrine of a thoroughly quantified predicate. Propositions, such as *Every A is B* and *some Bs are not A*, “convey,” you say, “*no idea of convertibility*,”—a doctrine (by the way) held also in your letters *subsequently* to the receipt of the Requirements [VI.].—But the doctrine of a thoroughly quantified predicate recalls the propositional terms to a reciprocal *equivalence*; converts *every* proposition; and reduces the *three* species of conversion to *one*—Simple Conversion. And this view, embodied in the 2d and 3d Propositions of the Prospectus, was “subscribed to” and adopted by you.

2°, Further, you there maintain (p. 4), "As to the *predicate*, it is *universal in negatives*, but *particular in affirmatives*;"—A repetition of the old doctrine, and in direct conflict with the new.

3°, Throughout that whole paper, not only is there much in contradiction, there is absolutely nothing in (more than fortuitous) conformity, with the theory of a quantified predicate. And this leads me,

In the *third* place, to consider in special, the assertions hazarded in your present "Statement," regarding the virtual identity of doctrine, in what you call, your first and second systems; these I shall take in the order in which they there follow.

1°, You thus assert :—"The whole of my second system turns on defining the quantity of the terms in *particular propositions*" (p. 13). Were this correct,—did "the *whole* of your second system turn," as you here assert; how could you thus define it in your correspondence?—"I should call my [second] view, *that of annexing definite quantity to the subject and predicate of propositions*," &c, [24];—and how could you thus speak of it in the very first words of your Addition—"Since this paper was written, I found that *the whole theory of the syllogism* might be deduced from the consideration of propositions, in a form in which *definite quantity of assertion* is given both to the subject and the predicate of a proposition" [37]?

2°, Again you say, "Could I, after writing what is in my paper, want any thing necessary for the Addition? I will here observe, that my system in no degree depends upon giving definite quantity to the predicate *as predicate*. It consists entirely in the quantification of the *middle term*, be it subject or predicate. In fact, perfectly definite quantification destroys the necessity of distinguishing subject and predicate." (P. 14.)

Now, as to your first assertion, that your "system in *no degree* depends upon giving definite quantity to the predicate *as predicate*;" this is backing out, with a vengeance, from the definitions of your second system given in your Letters and Addition just quoted, and to which I again appeal, as containing the most explicit contradiction of your present convenient averment.—Touching your second assertion, that your system "*consists entirely in the quantification of the middle term, be it subject or predicate*," I beg leave to observe, that this doctrine is not only to be found, neither in your original Memoir nor your Addition, but is repugnant to all that is in these taught; and this I say, whether the expression "middle term" be used in its right signification, or abused (for *conclusion*) in a wrong. As this is important, in more points than one, I formally request you to point out any passage of your *previous* writings, in which this doctrine is contained.—In regard to your third assertion, that "*perfectly definite quantification destroys the necessity of distinguishing subject and predicate*;" this is altogether a mistake. It is not "definite quantification," (in whatever sense the word *definite* be employed,) but the quantification of *both the terms* which "destroys the necessity of distinguishing subject and predicate;" and this by showing, that propositions are merely *equations*, and enabling us to convert them *all—simply*.

3°, This I shall make my last extract under this head; "In the former [the original paper] I have only," you say, "quantified the middle term, be it subject or predicate; in the latter [the Addition] I have noted the manner in which that same canon, derived from the middle term, is expressed when there is the quantification of *all terms*." (P. 14.)

On this I remark :—As in the following sentence you admit, that in your original paper the "*conclusion*" is "there called the *middle term*;" the middle

term, being a *proposition*, must consequently involve both a subject and a predicate; and, therefore, your expression, "*be it subject or predicate*," is only an additional blunder. Overlooking this, however, there is not, I make bold to say, in your whole original paper a single word, touching the quantification of the predicate, which is not in conformity with the vulgar doctrine; this, indeed, you yourself explicitly acknowledge in the Correspondence [36*].

And now I would call it to your attention, that for your own sake, an explanation is imperiously required of the following difficulties, which may stagger many.—1°, How you came to *confound two such very different things*, as the *second* and *third* schemes of quantification; for confusion of thought may not be accepted by all.—2°, How you *did this*, after being fully and confessedly *aware of the distinction*; for the excuse of oblivion may not suffice. 3°, How you *did this*, in *opposition* to your *former statements*, and to the *manifest contrast* of your first and second doctrines; for you may not be allowed to back out from a charge of appropriating, by asserting to the ignorant reader, the identity of the second and third schemes, and of your earlier and your later views.

To this head—of confounding very different things, I may refer the assertion or insinuation which you *now* adventure,—that *I have not preceded you, not having, in fact, promulgated the doctrine, which I said that I had for many years academically taught*. In itself, this statement is, perhaps, too contemptible for refutation; and in refuting it, I am not unconscious that I may deserve the ridicule of my pupils and of those around me. Still it shall be done—but briefly.

1°, Your denial of my *priority* is refuted even by the Requirements; for did these, marvellously, refer to a doctrine which was never taught, still, they themselves contain the principle of a quantified predicate with its consequences, and date many months prior to any document which you can allege.

2°, But that the doctrine in question has been by me *academically taught for many years*; I need *only* refer to the evidence,—1°, of the *Prize Essays*, awarded in my class, and published by academic authority, annually in the newspapers, and—2°, of the *Questions* proposed at the close of each Session, to candidates coming up for a Degree in Arts, and printed by the University.—They manifest either the doctrine at large, or its principal and exclusive results. Of these, I happen to have at hand, only some, for certain years; but that some, I presume, will be found far more than enough.—In Session 1841—2, I see from the general list of Honours, that there was an "Extra Prize awarded by the Professor, for a detailed exposition of a New Theory of Categorical Reasoning, to Alexander Higgins Burn, and Brinsley Nicholson, (equal)."—In the same list, for Session 1843—4, there stands,—"*Extraordinary Prize, awarded by the Professor, for an Essay on the advantages of a certain New Analytic of Propositional and Syllogistic forms, as given in the Lectures. 1. William Peddie; 2. James Hamilton.*"—At the close of the same Session, I find the following consecutive questions in Logic were proposed to the candidates for a Degree:—"3. How many kinds of propositions do the Quantity, and Quality taken together, afford?—a) On the doctrine of the Logicians,—b) On the doctrine taught in the Lectures.—4. Conversion of Propositions; state what it is; and of how many kinds (defining each) according to the Logicians.—5. Are all these reducible to one,—and in what way?—6. Is the *Conversio per Accidens* of the Logicians an error,—and how?—7. Figure in Categorical Syllogism:—What; and how manifold?—8. Mood in same:—What; and how manifold in the several figures, according to the

Logicians?—9. Is this correct? If not, what is the true number?—10. What are the four rules, which if obeyed, no categorical syllogism can be formally wrong?" (These four were subsequently reduced to *two*, and to *one*.)—After Session 1844—5. there stands the following question.—"7. (For those Candidates who have attended a course of Logic in this University.)—How does the Professor supersede all the special rules of the several Syllogistic Figures?" In like manner after Session 1845-6 there are found among the graduation questions the following:—"5. What is the Conversion of Propositions, on the doctrine of the Logicians?—6. What is the error of this doctrine?—and from what neglect did it arise?"—But, "Ohe jam satis est!"

3^o, In regard to Mr — and his Notes, I beg leave to say, that in my relative letters, neither to that gentleman nor to you, did I ever refer to his *Notes* of my lectures, but exclusively to his *personal information* in regard to them. And for a sufficient reason. The Paragraphs on Logic dictated to, and taken down by, my students, on which I afterwards prelect, were written so far back as the year 1837, and prior to many of my new views, and to the *whole* doctrine of a quantified predicate. These views, as developed, were, and are, introduced in a great measure as corrections of the common doctrine; in the older Notes especially, they may, therefore, not appear in the dictated and numbered Paragraphs at all; whilst, frequently, (particularly at first,) they were given out as data, on which, previous to farther comment, the students were called on or excited to write expository Essays. I distinctly recollect, that in the Session during which Mr — attended my course of Logic (1840-1) it was required, on the hypothesis of a quantified predicate,—to state in detail, the valid moods of each syllogistic figure; and I, further, distinctly recollect, that Mr — was one of those who essayed this problem. If wrong on this point, I shall admit that my memory is as treacherous as yours. It was, indeed, quite natural, that Mr — should give, and that you should receive, his Notes; but, of course, you could have sought or obtained no personal information from him, in reference to the point in question, without mentioning the fact. In his answer to me, that gentleman, indeed, says,—"I am afraid that my metaphysical studies have now received such an incrustation of law, that, verbally, I shall be able to do little more than whet his (Mr De Morgan's) curiosity;" and it was chiefly as an old pupil of your own, and from a remembrance of his general ability, that I left him to afford you what information he could, in regard to my logical doctrines, as they were taught "*so far back*." Indeed, I did not, at the time, recollect any other pupil to whom I could so well apply. Were it, however, requisite to give proof from *Notes* of so manifest a fact, I doubt not that scores of students would be willing to place theirs at my disposal. I now proceed to the last head,—

III. *Your other and miscellaneous misrepresentations.*

You stumble,—you break down, at the very first step,—even in the starting of your "*Statement*." The following is the opening paragraph of its proof:—

"There is something to say which is not evidence, except to those who know me. [1^o] I think I can show that I never had the *opportunity* of copying Sir William Hamilton's ideas; but, be this how it may, I declare upon my honour that I never derived a single hint of any kind from his communications. [2^o] I declare also, that I remember (and what is more to the purpose, that I remembered in February, when I printed it,) that the papers which I gave the means of identifying in January, were written before

I received any communication from Sir William Hamilton, except a civil note, promising to answer certain questions on the *history* of logic when he returned to Edinburgh. In any question of mere priority which may arise, these papers of course can only date, at the earliest, from the time at which I announced their existence. But as regards my own integrity in this matter, the two declarations above made will, as I said, be enough with those who know me" (p. 2).

All this you assert with apparent confidence, and I believe, with a clear conscience, that what you say is true. But out of your own mouth I shall prove, and prove, I hope, to your personal conviction, that neither of these declarations is to be relied on; adding, (in your proper words,) that "as regards your own integrity in this matter, these two declarations will be enough with those who know you,"—to show that you have here, as usual, with the most perfect veracity, made declaration of the most palpable untruths!

In regard to the *first*, how could you say—"I declare, upon my honour, that I never derived a single hint of any kind from his (Sir William Hamilton's) communications," when you afterwards proclaim,—*"I will not admit his (Sir William Hamilton's) own simple assertion: for, though I have full reliance on his veracity, yet he is as liable as myself to the well-known mistake of investigators, upon whose previous views the approach of those of others frequently throws such a new and sudden light, that they think they always must have seen that which they do then see"* (p. 15)? This first declaration, though "upon his honour," is thus cancelled by the declarant himself. I proceed to the other, and shall prove it, still more articulately, to be false.

In regard to the *second*, you say:—"I declare also, that I remember (and what is more to the purpose, that I remembered in February, . . .) that the papers . . . were written before I received any communication from Sir William Hamilton except a civil note, promising to answer certain questions on the history of logic when he returned to Edinburgh" (p. 2). But this declaration, now emitted, is contradicted by that very declaration, emitted in February; the former purporting that the papers in question were written *after* the receipt of my first note, the latter, that they were written *before*. For, in the "Addition" made in February to your Cambridge paper, you say:—"I had committed this [that is the new] view to paper, when I learned from Sir William Hamilton of Edinburgh, that he had for some time past publicly taught a theory of the Syllogism differing in detail and extent from that of Aristotle." Now, as this information and that promise were both exclusively contained in my first letter, your two statements are mutually contradictory; the one, extending the time of your discovery to the 4th November, the other, throwing it back before the 9th October; in both cases, however, it being not previous to the 3d of October. Now, one of those at least must result from an illusion, and there is no reason for surmising that both should not.

It is thus evident, from the contradiction of your own two statements, that you do not, in fact, know whether your new view was written out previously to the 9th of October, or previously to the 4th of November. Indeed you know, and can truly say, nothing determinately about it. You cannot, therefore, now pretend to assert, that it might not have been written subsequent to the latter date. But all I asked of you, to enable me to withdraw my hypothetical charges, was an admission, on your part, of the possibility of error in your recollection of the date of that writing; and this is now obtained from the contradiction of your two deliberate statements themselves, touching that very point.—And what reason do you assign, for refusing this acknowledg-

ment?—"I could not remember what did not happen . . . those papers *were* written before the letter of November arrived" (p. 6). You were never asked, be it observed, to "*remember what did not happen*," but simply to admit, if you could, that your *remembrance of the matter in question might possibly be wrong* [XXI.]; and *this*, I presume, you will *not now venture to deny*. You were required, however, (what you omit to mention,) to *publish* this admission, &c. in the volumes of the Transactions containing your paper on the Syllogism.

But the treachery of your recollection is, farther, still more instructively displayed, in regard to this the very cardinal point of the case,—the question, to wit, whether your new views were, or were not, thought and written out *before* the receipt of my letter of 2d November, enclosing the Requirements. I have shown (pp. 7, 17) that you *could not* then have had a doctrine founded on or involving the quantification of the predicate; and my inference from the state of facts displayed in your correspondence, you now virtually admit. The following passage thus affords, at once, another specimen of your habitual mnemonic confusion and confidence, with an acknowledgment, by yourself, of the cogency of my refutation of your claim. This is a most important statement, on the most important point in the discussion; and, requesting your attention, *I declare that I have no objection to allow the determination of the controversy in your favour, to depend upon the truth of this your avowment*.

You say:—"When the *Requisites* above cited first reached me, and I *saw* mention of the quantities which both subject and predicate *always have* in thought, all I could do was to wait for the *more articulate statement* which was promised me. It remained to see, whether Sir William Hamilton was really speaking of what they *always have* in thought, which the common system represents, [!] or of what they *have not*, but *always would have if our knowledge were exact enough*, which is what my extension of it supposes." (P. 14.)

Now, this is not only inaccurate,—is not only (you, I doubt not, firmly believing what you strongly state) untrue,—but the very reverse of truth; the decisive fact being, as your Letter [VI.] proves, that you *not only did not specify the proposition* [V. pr. 2] *containing the principle in question, but passed it over, specifying, however, two propositions* [V. prs. 4, 7.] *on minor points, as those, in regard to which you wished to keep reserved your claim "to a secondary originality," and in reference to which alone, you say that you "must wait for further information."* [14.] On that letter, as already said, I establish a proof, that at the receipt of the Requirements (4 Nov.), nay, ten days thereafter, you *could* have had *no* doctrine involving a thorough-going quantification of the predicate (pp. 7, 17). That proof (fact and inference) is here implicitly *admitted by yourself*; for, on the now "*believed fiction*," of *having, before that time, been in possession of such a doctrine, you imagine, and think that you recollect, a conformable state of facts—a state of facts, however, unfortunately exactly the converse of the real*. I suspect, that you will *now* regret, not having acquiesced in my proposal of an interchange of statements; for had this been done, and you become aware of the comparative weakness of your claim, I hardly think, that you would have courted the expense and exposure of a public controversy.—In point of fact, your remembrance is manifestly so fallacious, that you ought to rest nothing on it, either in your own favour, or against others. You forget or confound even the most important matters. Imagination and memory are for you the same; and your fictions, *bond fide*, become your facts. I have already had occasion to

commemorate your confession of an almost Lethæan oblivion ; and yet, at the same time, to show that your remembrance is, sometimes, more to be trusted than your judgment. (P. 33.)

You have taken the trouble to write much that I shall certainly not take the trouble to quote,—and not a little that it is most irksome, though most easy, to refute.

“ Scripta pudet recitare, et nugis addere pondus.”

A large proportion of your “ Statement ” is made up of matters, which, were your assumptions right, as they are wrong, would neither benefit your case, nor damage mine.

These are not always merely gratuitous assertions, nor erroneous inferences ; they are sometimes even (unintentionally, of course,) falsifications of fact. For example : you give (p. 5) as a quotation from my letter of 27 April,—“ I cordially sympathize with your feelings on the occasion ;” and on this text you make the following comment :—“ I wonder how ; being sure that he believed his own charge, I should have thought he would have taken me to be putting a false front of integrity over consciousness of wilful theft.” Now, my letter expressed sympathy with “ your *manifested* feelings ;” and sincerely ; whereas if I had, as you make me to do, expressed sympathy “ with your feelings ” simply ; the expression would in my mouth have been at once false and foolish. It surely cannot be, that my very attentive amanuensis has, in copying my scroll, omitted the adjective ? In your promised reply you will, of course, say, if the inaccuracy do not rest with you.

In your “ Statement ” you assert “ that no *intelligible* communication was ever made by him (Sir W. Hamilton) to me ” (p. 1.)—I have already said enough of your asserted non-comprehension of the Requirements (pp. 7, 10, 11, 20) ; but you now transcend that averment, by declaring your absolute non-intelligence of the Prospectus, nearly the whole propositions of which you made no scruple “ to subscribe to ” as expressions of your own new doctrine—Verily, in intrepidity of assertion, you stand alone ; “ None but yourself can be your parallel.”

You think it of importance,—in your “ Statement,” you mark it once and again in *Italics*,—that you first wrote to me “ to gain some information on existing sources as to the *history* of technical logic.” (Pp. 3, 2.)—To me it, certainly, did appear strange, that writing professedly for information on the *history* of Logic, and when I had stated both my ability and readiness to afford you such information, on any point which you might specify ; that you never afterwards alluded to this matter, but caught at my offer of communicating to you the purport of my own doctrine. Perhaps, as you have thought it important to state a *part*, you will not deem it unimportant to explain the *whole*, of your conduct in this respect.

It would be all too trifling to gainsay the following, for any bearing it may have on me :—“ Sir William Hamilton has by this time, no doubt, put it out of his own power to silence whispers, those whispers which sometimes circulate for a quarter of a century, and are then published in recollections and memoirs, perhaps at a time when neither party can correct them. Of course, *I* have widely announced the purport of the letter of March 13th.”—As to the “ *no doubt*,” &c., your certainty is here, as, indeed, too frequently elsewhere, altogether at fault. I whispered the affair to none, beyond the narrow circle of my own family, with the one exception of my friend Dr ———, who the first spoke of it to me, in consequence of the letter, formerly noticed, from his and your friend,

Dr ——. On your "*wide announcements*" I make two remarks. The first—That these your "*wide announcements*" show out, in still stronger relief, your unnatural silence,—to all the world,—for a quarter of a year,—of the great discovery of your second system (see pp. 8, 11, 19.)—The second—That by these "*wide announcements*" you had, in a manner, shut yourself out from private accommodation. Of this I was not aware, until the publication of your "Statement;" but when no longer wholly ignorant, but *still* not fully apprised, of your defect of memory, I had thought, in my simplicity, that you (of course as an honourable man) would find no difficulty to admit its fallibility—all I ever asked; in preference to the public and painful annihilation of your claim, which the exhibition of your own Letters (could I imagine you had so forgotten even them?) would necessarily entail. "*Tantus amor nihili!*" I, therefore, more than hoped, that through the intervention of our two several friends, a reasonable admission would terminate a private difference; and was, certainly, never more surprised than by your note of the 13th [XXII.] followed by that of the 23d of April [XXVI.] suddenly changing your style and tone, and reproaching me with delay in publicly stating and establishing my charges. Since that, at least, there has been no tardiness on my part; and I suspect that you, and am confident that others, will attribute my confessed dislike of the polemic, to any thing rather than a want of means whereby to vindicate my allegations.

In conclusion:—My whole argument goes to prove, that you *are*, what you yourself admit that you *may be*,—suffering under "the well-known mistake of investigators, upon whose previous views the approach of those of others frequently throws such a new and sudden light, that they think they always must have seen that which they do then see" (p. 15). And having now convicted you of so many and such momentous—treasons of memory—in-accuracies of observation,—confusions of thought,—and self-contradictions; you cannot wonder, should I or others decline you as a competent witness,—and that, too, on your own behalf,—even were there in existence no counter testimony whereby your evidence was at once nullified. At the same time, I am, and trust that others may be, convinced (on the ground even of your marvellous mistakes, and independently of your high character for integrity) that, in advancing and maintaining your *claim*, there was no dishonourable intention on your part. I am, therefore, happy now again, unreservedly, to withdraw any derogatory supposition of which I may have, formerly, surmised the possibility.

I remain, Sir,

with much respect,

Your most obedient Servant,

W. HAMILTON.

NOTE.

I have avoided, in the previous Letter and Postscript, all details in regard to the *third* scheme of quantification (p. 32); because that scheme, except in so far as it is confounded with the *second*, has no bearing in the controversy; and I admit that whatever Mr De Morgan has therein accomplished, he has accomplished independently of me. Further, I shall not deny him any claim of priority to whatever he may have stated in our correspondence, in reference to this third scheme. Finally, I shall acknowledge, for I think it not improbable, that his syllogism (p. 19) suggested a reconsideration, on my sick-bed, of a certain former speculation, in regard to the ultratotal quantification of the middle term in both premises together;—a speculation determined by the vacillation of the logicians, touching the predesignations *more*, *most*, &c., but which I had laid, aside, as a useless and cumbrous subtlety.

Aristotle, followed by the logicians, did not introduce into his doctrine of syllogism, any quantification between the absolutely universal and the merely particular predesignations, for valid reasons.—1°, Such quantifications were of no value or application in the one whole (the universal, potential, logical), or, as I would amplify it, in the two correlative and counter wholes (the logical,—and the formal, actual, metaphysical,) with which Logic is conversant. For all that is out of classification, all that has no reference to genus and species, is out of Logic, indeed out of Philosophy; for Philosophy tends always to the universal and necessary. Thus the highest canons of deductive reasoning, the *dicta de Omni et de Nullo*, were founded on, and for, the procedure from the universal whole to the subject parts; whilst, conversely, the principle of inductive reasoning was established on, and for, the (real or presumed) collection of all the subject parts as constituting the universal whole.—2°, The integrate or mathematical whole, on the contrary, (whether continuous or discrete) the philosophers contemned. For whilst, as Aristotle observes, in mathematics genus and species are of no account; it is, almost exclusively, in the mathematical whole, that quantities are compared together, through a middle term, in neither premise, equal to the whole. But this reasoning, in which the middle term is never universal, and the conclusion always particular, is,—as vague, partial, and contingent,—of little or no value in philosophy. It was accordingly ignored in Logic; and the predesignations *more*, *most*, &c., as I have said, referred, to universal, or (as was most common) to particular, or to neither, quantity. This discrepancy among Logicians long ago attracted my attention; and I saw, at once, that the possibility of inference considered absolutely, depended, exclusively on the quantifications of the middle term, in both premises, being, together, more than its possible totality—its distribution, in any one. At the same time I was impressed—1°, with the almost utter inutility of such reasoning, in a philosophical relation; and 2°, alarmed with the load of valid moods which its recognition in Logic would introduce. The mere quantification of the predicate, under the two pure quantities of *definite* and *indefinite*, and the two qualities of *affirmative* and *negative*, gives (abstractly) in each figure, *thirty-six* valid moods; which, (if my present calculation be correct,) would be multiplied, by the introduction of the two hybrid or ambiguous quantifications of a *majority* and a *half*, to the fearful

amount of *four hundred and eighty* valid moods for each figure. Though not, at the time, fully aware of the strength of these objections, they however prevented me from breaking down the old limitation; but as my supreme canon of Syllogism proceeds on the mere formal possibility of reasoning, it of course comprehends all the legitimate forms of quantification. It is;—*What worst relation of subject and predicate, subsists between either of two terms and a common third term, with which one, at least, is positively related;—that relation subsists between the two terms themselves*: in other words;—*In as far as two notions both agree, or one agreeing, the other disagrees, with a common third notion;—in so far, those notions agree or disagree with each other.* This canon applies, and proximately, to all categorical syllogisms,—in extension and comprehension,—affirmative and negative,—and of any figure. It determines all the varieties of such syllogisms; is developed into all their general, and supersedes all their special, laws. In short, without violating this canon, no categorical reasoning can, formally, be wrong. Now this canon supposes, that the two extremes are compared together, through the *same common middle*; and this cannot but be, if the middle, whether, subject or predicate, in both its quantifications together, exceed its totality, though not taken in that totality in either premise.

But, as I have stated, I was moved to the reconsideration of this whole matter; and it may have been Mr De Morgan's syllogism in our correspondence (p. 19), which gave the suggestion. The result was the opinion, that these two quantifications should be taken into account by Logic, as authentic forms, but then relegated, as of little use in practice, and cumbering the science with a superfluous mass of moods. As to Mr De Morgan's statement in our correspondence (p. 21) of the principle on which (by his later system) such syllogisms proceed, this, to use his own expression, "I did not comprehend at all;" nor do I now, having, to speak with the Rabbis, "reserved it for the advent of Elias." I saw, however, that, be it what it might, it had no analogy with mine; indeed, even from the fuller exposition of his doctrines, contained in the body of the Cambridge Memoir and its Addition, which I afterwards received, I can find no indication of his having generalised either, 1° *the comprehensive principle of all inference,—that the two quantifications of the middle term, should, together, exceed it as a single whole*; or, 2°, *under a non-distributed middle, the two exclusive forms of its quantification.* On receipt, however, of Mr De Morgan's Cambridge Memoir, I saw, or thought I saw, in the body of the paper, on his *old* view, some manifestation of a less erroneous doctrine upon this point, than that afterwards contained in his Letters and Addition, upon his *new*. Accordingly, to obviate all misconstruction, I wrote immediately the following letter, of which an account has been previously given (p. 26, note.)

EDINBURGH, 30th March 1847.

DEAR SIR,

After despatching my last note, it occurred to me, that in saying "my co-operation," &c., I was hardly explicit enough; and that you might not take it, as I meant it, to include a half of the expenses of publishing the relative documents. I delayed, however, writing till the arrival of your treatise, which was only this afternoon. I had supposed it to be a separate publication, and not your paper in the Transactions of the Cambridge Philosophical Society. In the Addition, I see you advance the same claim as in

your letters. In this I feel convinced, and feel convinced of proving, that you are wrong; but still I am more and more inclined to refer this, (supposing it established,) to some confusion of thought, rather than to any obliquity of intention.

Your paper read to the Society I have cursorily perused; but though opposed to many of its doctrines, I admire the ingenuity which characterises it throughout. On one point, I find we coincide, in principle, at least, against Logicians in general. They have referred the quantifying predesignations *plurimi*, and the like, to the most opposite heads; some making them universal, — some, particular, — and some between both; (for you are not correct in saying, (p. 6), that Logicians are unanimous in regarding them as particular, [though most do.]) This confliction attracted my attention; and a little consideration shewed me, that besides the quantification of the pure quantities, *universal* and *particular*, (which I call *definite* and *indefinite*,) there are two others of these, mixed and half developed, which ought to be taken into account by the Logician, as affording valid inference; but which, without scientific error, cannot be referred either to *universal*, (definite,) or to *particular*, (indefinite) quantity, far less left to vacillate ambiguously between these. I accordingly, introduced them into my modification, in English doggerel, of "*Asserit A.*" &c., which [in the original cast] I formerly said was at your service; and as it affords a brief view of my doctrine on this point, I may now quote it.

A, it affirms of *this, that, all*,*
 Whilst E denies of *any*;
 I, it affirms, whilst O denies,
 Of *some* (or few or many).

Thus A affirms, as E denies,
 And definitely either;
 Thus I affirms, as O denies,
 And definitely neither.

A *half*, left semi-definite,
 Is worthy of its score;
 U, then, affirms, as Y denies,
 This, neither less nor more.

Indefinito-definites,
 To UI, YO, last we come;
 And that affirms, and this denies,
 Of *more, most*, (half plus some).

"The rule of the Logicians, that the middle term should be once at least distributed [or indistributable,] (i.e. taken universally, or singularly, = definitely,) is untrue. For it is sufficient, if, in both the premises together, its quantification be more than its quantity as a definite whole. (Ultratotal)" - - -

- - - "It is enough for a valid Syllogism, that the two extreme notions should (or should not), of necessity, partially coincide in the third or middle notion; and this is necessarily shown to be the case, if the one extreme coincide with the middle, to the extent of a half, (dimidiate quantification); and the other, to the extent of aught more than a half, (ultradimidiate quantification)" - - -

"The first and highest quantification of the middle term (.) is sufficient not only in combination with itself, but with any of all the three inferior.

* Better: 'A, it affirms of *this, these, all*.'

The second (..) suffices, in combination with the highest, with itself, and with the third, but not with the lowest. The third (.) suffices, in combination with either of the higher, but not with itself, far less with the lowest. The fourth and lowest (,) suffices only in combination with the highest." [1. Definite; 2. Indefinite-definite; 3. Semi-definite; 4. Indefinite.]


Of the effect of this new system of quantification in amplifying the syllogistic moods, (which in all the figures remain the same,) I say nothing. It should be noted, however, that the letters A, E, &c. do not mark the quantification [and qualification] of *propositions*, (as of old), but of *propositional terms*. The sentences within inverted commas are taken from notes for the "Essay towards," &c.

Before concluding, I ought to apologise, in the circumstances, for the details, that have insensibly lengthened out, of a part of my doctrine, which I have found, to a certain extent, coincident with what appears in your paper. I was anxious, however, that you and others should have no grounds for surmising, that I borrowed any thing from my predecessors without due acknowledgment.—On second thoughts, however, I deem it more proper to make this communication through a third party.

I remain, dear Sir,

Your most obedient servant,

W. HAMILTON.

 In appending to this pamphlet the (already printed) Prospectus of the "Essay," &c., I have not thought it proper to retrench from it the announcement of the Contributions, &c. I must, however, express my regret, that first, engrossing business, then, a tedious inflammatory illness, and latterly, since my recovery, the present necessary defence, have prevented me, for the last eight months, from even thinking of my obligation to execute the latter. I hope now, however, to be able to fulfil my promise, leisurely, but without further delay.

PREPARING FOR PUBLICATION

BY

SIR WILLIAM HAMILTON.

I.

ESSAY TOWARDS

A NEW ANALYTIC OF LOGICAL FORMS.

"Now, what has been the source of all these evils, I proceed to relate, and shall clearly convince those who have an intellect and a will to attend,—that a trivial slip in the elementary precepts of a Logical Theory, becomes the cause of mightiest errors in that Theory itself."—GALEN. (De Temperamentis, l. i. c. 5.)

As my peculiar views on Logic have, for years, been academically published, and, long adequately tested and matured, should before now have been given to the world through the press; whilst, at the same time, circumstances may prevent, at least for a season, my intentions in this respect from being carried into effect: I take the present opportunity, (in order formally to establish my right of authorship,) of more widely publishing the prominent results of my doctrine, especially of Syllogistic; results, the nature, novelty, and importance of which, those competently versed in logic will be able to estimate, apart even from the exposition which the proposed Essay will contain.

This New Analytic is intended to complete and simplify the old;—to place the keystone in the Aristotelic arch. Of Abstract Logic, the theory, in particular, of Syllogism, (bating some improvements, and some errors of detail,) remains where it was left by the genius of the Stagirite; if it have not receded, still less has it advanced. It contains the truth; but the truth, partially, and not always correctly, developed,—in complexity,—even in confusion. And why? Because Aristotle, by an oversight, mar-

vellous certainly in him, was prematurely arrested in his analysis; began his synthesis before he had fully sifted the elements to be recomposed; and thus, the system which, almost spontaneously, would have evolved itself into unity and order, he laboriously, and yet imperfectly, constructed by sheer intellectual force, under a load of limitations and corrections and rules, which, deforming the symmetry, has seriously impeded the usefulness, of the science. This imperfection, as I said, it is the purpose of the New Analytic to supply.

In the *first* place, in the Essay there will be shown, that the Syllogism proceeds, not as has hitherto, virtually at least, been taught, in one, but in the *two* correlative and counter *wholes*, (Metaphysical) of *Comprehension*, and (Logical) of *Extension*;—the major premise in the one whole, being the minor premise in the other, &c.—Thus is relieved, a radical defect and vital inconsistency in the present logical system.

In the *second* place, the self-evident truth,—That we can only rationally deal with what we already understand, determines the simple logical postulate,—*To state explicitly what is thought implicitly*. From the consistent application of this postulate, on which Logic ever insists, but which Logicians have never fairly obeyed, it follows:—that, logically, we ought to take into account the *quantity*, always understood in thought, but usually, and for manifest reasons, elided in its expression, not only of the *subject*, but also of the *predicate*, of a judgment. This being done, and the necessity of doing it, will be proved against Aristotle and his repeaters, we obtain, *inter alia*, the ensuing results:—

1°. That the *preindesignate terms* of a proposition, whether subject or predicate, are never, on that account, thought as *indefinite* (or indeterminate) in quantity. The only indefinite, is *particular*, as opposed to *definite*, quantity; and this last, as it is either of an extensive *maximum* undivided, or of an extensive *minimum* indivisible, constitutes quantity *universal* (general,) and quantity *singular* (individual.) In fact, *definite* and *indefinite* are the only quantities of which we ought to hear in Logic; for it is only as indefinite that particular, it is only as definite that individual and general, quantities have any (and the same) logical avail.

2°. The revocation of the *two Terms of a Proposition* to their *true relation*; a proposition being always an *equation* of its subject and its predicate.

3°. The consequent reduction of the *Conversion of Propositions* from three species to *one*—that of Simple Conversion.

4°. The reduction of all the *General Laws of Categorical Syllogisms* to a *Single Canon*.

5°. The evolution from that *one canon* of all the *Species and varieties of Syllogism*.

6°. The *abrogation* of all the *Special Laws of Syllogism*.

7°. A demonstration of the *exclusive possibility of Three syllogistic Figures*; and (on new grounds) the scientific and final *abolition of the Fourth*.

8°. A manifestation that *Figure* is an *unessential variation* in syllogistic form ; and the consequent *absurdity of Reducing* the syllogisms of the other figures to the first.

9°. An enunciation of *one Organic Principle* for each *Figure*.

10°. A determination of the true *number* of the legitimate *Moods* ; with

11°. Their *amplification* in number ;

12°. Their numerical *equality* under all the figures ; and,

13°. Their *relative equivalence*, or virtual identity, throughout every schematic difference.

14°. That, in the *second* and *third* figures, the extremes, holding both the same relation to the middle term, there is *not*, as in the first, an *opposition and subordination between a term major and a term minor, mutually containing and contained, in the counter wholes of Extension and Comprehension*.

15°. Consequently, in the *second* and *third* figures, there is *no determinate major and minor premise*, and there are *two indifferent conclusions* ; whereas, in the *first* the *premises* are *determinate*, and there is a *single proximate conclusion*.

16°. That the *third*, as the figure in which *Comprehension* is predominant, is more appropriate to *Induction*.

17°. That the *second*, as the figure in which *Extension* is predominant, is more appropriate to *Deduction*.

18°. That the *first*, as the figure in which *Comprehension* and *Extension* are in equilibrium, is common to *Induction* and *Deduction*, indifferently.

In the *third* place, a scheme of Symbolical Notation will be given, wholly different in principle and perfection from those which have been previously proposed ; and showing out, in all their old and new applications, the propositional and syllogistic forms, with even a mechanical simplicity.

This Essay falls naturally into two parts. There will be contained—in the *first*, a systematic exposition of the new doctrine itself ; in the *second*, an historical notice of any occasional anticipations of its several parts which break out in the writings of previous philosophers.

Thus, on the new theory, many valid *forms* of judgment and reasoning, in ordinary use, but which the ancient logic continued to ignore, are now openly recognised as legitimate ; and many *relations*, which heretofore lay hid, now come forward into the light. On the one hand, therefore, Logic certainly becomes more complex. But on the other, this increased complexity proves to be only a higher development. The developed Syllogism is, in effect, recalled, from multitude and confusion, to order and system. Its laws, erewhile many, are now few,—we might say one alone,—but thoroughgoing. The exceptions, formerly so perplexing, have fallen away ; and the once formidable array of liminary rules has vanished. The science now shines out in the true character of

beauty,—as *One at once and Various*. Logic thus accomplishes its final destination; for as “Thrice-greatest Hermes,” speaking in the mind of Plato, has expressed it,—“*The end of Philosophy is the intuition of Unity.*”

In conclusion: I am fully conscious of the boldness, of the apparent arrogance of the pretension,—To illustrate what was left obscure by the brightest luminary ever rising on the horizon of philosophy, and to supply what has remained imperfect during more than two thousand years, after the published labours of far more than two thousand Logicians. Not that, for a moment, I would compare my weakness with Aristotle's strength: his bow, I have never thought to bend. If any thing is here accomplished over Aristotle, to Aristotle's method, precept, discipline, and example—to his spirit, if not to his letter, be it all ascribed. To the Stagirite—and I rejoice in the acknowledgment—I owe more than to all other philosophers together. But this obligation I would not discharge by a blind sequacity. “*Non imitando, imitamur.*” In a certain sense, therefore, I may profess:—

“Te sequor, o Graiæ gentis decus! inque tuis nunc
 Fixa pedum pono pressis vestigia signis;
 Non ita certandi cupidus, quam propter amorem
 Quo te imitari aveo. Quid enim contendat hirondo
 Cyenis? aut quidnam tremulis facere artubus hædei
 Consimile in cursu possint, ac fortis equi vis?
 Tu Pater, et rerum Inventor! Tu patria nobis
 Subpeditas praecepta; tuisque ex, inclute, chartis,
 Floriferis ut apes in saltibus omnia limant,
 Omnia nos itidem depascimur aurea dicta,
 Aurea, perpetua semper dignissima vita.”

Neither is the pretension rash or indeliberate. I have not speculated without preparation; nor hastened to lay the result before the world. I have been diligent in collecting all works of a logical import; have read many, and examined most. In an academical experience, too, as long at least as the Horatian term, and during which my system has been gradually matured, I have proved that its principles are, with their applications, easily, nay eagerly, apprehended by logical learners; and have, indeed, to thank the delicacy of my pupils, for not precipitating a publication through the press of those doctrines, which so many showed themselves well qualified to appreciate. Neither is the pretension shielded from opprobrium—if opprobrium be deserved. After the indications now given, touching what is to be found, and the mode of finding it,—after these alone, it would not be difficult for any respectable proficient in logic to reproduce, with competent exactness, that system in detail, and to apply to it the test of criticism. But I confidently challenge criticism to show, that, in comparison with the Old, the New Analytic is not, both more correct in theory, and preferable in practice.

The following must, however, take precedence of all else:—

II.

CONTRIBUTIONS TOWARDS

A TRUE HISTORY OF LUTHER AND THE LUTHERANS.

PART FIRST.

CONTAINING NOTICE OF

THE VENERABLE ARCHDEACON HARE AND HIS POLEMIC.

"Melanchthon is dear, Luther is dearer, but dearest is the Truth for which they both contended."—LUTHER. (De Servo Arbitrio, P. i. s. 6.; Proverb paraphrased.)

"Censor! For shame! Thy Note, it brands thyself."

LUTHER (quotes passim.)

It was recently, and by accident, that I became aware of the attack made on me by Mr Hare, through seventy-six dense pages of his "Mission of the Comforter," published, I believe, several months previously. I am there charged with archidiabolic ingenuity, logic, and learning, but eke with ignorance, false reasoning, and stupidity; affronting an attack, yet conscious that there were no means of defence; and calumniating Luther and Melanchthon, through statements false in fact, and false in intention. Alas!—if these conflicting accusations be not, and be not evinced to be, one and all, unfounded. For the only logic and learning I do not scorn, is the ability wherewithal *to seek*, and I prize no ingenuity, apart from the disposition ingenuously *to speak the truth*. But to establish my veracity, to roll back, from myself at least, the imputation of bad faith, is easy—in fact, too easy. It would, assuredly, please me better had I been called to vindicate the truth against a more puissant controversialist; for Mr Hare is strong only in maligning. But, such as he is, there is no alternative; contemned he cannot, answered he must, be. For, his position in the Church,—his reputation, I believe, for learning,—and above all, the purport of his "Note W," would make silence on my part tantamount to a defeat; and, in the circumstances, defeat would be tantamount to criminality.

I only regret, that my rejoinder cannot be altogether so prompt as the assault deserves, and as I could wish it to be. Hitherto, my attention has been engrossed with more important—at least, more urgent, matters; and the whole almost of my disposable exertion is, for some months to come, necessarily preoccupied. At the same time, as what is personal in the affair is of a narrow and transitory interest, whilst the question itself is of wide and permanent import; it becomes requisite

to detail the evidence in the cause more fully, than might otherwise suffice to settle the comparative value of Mr Hare's authority and mine. In other respects, were it not always painful to expose the faults and follies of the good and wise; painful to exhibit any one, far more a christian minister, in the colours in which I shall be forced to make Mr Hare reveal himself; and, now especially, to me a tedious drudgery to dictate the (translated) passages which detail the proof;—were it not for these abatements, the work would be one of mere amusement. For the evidence is in my mind; I know it to be resistless; recollecting, amain, both what the passages contain, and where they should be looked for. A thing, however, is soon enough done, when it is done well, and as to that in hand, no time, I promise, shall be lost by me, in performing it effectually.

As to the objection of *Scandal*;—this I hold, and have always held, as of the lightest—indeed, as of no weight at all. In sooth, against the promulgation of the true, the objection is itself a scandal.—I am well aware that false opinions are prevalent, that false opinions have been industriously inculcated, touching the Reformers, and this not alone by enemies of the Reformation. I am consequently well aware, that the propagation of the true opinion will give pain—will give offence to many. But I know also that men ought to be disabused of their errors; and that it is the duty of every one capable of this, so to disabuse them. In the words of an illustrious Father:—"He is not alone a traitor to the truth, by whom the false is spoken for the true; but he also who does not proclaim, who does not vindicate, the truth, as proclaimed and vindicated it ought to be."—If, indeed, scandal could be taken at the truth, before the truth the scandal would sink to insignificance. "In so far," says another great Saint and Father, "as this can be done without sin, we should refrain from affording scandal to our neighbour. But should our neighbour conceive scandal at the truth, *better allow scandal to arise, than truth to be abandoned.*"—But, in reality, truth can never be a ground of *legitimate* scandal. No man is, no man can even pretend to be, a Christian, unless actuated (in reality or profession) by the spirit of truth; and he who does not love to speak, he who does not love to hear, the truth, is a renegado, at once, of truth and Christianity. To say that truth, as truth, may justly scandalise a believer, is, in effect, to blaspheme. For what is this, but to denounce our faith as false?—what is this, but "to turn the truth of God into a lie?" Well, therefore, is it declared by Luther:—"Truth should be proclaimed, in all ways, to all persons, and at all times; never should it be contorted, never should it be concealed. For why? Truth is 'the rod of right;' it cannot, therefore, be a source of scandal."

W. H.

Nov. 1846.

ART. III.—*A System of Logic, Ratiocinative and Inductive, being a connected View of the Principles of Evidence, and the Methods of Scientific Investigation.* By John Stuart Mill. 2 vols. Parker : London.

It is impossible for Mr. Mill to write any thing, except on one subject, which will not well repay a careful and attentive perusal, more than once repeated. On the field of speculation which he has in the main chosen for himself, what living writer is there, who can fairly be said to approach him in his peculiar qualities? in uniting the perception of theoretical principles with the knowledge of things as they really are; in putting forth views at once so cautious and accurate, and yet so original and sagacious; in realizing vividly and distinctly social evils, without being thrown off his balance by the dizzy prospect, or clamouring at random for the first measure which may seem to promise some possible alleviation, and so proposing, it may be, remedies which would be worse than the disease? Others may be more profound and subtle philosophers; others, again (though this we doubt), may have a more extended or more intimate knowledge of facts past and present; but in the *union* of these, in perceiving and proclaiming the connection of philosophical speculation with the practical business of life, in this Mr. Mill reigns absolutely without a rival, as compared, to say the very least, with any living writer on secular subjects.

The present work, though we do not think it does him on the whole so much justice as some other things he has written,* still is in many ways very characteristic of Mr. Mill's habit of mind. Those who have given themselves up exclusively to abstract and deep thought, will regard it, to a certainty, as shallow and unphilosophical; those who are plunged in active life will look on its suggestions as crotchety, unmeaning and "theoretical;" and yet, or rather in consequence of the very circumstance that their respective tendencies lead them so to consider it, the general principles, which it enunciates and enforces, have a strong claim on the attention of both these classes of men, if they would avoid one-sidedness and narrowness of mind. Not that it is *mere* prejudice, which will induce the former class of men to think meanly of the work before us. On the contrary, the author has been, we

* The writings which do Mr. Mill most justice are, we think, his articles on "Civilization" (London and Westminster Review for April, 1836); "Bentham" (January, 1837); "Coleridge" (March, 1840); and one on "Political Economy," to which we are not able at the moment to refer, but which must be carefully distinguished from "A Dialogue on Political Economy," which is by a different hand. We should add, that both here and in the body of the article we much fear that these references may be found incorrect, as the volumes to which we have access seem in some way incorrectly bound up.

cannot but think, in one or two particulars, betrayed into a certain shallowness of thought, as we shall presently endeavour to show, which might well prejudice against him profound and accurate thinkers, who may have no special sympathy with the practical and (in a high sense) political tendency of Mr. Mill's mind.

Yet let us not be supposed to imply that *all* who will think the book shallow are profound and accurate thinkers. Mr. Mill, in whatever he writes, displays one quality, which strikingly characterizes the reverse of a "humbug." He is sincerely desirous to know his own mind, and to convey it to others in the most simple, unaffected and expressive language he can command. His style is unusually felicitous, as the appropriate vehicle of his thought; and his thought at all times peculiarly accurate and consistent with itself. Now, it does appear to us, that there is a tendency in a certain class of writers at the present day (a tendency which probably had its origin in the reaction against the philosophy of the last century) to look with great suspicion on clear, consistent, straightforward thought and language; a tendency to admire the self-contradictory, as being all one with the mysterious; and to regard the pursuit of system, as betokening in itself somewhat of a rationalistic and dangerous disposition. Yet, surely, as has been said, "system is the very soul of philosophy," and is in no other sense rationalistic than philosophy itself is so. To form a system, indeed, prematurely, and on insufficient data; to refuse to profit by the labours and the wisdom of past ages; to fix attention on one class of truths, so as to disparage another; in a word, to pursue system at the expense of facts; this is, indeed, a very shallow, a very presumptuous, and very dangerous philosophy. But not to systematize *at all* is not to philosophize at all; without systematizing, there may be much poetical feeling, much religious musing, much accumulation of *materials* for a philosophy, but true philosophy there can be none; for philosophy has its very essence in impartially surveying all the phenomena within us and around us, in analyzing their real nature, in defining those truths (if any) which they evidence, and in reducing both truths and phenomena to general principles or classes, as far as they admit of such reduction. The true philosopher, so far as he is such, "refers every thing to its true place in the universal system," and "communicates the whole body" of truth "to every separate member;" "he cannot be taken by surprise," and so cannot be inconsistent or one-sided in his views. Now this seems overlooked by the writers in question: prove to them that they have urged against one class of opponents to-day the very contradictory of what they urged against the opposite class yesterday, they will take it rather as a compliment than otherwise; as though such a procedure were a

proof of depth. They find a peculiar pleasure in criticizing unfavourably the visible effects of systems which live and energize: secure themselves against any danger of retaliation; maintaining as they do a verbal theory, which can lead to no corruptions, for it leads to no results, and is preserved from all danger of further development, for that which is nothing remains its own development. They shrink, with the sure instinct of self-preservation, from any reality, from any exhibition of life or promise of progress. No wonder that, with writers of this class, it is a very favourite practice to call the Roman Church herself "rationalistic;" though on what principle they defend the early Church against the same charge, does not so readily appear;—except, indeed, which we had forgotten for the moment, that to be inconsistent with themselves is their principle. ✕

In saying this, we do not, of course, forget that the world, spiritual, moral and physical, abounds in mysteries; and that it is in real truth the nature of a mystery to be *primâ facie* self-contradictory: but in such cases a sound thinker fixes his mind carefully, intently and habitually, as it were, on both sides of the picture, until they make on it respectively their appropriate impression. If he profess to be a philosopher, he then proceeds to aim at developing, to his own consciousness by accurate thought, and afterwards to the world by accurate language, that wide and expansive field of truth which shall embrace both these seeming contradictions. And on the other hand, to contemplate only one side of the mystery, and carry it on into consequences as though it were the whole account of the matter, this, we fully acknowledge, is the fruitful parent of error and mischief. But if the pursuit of truth by means of the intellect be the object proposed, one really does not see what great advantage the method, we are now criticizing, has even over such baseless theories as we just mentioned. To view one side of truth, for the time, as though it were the whole, and this in order to answer an immediate purpose, to silence some perplexing inquiry, or throw cold water on some plausible innovation; and the next moment to use the opposite side of that truth for a purpose precisely similar, and with a view equally one-sided; this method of procedure surely implies, in its very statement, that neither side has been really embraced by the mind, and is thus not less shallow than the former, while it adds the additional merit of being self-contradictory. And since, whatever amount of declamation be adopted against the desire of consistency, very few persons are willing fully to plead guilty, in their own consciousness, to this latter allegation; they proceed to adopt a convenient mistiness of language, which, being in them

✕ if his principles may be inferred from them
uniform practice.

but a symbol of shallowness, they impose on themselves and on others as being the necessary concomitant of depth.

This reckless disregard of consistency and theoretical truth is just now, in various shapes and in various quarters, putting forward such bold pretensions, and appears really a cloak for so much of secularity and indolent conservatism, that it seems a positive duty to speak strongly on the subject; though it is most difficult to give a picture, which shall be at once recognized as both sufficiently general and sufficiently accurate. We are not alluding for a moment to such divines as, avoiding angry and personal attacks, have occupied themselves in giving a positive shape to the opinions they have inherited. How could we, though we were to wish it, avoid entertaining great and sincere respect for such writers, even where non-concurrent with them? Nor, much less, are we blaming those gentle and humble souls, whom the present course of opinion fills with distress and alarm; though we cannot but think such distress and alarm groundless. We have again and again expressed our deep sympathy, in many cases even reverence, for such persons; and have from time to time ventured to throw out suggestions, which might tend to tranquillize and reassure them. But those, of whom we speak, are the very first themselves to repudiate such sympathy. They are not among those who shrink from positive and decided statements, but who delight in nothing so much; they do not profess to be perplexed, but to see their way clearly, to possess with undoubting certainty a definite scheme of doctrine, nay to trace, with penetrating glance, what they are pleased to call "extreme opinions" to the various evil qualities and tempers in which they originate. Such writers as these, if their doctrines themselves only had some substantive existence, as well as the vociferousness with which they maintain them,—if the wideness of their view bore at least some discernible proportion to the comprehensiveness of their denunciations,—would be really entitled to a hearing. We will not ask them at present to prove their positions: it will be sufficient for us, and much more than they have yet displayed the power of doing, if they will only state them. Sometimes indeed they have not so much to say in their defence, as that their subjects *are* in themselves mysterious; those subjects, on which they snatch up at random such assumptions as may serve a momentary purpose, and which may give the appearance of argumentative discussions to mere declamations and expressions of individual feeling.

No one of course denies, that expressions of individual feeling, on the part of men of character and credit, will have very great weight with reasonable minds; but let them profess to be what they really are, and let not those who put them forth assume the

character of philosophers or divines. But now-a-days these are your only true divines and philosophers. Let any writer weigh his words, define his thoughts, and endeavour to realize all that is involved in them ; nay, let him take any one principle which can possibly impinge upon practice, and honestly keep to it ; he will hardly escape the imputation of propounding "rash and unguarded statements," of being a "wild theorist," a "dangerous and objectionable speculator." On the contrary, let persons advocate any set of principles they may choose, and that with any amount of feebleness, inconsistency, violence and extravagance, so only that they have this one indispensable qualification, of defending things exactly as they are, here are your true moderate theologians, here are the men needful for these times, here are the men who will steer you safely "along the channel of No-meaning between the Scylla and Charybdis of Aye and No." The very name moderate, that sacred word, is profaned in modern controversies ; it is refused to those who realize any one principle, though they may be anxious to take the most favourable view of all human beings, to make allowance for their circumstances, to sympathize with their difficulties ; it is granted to persons, who may defend positions not held by ten persons besides themselves in the whole world (so only that those positions result in seeming to establish the absolute perfection of our present system), and who yet can never allude to their opponents after any other fashion, than that of characterizing them as fit subjects for Bedlam or the gallows.

We have, of course, in order to colour the picture, taken extreme cases ; would we could say imaginary ones. But numbers, who are far from going the whole length of this extravagance, and who originate many true and valuable thoughts, still seem far from sufficiently alive to the paramount importance, in speculation, of accurate and consecutive thought. And as persons of this kind peculiarly need such special discipline as would be supplied by Mr. Mill's writings, so they are, from that very circumstance, little likely to appreciate them fully. In truth, Mr. Mill is very far from obtruding into notice the elaborate process of thought, through which he has gone. The very completeness with which he does his work, the definiteness of shape into which he has brought his views, by no means impresses on the ordinary reader a notion of the careful, laborious, and anxious thought, of which that definiteness is the result. No living writer of whom we know, with but one exception, comes near him in the power of viewing each fact, as it arises, by the light of all the other facts (in the largest sense of that word) of which he is cognizant ; or again of analyzing complex ideas into their simplest elements,

and so bringing into full light their mutual relations and adjustments.

"No question with him is ever an isolated one; he sees every subject in connection with all the other subjects, with which, in his view, it is related, and from which it requires to be distinguished. . . . He does not . . . forget and overlook a thing on one occasion to remember it on another. Hence there is probably no philosopher of so wide a range, in whom there are so few inconsistencies."

Such is Mr. Mill's own language in speaking of Mr. Bentham; and such is eminently his own character of mind: but in him it is, beyond any possible comparison, more praiseworthy (intellectually) than in the other, by how much the range of things observed by him is wider, and his observation of them deeper. True indeed it is, as we hope to point out before we conclude the present notice, that at last his contemplation of the field of knowledge is incomplete, and his perception of the sources of knowledge inadequate; but even in regard to this, we may, in some small degree, console ourselves by resuming his eulogium of Bentham, and still applying it most deservedly to himself.

"If any of the truths which he did not see had come to be seen by him, he would have remembered it everywhere and at all times, *and would have adjusted his whole system to it*. And this is another admirable quality which he has impressed upon the best of the minds trained in his habits of thought: when these minds do open to receive new truths, *they digest them as fast as they receive them*."*

A very characteristic excellence of Mr. Mill's is implied in what we have said, but deserves more specific notice. He is peculiarly free from what is called "crotchettiness," or from being, as the French say, the victim of "illusions." Intellectually his mind exerts its functions in an eminently healthy and vigorous manner, and exercises a natural straightforward unperverted judgment on the subjects which it contemplates. A contrast will make clearer what we mean, and, we hope, may not give offence. Mr. Maurice (of Guy's Hospital) has written, on the whole, not a little on matters closely connected with practical life; and the fundamental principle of his philosophy, as we understand it, is essentially true and important: the principle, we mean, of taking for granted, that every sect and party must have within it something of real and substantial truth, or else could not have had existence. For the real distinction which separates Catholicism from a faulty eclecticism, consists not in denying so just a principle, but in this; that the right minded inquirer looks on men and parties with the eye of a learner, not as judge or umpire; tests truth by his con-

* London and Westminster Review, January, 1837, p. 479.

science, not by his intellect ; and believes fully, moreover, since the Gospel has come, that there exists a divine system, the very correlative of human nature in all its fulness, and that such increased illumination of mind as he receives from observation, will enable him to see the more fully into all the marvellous properties and capabilities of that system. Still, whatever benefit Mr. Maurice might have been expected to derive from his method of philosophizing, is lost to him by the sort of indescribable haze which seems to invest all objects, in their way to his apprehension. We cannot read any of his writings, without seeing that the world, of which he speaks, is something quite heterogeneous from the world around us ; that men and things in his pages are quite different from men and things in real life ; and whatever the incidental value of his thoughts (which is often considerable) they all require to be transmuted and recast, before they can serve us for available maxims of truth or practice. Now it is the reverse of this which we eulogize in Mr. Mill. His sad ignorance indeed, on the most important element of the human mind, makes his observation of one class of phenomena grossly inadequate ; but his perception even of these is quite accurate as far as it goes. And of facts in the mass, as they are found in the busy scene of life, his view is not only accurate but very sufficiently complete.

We had hoped to discuss in some little detail with Mr. Mill a few matters of logical principle ; but find so much else which it is indispensable to notice in his work, that we must give up the intention. We will only say, that there are very many observations he has made, both on the general province and on the details of *Logic*, which will claim the careful attention of succeeding inquirers. Still his system, taken as a whole, depends so entirely for its very existence, we may say, on a denial of those truths, on which alone religious faith, or even moral practice, can be reared, that we must perforce apply ourselves to the defence of those truths. We regret indeed, that this must give our observations a far more controversial aspect than we could have wished : but the merits of the work will display themselves to any one who may peruse it ; its defects require to be specifically pointed out. Mr. Mill has professed to make "*Logic*" a "common ground on which the partisans of Hartley and of Reid, of Locke and of Kant, may meet and join hands." (p. 15.) But even as against Reid and Kant he has not kept the peace ; for he introduces in two different places a lengthened discussion, with the view of proving that geometry and arithmetic are experimental sciences. And as regards far

more important interests than Reid or Kant have taken on themselves to defend, it is sufficient here to say, that if Mr. Mill's principles be adopted as a full statement of the truth, the whole fabric of Christian Theology must totter and fall. We shall make it then our principal object to aim at refuting Mr. Mill's fundamental position; viz. that all additions to our stock of knowledge must be derived from experience. And though fully sensible how infinitesimal is the importance of just views on mathematical science, as compared with moral and theological, still it will be much more satisfactory to include the former also in our range; both in order to make that range complete, and also with the hope of inducing a belief, that if Mr. Mill has been carried so far away by his ardour for the cause of experiment, as to fall into a metaphysical error, which cannot be accounted trifling, on a simply intellectual question, there is nothing improbable, at first sight, in supposing that this may be much more the case, where moral elements are the chief matter for consideration.

It will be only fair, however, to Mr. Mill, to precede our discussion by a few extracts from his work, which may both show the general nature of his design, and also furnish in several instances matter for examination in what will follow. And we shall see at once that the province of logic, according to his conception, takes a far wider range than has usually been assigned to it.

"The sole object of Logic is the *guidance of one's own thoughts*; the communication of those thoughts to others falls under the consideration of Rhetoric, in the large sense in which that art was conceived by the ancients; or of the still more extensive art of education."—p. 5.

"Logic is the science of the operations of the understanding which are subservient to the estimation of evidence; both the process itself of proceeding from known truths to unknown, and all intellectual operations auxiliary to this. It includes, also, Definition and Classification. For the use of these operations (*putting all other minds than one's own out of consideration*) is to serve not only for keeping our evidences and the conclusions from them permanent and readily accessible in the memory, but for so marshalling the facts which we may at any time be engaged in investigating, as to enable us to perceive more clearly what evidence there is, &c."—p. 13.

"I shall attempt to analyze the process of inference and the processes subordinate to inference, so far only as may be requisite for ascertaining the difference between a correct and incorrect performance of these processes. . . . Any ulterior and minuter analysis must be left to transcendental metaphysics."—pp. 14, 15.

The first book is on Names and Propositions; a subject necessary for him to treat, because,

"since Reasoning or Inference, the principal subject of Logic, is an operation which usually takes place by means of words, and in all

complicated cases can take place in no other way; those who have not a thorough insight into the signification and purposes of words, will be under almost a necessity of reasoning or inferring incorrectly."—p. 20.

In this book the greater detail has been used,

"because many useful principles and distinctions which were contained in the old Logic, have been gradually omitted from the writings of its later teachers; and it appeared desirable both to revive these, and to reform and rationalize the philosophical foundation on which they stood." (Preface, p. iv.)

Aristotle's categories are thus criticised:—

"The imperfections of this classification are too obvious to require, and its merits are not sufficient to reward, a minute examination. It is a mere catalogue of the distinctions rudely marked out by the language of familiar life, with little or no attempt to penetrate, by philosophical analysis, to the rationale even of those common distinctions. Some objects are omitted, and others repeated several times under different heads."—p. 60.

And then he proceeds to substantiate his charge. The author then hopes to "recommence under better auspices" the "enumeration and classification of all nameable things;" and having first illustrated at some length the following table for that purpose:—

FEELINGS, or States of Consciousness.				SUBSTANCES, or Unknown Causes.		ATTRIBUTES.		
Sensations. Thoughts. Emotions. Volitions.				Of External Sensations. Of Internal Consciousness.		Quality. Quantity. Relation.		
				Bodies.		Minds.		

by a further analysis he reduces the latter head to "the Successions and Co-existences, the Likenesses and Unlikenesses, between feelings or states of consciousness." (p. 101.)

It is alas! very characteristic, that he has literally left no possible place for the introduction of such "nameable things" as "God," "Angelic Being," or the like; or, again, of "right" and "wrong" as qualities inherent in actions, without reference to our consciousness of them. Here again what would "Reid and Kant" say?

On coming to propositions, Mr. Mill protests in very just and forcible language against what he considers "one of the most fatal errors ever introduced into the philosophy of Logic," (p. 119,) viz. that a proposition "consists in affirming or denying one *idea* of another."

"In order to believe that gold is yellow, I must indeed have the idea of gold, and the idea of yellow, and something having reference to those ideas must take place in my mind; but my belief has not reference to the ideas, it has reference to the things. What I believe is a fact relating to the outward thing gold, and to the impression made by that outward thing upon the human organs; not a fact relating to my conception of gold, *which would be a fact in my mental history, not a fact of external nature...*

I cannot dig the ground unless I have the idea of the ground, and of a spade . . . and unless I put those ideas together. *But it would be a very ridiculous description of digging the ground to say that it is putting one idea into another.*"—p. 118.

And next against the equally fatal error introduced apparently by Hobbes, that predication is mainly a matter of names.

"That the diamond is combustible, was a proposition certainly not dreamt of when the words Diamond and Combustible received their present meaning; and could not have been discovered by the most ingenious and refined analysis of the signification of those words. It was found out by a very different process; namely, by exerting the five senses. . . . The assertion is not a question of the signification of names, but of the laws of nature."—p. 124.

Mr. Mill's use of the word Connotation throughout his work (an use for which he professes himself indebted to the schoolmen) is peculiarly useful, as in other ways, so also in protecting his readers from the latter of these fallacies. His zeal, however, for realities, as opposed to shadows, has led him a good deal too far. He speaks (p. 156) of "a class of propositions" as being "those only which are themselves instructive, or from which any instructive propositions can be inferred," viz. that class from which "we learn a new fact; a fact not included" in short in our previous consciousness: all other propositions he denominates merely verbal. Now there is a distinction drawn by Kant,* of very great importance in logical inquiries, between "analytical" and "synthetical" propositions: the latter being pretty much the same as those called by Mr. Mill real; such as that "diamonds are combustible," or that "Greece flourished for a very short period." But the former play a very much more important part in philosophy, than Mr. Mill seems to think; and reach to a far greater extent than "asserting of a thing under a particular name, only what is asserted of it, in the fact of calling it by that name." The principal cause of their importance is the prodigious complexity of human consciousness; the incalculable number of synthetical judgments, which we are forming *inadvertently* every hour of every day. For instance. I am going to India in a steam-vessel, and mix a good deal with the other passengers; not that I trouble my head about them, for my thoughts are otherwise occupied. At my voyage's end a friend asks me my opinion of Mr. Thompson, with whom I have come over; the subject has never so much as once, to my

* As we may mention this philosopher's name more than once in this article, we particularly desire it to be understood, that we are expressing no opinion whatever on the value of his general system. The present writer has indeed no sufficient knowledge of it, extensive or intensive (as Kant would himself say), to warrant any opinion, beyond the very ordinary one, that his works are very difficult reading. Of course his avowed scepticism makes great caution in reading them a plain duty.

knowledge, entered my thoughts; still, on consulting my own consciousness, I find a sufficiently vivid impression of his character, and endeavour to put it into words as best I can. The mental form which precedes the verbal expression is an analytical judgment, as the expression itself is an analytical proposition: and in order to help me in testing its correctness, rules must be given on two widely differing subjects; the one, how to put rightly into shape the existing impression; the other, how to form such impressions justly and legitimately. And it is plain at once that the first of these, by itself, is a matter requiring no ordinary skill and delicacy of mind. Now in estimating the value of such judgments and propositions, we have no wish at all to be unfair to the author. We fully grant that a marked distinction exists, between sciences founded on synthetical propositions, and any others which may exist. We fully grant (in opposition, we believe, to the opinion generally current in England), that the foundations of mathematical science are synthetical and not analytical; for this Mr. Mill has, we think, fully established (were it otherwise doubtful), (p. 194—202, 328—336), and our only question with him is, whether these synthetical propositions be from *experience*. Nay, we fully grant that even psychology necessarily includes such propositions; for there is at least the implied judgment, "what is true of my mind is true of other men's," (however such judgment be obtained, and within whatever limits it be true); and moreover any result obtained, whether from observation on others, or by mental *experimentation* on ourselves, is synthetical also. Still, on the other hand, by far the most important theorems in psychology are surely analytical; derived not from trying new experiments on our mind, but by examining our past or present consciousness. And, again, the architectonic philosophy itself, that which assigns to all sciences and to all known things their relative position and value, must, from the nature of the case, be wholly drawn from the resources of the mind itself; it is a theorizing on existing knowledge, not an adding to its bulk. So much must be granted on Mr. Mill's own principles; but if it be further conceded that our moral faculty is the recipient of truths the most ineffable and transcendent, the attempt so to depict them on the intellect, that the representation may be the most just, complete, and vivid of which the case admits, may conceivably call into being a science, which shall enlist in its service, age after age, all the energies of the most gifted, subtle, and capacious minds. But we must not anticipate the course of our remarks.

The division of propositions into analytical and synthetical is not, however, exhaustive. I know, e. g.* these three laws of

* See vol. i. p. 382.

nature ; 1. that air has weight ; 2. that pressure on a fluid is propagated equally in all directions ; and 3. that pressure in one direction, not counteracted, produces motion, which does not cease till equilibrium is restored. These three judgments, when I formed them, were synthetical ; and by means of them I should be able to predict the rise of the mercury in the Torricellian tube. Can we call this latter an analytical judgment ? surely not ; it is obtained by ratiocination, not by self-inspection. Can we call it synthetical ? no, again ; for no fresh experimentation is necessary to discover it : it is the result, by strict syllogistic process, of truths already known to me. Let us be allowed then to call such judgments and propositions “deducible ;” and these three classes will make the division exhaustive. It is plain, at once, that as nearly as possible the whole of mathematical science consists of these deducible propositions ; while in every single case, as Mr. Mill shows in one instance, (p. 285, 286,) if the demonstration be expressed syllogistically, the ultimate major premisses, to which all is referred, will be those few axioms and so-called definitions, which are the real synthetical truths, on which the whole science is based. (See vol. ii. p. 173—175.) That a straight line lies always in the same direction, is an analytical proposition ; that two straight lines cannot enclose a space, a synthetical ; that the base angles of an isosceles triangle are equal, a deducible proposition.

Mr. Mill occasionally seems involved in a confusion, between the logical process by which man collectively gains knowledge, and that by which the individual gains it. This is exemplified when he says (p. 126) that the reference of objects to a class, is an operation necessarily subsequent in order to our discerning in them those properties, which we finally take to characterize the class. This is surely not true of men, one by one. When I am told that my cousin is gone into the army, it would be difficult to assign any new *quality*, which I believe him to possess ; I refer him in my mind to a certain class, and that is all. In the sense however in which Mr. Mill means it, the observation is not only very true but of great importance.

The author finally classifies the import of propositions thus :—

“ Existence, Order in Place, Order in Time, Causation, Resemblance ; one or other of these is asserted (or denied) in every proposition without exception. This five-fold division is an exhaustive classification of matters of fact ; of all things that can be believed, or tendered for belief.”—p. 139.

This will not hold if Christianity be true ; but it is not necessary to insist on this.

Mr. Mill acknowledges distinctly, that a certain classification of things is made by Nature ; and that the "infima species" is not an arbitrary creation of the philosopher for his own purpose :—

"Some classes have little or nothing in common to characterize them by, except precisely what is connoted by the name ; *white things*, e. g. . . . *But a hundred generations have not exhausted the common properties of animals or of plants, of sulphur or of phosphorus. . . . These classes, distinguished by unknown multitudes of properties, and not solely by a few determinate ones*, are the only classes which by the Aristotelian logicians were considered as genera or species."—pp. 166, 167.

These particular classes Mr. Mill calls "kinds," and gives to them a great prominence throughout his work.

"The lowest kind to which an individual can be referred is called its (infima) species. . . . The differentia of a species (is) that part of the connotation of the specific name, . . . which distinguishes the species in question from all other species of the genus to which . . . we are referring it.—A proprium of the species (is) any attribute which belongs to all the individuals included in the species, and which, although not connoted by the specific name, *follows* from some attribute which the name connotes.—Thus the attribute of being capable of understanding language is a proprium of the species man ; since, without being connoted by the word, it follows from an attribute which the word does connote, viz. rationality.—Inseparable accidents are those which, although we know no connection between them and the attributes constitutive of the species . . . are yet in fact never known to be absent ; as blackness to a crow. Separable accidents are those which are found in point of fact to be sometimes absent from the species."—p. 168—181.

The whole chapter "On the Functions and Logical value of the Syllogism" (p. 244—274) is admirable.

"Every induction which suffices to prove one fact, proves an indefinite multitude of facts ; the experience which justifies a single prediction must be such as will suffice to bear out a general theorem. This theorem *it is extremely important to ascertain and declare in its broadest generality ; and thus to place before our minds, in its full extent, the whole of what our evidence must prove, if it proves any thing. . . . The advantage, in judging whether any controverted inference is legitimate, of referring to a parallel case, is universally acknowledged. But, by ascending to the general proposition, we bring under our view not one parallel case only, but all possible parallel cases at once.*"—p. 264—6.

Mr. Mill's language, it is true, is based throughout on his assumption, that all real increase of knowledge is derived from induction ; but its substantial truth is not interfered with by that circumstance. We are convinced, indeed, that it is impossible to overstate the importance, even in the most honest reasoners, of stating explicitly their major premisses, where they may have reason to doubt the completeness of their argument. It is a very common

case that a conclusion shall be perfectly correct, and the reasons on which it is grounded amply sufficient, and yet, from want of this process, those reasons shall be by no means sufficiently evolved and brought to light. We may give an instance, which tells against our own friends. Less than two years ago, it was urged in many quarters with great earnestness, that religious differences should not interfere in the election to a non-theological chair. Now, few men indeed, we may hope, through all England, would acquiesce in the major, "*no* religious differences ought so to interfere." The real major premiss implied by these advocates of a good cause, was that "the religious opinions of a certain candidate were not such as ought to be in his way;"—a most just proposition, doubtless, but one which it was neither wonderful nor censurable that the other party should deny and repudiate. Of so much importance, then, is this rule, even for honest reasoners: but as to those confused and misty thinkers, of whom we spoke a few pages back, were they only to catch a distant glimpse of the major premisses, which are necessarily implied in their reasonings, they would recoil in dismay. That "the English Church is infallible," is a sufficiently strong sentiment: particularly considering that she has issued her "infallible" disavowal of such a claim; which would seem to involve its advocates in the fallacy of Epimenides the Cretan. But even this vast assumption will not suffice for their purpose; nor can we, looking on their works with the unbiassed eyes of mere logicians, devise any major premisses which can sustain the weight of their reasonings, except such as the following:—1. "The English *practical system* is infallible."—2. "The English practical system suits me," joined to "I am infallible."—3. "The primitive Church is infallible," joined with "The English system appears to me a faithful copy of that Church," and "I am infallible;" or, 4. "All persons all their life long ought to think that system perfect, under which they were born." If any other major premiss, or combination of majors, would really serve as a substratum for their reasonings, they will do a great benefit, both to the world at large and to their own cause, by stating them.

On the contrary, and here we differ from Mr. Mill, whether faulty *reasoning* often, or indeed ever, takes place (and this is not our own original observation), we are very much inclined to doubt. That any one out of Bedlam, having placed clearly before his mind major and minor, will come to a wrong conclusion, is to us extremely questionable;—a consideration, which, if true, is important, as bearing on the time usually devoted to the canons, &c. of syllogisms.

The chapter on "Trains of Reasoning and Deductive Sciences"

(p. 274—295), is of equal merit with the preceding; evincing, we think, consummate perspicuity and power of thought. And from thence we are speedily brought to the philosophy of induction, now for the first time introduced into a logical treatise. To consider how far such an introduction is appropriate, is a question which we have expressly declined; but that the subject itself is of great practical importance we have no doubt whatever. Mr. Mill's pages furnish abundant proof, how erroneously physical inquirers have estimated the respective certainty of their own discoveries; and in like manner the advocates of phrenology, animal magnetism, cold water cure, and the like, require the strenuous inculcation of true principles on evidence. We need hardly add how much more this is the case with proclaimers of political nostrums and universal specifics; a class who meet with the most masterly treatment at Mr. Mill's hands. But in their daily conduct men need no less the practical knowledge of these principles; whether they are considering the possible advantages of some commercial speculation, or passing judgment on the faith of a people, on the strength of the random and chance observations of a tour designed chiefly for pleasure. Here however even so slight a sketch of the book as we have hitherto given must come to an end, and our extracts will be wholly miscellaneous. The following is Mr. Mill's account of that class of inductive reasonings with which social and historical inquiries are, in his view, exclusively concerned:—

“The problem of the deductive method is, to find the law of an effect, from the laws of the different tendencies of which it is the joint result. The first requisite is therefore to know the laws of those tendencies; the law of each of the concurrent causes: and this supposes a previous process of observation or experiment upon each case separately; or else a previous deduction, which also must depend for its ultimate premises upon observation or experiment. Thus, if the subject be social or historical phenomena, the premisses of the deductive method must be the laws of the causes which determine that class of phenomena; and those causes are human actions, together with the general outward circumstances under the dominion of which mankind are placed, and which constitute man's position in this world. . . . Some of these general truths will naturally be obtained by observation and experiment, others by deduction: the more complex laws of human action, for example, may be *deduced from the simpler ones*; but the simple or elementary laws will always, and necessarily, have been obtained by a directly inductive process.”—Vol. i., pp. 534, 535.

“When the laws of the causes have been ascertained, and the first stage of this great logical operation satisfactorily accomplished, the second part follows; that of determining, from the laws of the causes, what effect any given combination of those causes will produce. This is a process of calculation, in the wider sense of the term.”—p. 541.

"Verification is the third essential component part of the deductive method ; without which all the results it can give have little other value than that of guess-work. To warrant reliance upon the general conclusions arrived at by deduction, these conclusions must be found, on a careful comparison, to accord with the results of direct observation, wherever it can be had. . . . Nor is the verification complete, unless some of the cases in which the theory is borne out by the observed result, are of at least equal complexity with any other cases in which its application could be called for."—p. 544.

"To the deductive method, thus characterized in its three constituent parts, induction, ratiocination, and verification, the human mind is indebted for its most glorious triumphs in the investigation of nature."—p. 546.

"It is destined irrevocably to predominate in the course of scientific observation from this time forward. *A revolution is peaceably and progressively effecting itself in philosophy, the reverse of that to which Bacon has attached his name.* That great man changed the method of the sciences from deductive to experimental, and it is now rapidly reverting from experimental to deductive. But the deductions which Bacon abolished were from premisses hastily snatched up or arbitrarily assumed. The principles were neither established by legitimate canons of experimental inquiry, nor the results tested by that indispensable element of a rational deductive method, verification by specific experience."—p. 579.

This is the light, in which Mr. Mill regards historical facts ; as verifications of those results, which are drawn deductively from the laws of human nature. Such facts, according to his view of the case, are most indispensable for us to contemplate, that they may every moment check and confront our deductions, and compel us to look more deeply into our own minds, and more observantly on those of others. But the idea of building a political philosophy on those facts, he frequently characterizes as inexpressibly puerile and shallow. To our own minds we confess that Mr. Mill appears the founder of the real science of history ; though, of course, by such a sentiment, we do not pledge ourselves to a servile concurrence with all that he has advanced on the subject. It is very plain indeed, and Mr. Mill would be the last to deny, that really great men, men endued with an extraordinary degree of genius and imagination, with a vivid and accurate impression of the real circumstances of their own time, and that power of throwing their mind into scenes wholly differing from their own experience which so few possess, may at all times have obtained from history the most useful and exalted lessons of an immediately practical character. But philosophical truths, accessible to the mass of educated men, can never be derived from it, until it shall be cultivated on those principles, which the present author has so admirably conceived and expressed. 'This is very far from tantamount to an assertion, that the study of history has been hitherto

useless, or nearly so, to ordinary minds ; the case is far otherwise. One benefit, which all derive from it has been frequently mentioned by Mr. Mill ; viz. the unconscious correction of a certain narrowness, incidental to personal experience. Its study "familiarizes them with the action of great causes," and "brings vividly home to them the infinite varieties of human nature."

Another great benefit, which history is calculated to confer, arises from a fact in human nature, which we have been at times surprised not to see more distinctly recognized in Mr. Mill's writings ; we allude to the very serious evil which arises, when the mind is prematurely introverted on itself. Intellectual, like moral unconsciousness, is the great charm of youth ; and the interruption of it prevents the equable and healthy growth of those very phenomena, which at a later period it may be well for the mind to contemplate. Now historical lessons, well chosen, have a place, quite peculiar to themselves, in assisting to draw forth the judgment and affections unconsciously in a right direction, and afford them fresh nutriment, as they mature in growth. The circumstance of Mr. Mill not having sufficiently realized the importance of this unconsciousness, will perhaps account for another opinion of his ; the only particular, we think, in which we differ from the generally excellent statement on the various modes of intellectual culture, which he has put forth in his article on Professor Sedgwick. We allude to his comparatively low appreciation of mathematical studies. While that science keeps the student's mind wholly fixed on matters external to himself, we know of none which can even pretend to supply its place, in fostering e. g. the power of separating in our imagination, accurately, vividly and consistently, distinct elements, which are never found by experience otherwise than in union ; or, again, of keeping steadily before the mind, so as fully to scan and realize its appearance, some group of ideas, each even in itself difficult to be clearly conceived, and which all of them use, as it were, their utmost endeavours to escape from the grasp of him who is viewing them ;—powers both of these (to mention no others) pre-eminently requisite, in dealing with phenomena so subtle, fleeting and evanescent, as those with which metaphysical researches have to do. An article in the *Edinburgh Review* some time since, which was occupied in disparaging mathematics as an instrument of education, laid great stress on the fact, that the greatest mathematicians have frequently ceased from those studies at an early age, and betaken themselves to metaphysical inquiry. Can there be a stronger confirmation of the view we have here taken ? There cannot, we are convinced, be any preparation for those inquiries nearly so good, as an early training in mathematics ; few so bad, as an early application to metaphysics. Nor are

we able to follow the very high authority of Mr. Froude, in considering the new mathematics less valuable, as an intellectual culture, than the ancient methods; so far from it, that we think the superiority of the former over the latter, in that point of view, to be so immensely great, as to be one of kind rather than degree.

To return, however, from this short digression. The following criticism on the ordinary habit of arguing directly from historical facts, is worthy of being well weighed:—

“The condition of politics, as a branch of knowledge, was until very lately, and has scarcely even yet ceased to be, that which Bacon animadverted upon, as the natural state of the sciences, while their cultivation is abandoned to practitioners; not being carried on as a branch of speculative inquiry, but only with a view to the exigencies of daily practice, and the *fructifera experimenta* therefore being aimed at, almost to the exclusion of the *lucifera*. . . . The only questions which engaged attention were, Is such an enactment or such a form of government, beneficial or the reverse, either universally or to some particular community? without inquiring into the general conditions by which the operation of legislative measures, or the effects produced by forms of government, are determined.”—p. 532.

“The laws of the phenomena of society are, and can be, nothing but the laws of the actions and passions of human beings united together in the social state. *Men, however, in a state of society are still men; their actions and passions are obedient to the laws of individual human nature.* . . . Now the method of philosophising” in question “overlooks this fact, and proceeds *as if the nature of man as an individual were not concerned at all, or concerned in a very inferior degree, in the operations of man in society.* All reasoning in politics or social affairs, grounded upon principles of human nature, is objected to by reasoners of this sort, under such names as ‘abstract theory.’ For governing their opinions and conduct, they profess to demand, in all cases without exception, specific experience.”—p. 537.

To refute this view the more completely, the author proceeds:—

“We shall suppose our inquirer acquainted with the true conditions of experimental investigation, and competent in point of acquirements for realizing them, if they can be realized in any case of the kind. He shall know as much of the facts of history as mere erudition can teach—as much as can be proved by testimony, without the assistance of any theory; and if these mere facts, properly collated, can fulfil the conditions of a real induction, he shall be qualified for his task.”—p. 539.

And then he goes through the various canons of induction, showing how absurdly inapplicable they are to the direct observation of historical facts. Proceeding from his refutation of others to his own statements, the following will be found interesting:—

“Since it is impossible to obtain really accurate propositions respecting the formation of character from observation and experiment alone,

we are driven perforce to that which, even if it had not been the indispensable, would have been the most perfect mode of investigation, and which it is one of the principal aims of philosophy to extend. . . . The laws of the formation of character are, in short, derivative laws, resulting from the general laws of the mind; and they are to be obtained by deducing them from those general laws; by supposing any given set of circumstances, and then considering what, according to the laws of mind, will be the influence of those circumstances on the formation of character. *A science is thus formed, to which I would propose to give the name Ethology.* . . . the science which corresponds to the art of education, in the widest sense of the term, including the formation of national character as well as individual."—vol. ii. p. 522.

We very readily accept Mr. Mill's admission (p. 527), that this science is as yet wholly unknown in modern times and in the philosophical world. Should his researches, however, be turned to the whole system of education existing in Catholic times, and of which the Church abroad still retains some traces, he must consider the inventors of that system no mean proficient in "ethology." The following remarks on Bacon seem of some importance:—

"Ethology stands to psychology in a relation very similar to that in which the various branches of natural philosophy stand to mechanics. The principles of ethology are properly the middle principles, the *axiomata media* (as Bacon would have said) of the science of mind: as distinguished on the one hand from the empirical laws resulting from simple observation, and on the other from the highest generalizations. And this seems a very proper place for a logical remark. . . . Bacon has judiciously observed that the *axiomata media* of every science principally constitute its value. The lowest generalizations, until explained by and resolved into the middle principles of which they are the consequences, have only the imperfect accuracy of empirical laws; while the most general laws are *too* general, and include too few circumstances, to give sufficient indication of what happens in individual cases, where the circumstances are almost always immensely numerous. In the importance, therefore, which Bacon assigns, in every science, to the middle principles, it is impossible not to agree with him. But I conceive him to have been radically wrong in his doctrine respecting the mode in which these *axiomata media* should be arrived at; although there is no proposition in his works for which he has been so extravagantly eulogised. He enunciates as an universal rule, that induction should proceed from the lowest to the middle principles, and from those to the highest, never reversing that order, and consequently *leaving no room for the discovery of new principles by way of deduction at all.*" . . . But "Bacon's greatest merit cannot consist, as we are so often told that it did, in exploding the vicious method pursued by the ancients of flying to the highest generalizations first, and deducing the middle principles from them; since this is neither a vicious nor an exploded, but the universally accredited method of modern science, and that to which it owes its greatest triumphs."—p. 524—6.

Bacon's method "entirely overlooked plurality of causes ; all his rules tacitly imply the assumption, so contrary to all we now know of nature, that a phenomenon cannot have more than one cause."—p. 373.

On the distinction between art and science :—

"The relation in which rules of art stand to doctrines of science may be thus characterized. The art proposes to itself an end to be attained, defines the end, and hands it over to the science. The science receives it, considers it as a phenomenon or effect to be studied, and having investigated its causes and conditions, sends it back to Art with a theorem of the combinations of circumstances by which it could be produced. Art then examines these combinations of circumstances, and according as any of them are or are not in human power, pronounces the end attainable or not."—p. 615.

"Art in general consists of the truths of Science, arranged in the most convenient order for practice, instead of the order which is most convenient for thought."—p. 619.

We will conclude with a series of quotations illustrative of the author's general views of political science. Certainly we claim no scientific proficiency in *these* matters for earlier times ; nor, as the reader will not fail to observe, are such views as some of those which follow, consistent with any real recognition of the unceasing agency of Providence. On this, indeed, we shall have something to say before the end of the article. Their conspicuous ability however admits, we conceive, of no possible question.

"If all the resources of science are not sufficient to enable us to calculate *à priori*, with complete precision, the mutual action of three bodies gravitating towards one another ; it may be judged with what prospects of success we should endeavour, from the laws of human nature only, to calculate the result of the conflicting tendencies, which are acting in a thousand different directions, and promoting a thousand different changes at a given instant in a given society. . . . But without dissembling the necessary imperfections of the *à priori* method when applied to such a subject, neither ought we, on the other hand, to exaggerate them. The same objections which apply to the method of deduction in this its most difficult employment, apply to it in its easiest ; and would even there have been insuperable, if there had not existed an appropriate remedy. This remedy consists in the process . . . of verification ; . . . of collating the conclusions of the ratiocination either with the concrete phenomena themselves, or, when such are obtainable, *with their empirical laws*. The ground of confidence in any concrete deductive science, is not the *à priori* reasoning, but *the consilience between its results and those of observation à posteriori*."—pp. 562, 563.

"There is one large class of social phenomena, in which the immediately determining causes are principally those which act through the desire of wealth. . . . I mean of course that portion of the phenomena of society which emanate from the industrial, or productive, operations of mankind ; and from those of their acts, through which the distribution

of the products of those industrial operations takes place, in so far as not affected by force, or modified by voluntary gift. . . . A science is thus constructed which has received the name of political economy. . . . Political economy . . . makes entire abstraction of every other human passion or motive besides the pursuit of wealth," "except those which may be regarded as perpetually antagonising principles to the desire of wealth, namely, aversion to labour, and desire of the present enjoyment of costly indulgences: . . . not that any political economist was ever so absurd as to suppose that mankind are really thus constituted, but because this is the mode in which science must necessarily proceed. When an effect depends upon a concurrence of causes, these causes must be studied one at a time, and their laws separately investigated."—pp. 569—572.

"It has been a very common error of political economists to draw conclusions from the elements of one state of society, and to apply them to other states in which many of the elements are not the same.

"Empirical laws of human nature are tacitly assumed by English thinkers, which are calculated only for Great Britain and the United States. Among other things an intensity of competition is constantly supposed, which, as a general mercantile fact, exists in no country in the world except those two."—pp. 574, 577.

"Those portions alone of the social phenomena can with advantage be made the subjects, even provisionally, of distinct branches of science, into which the diversities of character between different nations or different times, enter as influencing causes only in a secondary degree. Those phenomena, on the contrary, with which the influences of the ethological state of the people are mixed up at every step, . . . could not, without great disadvantage, be treated independently of political ethology, nor therefore of all the circumstances by which the qualities of a people are influenced. For this reason, . . . *there can be no separate science of government*; that being the fact, which, of all others, is most mixed up, both as cause and effect, with the qualities of the particular people, or of the particular age."—p. 578.

The French "are perpetually arguing that such and such a measure ought to be adopted, because it is a consequence of the principle on which the form of government is founded. . . . Inasmuch, however, as no government produces all possible beneficial effects, but all are attended with more or fewer inconveniences, and *since these cannot be combated by means drawn from the very cause which produced them*; it would be often a much stronger recommendation of some practical arrangement, that it does not follow from what is called the general principle of the government, than that it does."—p. 619.

"There are two kinds of sociological inquiry. In the first kind, the question proposed is, what effect will follow from a given cause, a certain general condition of social circumstances being presupposed? . . . But there is also a second inquiry.—What are the causes which produce, and the phenomena which characterize, states of society generally? *In the solution of this question consists the general science of society.*"—p. 585.

"The first branch of the science (statics) ascertains the conditions of stability in the social union; the second the laws of progress. Social dynamics is the theory of society considered in a state of progressive movement; while social statics is the theory of the consensus . . . existing among the different parts of the social organism; in other words the theory of the mutual actions and reactions of contemporaneous social phenomena: making provisionally, as far as possible, abstraction, for scientific purposes, of the fundamental movement which is at all times gradually modifying the whole of them."—pp. 594, 595.

"The empirical laws which are most readily obtained by generalization from history, . . . consist of certain general tendencies which may be perceived in society; a progressive increase of some social elements, and diminution of others, or a gradual change in the general character of certain elements. It is easily seen, for instance, that, as society advances, mental tend more and more to prevail over bodily qualities, and masses over individuals: that the occupation of all that portion of mankind which are not under external restraint is at first chiefly military, &c. and with generalizations of this description, ordinary inquirers, even of the historical school now prevalent on the continent, are satisfied. But these and all such results are still at too great a distance from the elementary laws of human nature, on which they depend. . . . They have therefore, in the minds of most inquirers, remained in the state of empirical laws, applicable only within the bounds of actual observation; without any means of determining their real limits, and of judging whether the changes which have hitherto been in progress are destined to continue indefinitely, or to terminate, or even to be reversed. In order to obtain better empirical laws, . . . it is necessary to combine the statical view of social phenomena with the dynamical, . . . and thus obtain empirically the law of correspondence not only between the simultaneous states, but the simultaneous changes of the different elements."—pp. 605, 606.

"The economical workings of society afford innumerable cases in which the effects of a cause consist of two sets of phenomena: the one immediate, concentrated, obvious to vulgar eyes, and passing in common apprehension for the whole effect; the other widely diffused, or lying deeper under the surface, and which is exactly contrary to the former. *Take, for instance, the vulgar notion, so plausible at the first glance, that encouragement is given to industry by lavish expenditure.* A., who spends his whole income, and even his capital, in expensive living, is supposed to give great employment to labour. B., who lives upon a small portion, and invests the remainder in the funds, is thought to give little or no employment. For every body sees the gains which are made by A.'s tradesmen, servants, and others, while his money is spending. B.'s savings, on the contrary, pass into the hands of the person whose stock he has purchased, who with it pays a debt he owes to some banker, who lends it again to some merchant or manufacturer; and the capital, being laid out in hiring spinners and weavers, or carriers, and the crew of merchant vessels, not only gives immediate employment to as much industry at once, as A. employs during the whole of his career, but

coming back with increase by the sale of the goods which have been manufactured or imported, form a fund for the employment of the same, and perhaps a greater quantity of labour in perpetuity."—pp. 399, 400.

In pursuing our proposed design, it comes first in order to consider Mr. Mill's paradoxical views on the origin of *mathematical* knowledge; which he boldly includes under the head of experimental. Now first let us observe this, that, putting aside for the present the question of *à priori moral* truth and its results, there is no one thing of which we are absolutely certain, except our *present* consciousness. *That* doubtless admits of no error; that at this moment of time I am writing, that my mind is occupied with certain ideas, that when I look at my desk I perceive the colour of green, that on looking up I receive an immediate impression which I take to be that of distance, that I believe in a certain chain of events as being those of my past life, all this is the mere statement of a matter of fact; but when I proceed beyond this, when I think that those events really had existence, when e. g. I call to mind that an hour ago I was taking a walk, and pursuing a certain train of thought, much more when I *infer* from past remembrances that walking is good for health, and also for thinking, and so on, for anything that the argument from experience has to tell me to the contrary, how can I know that I am not the victim of some miserable and complete delusion? how can I know that an hour since I was not seated on Mount Vesuvius, or carried beyond the bounds of space and time? how can I know that the mere inference from the premisses of a syllogism to its conclusion is not as extravagant, considered in the light of an approach to truth, as *we* should think it extravagant, from the size of a ship and the number of passengers to infer *à priori* the name of her captain? But if such an inference as this implies a confidence in our intellectual powers, wholly indefensible on grounds of experience, how far more is this the case (did impossibility admit of degrees) when we believe in the consistent and harmonious course of nature; which belief, Mr. Mill tells us, is an absolutely necessary condition to all argument from experience, and is nevertheless not to be justified, if we still take Mr. Mill for our guide, except by an intellectual process of very considerable extent and delicacy. Experience, in a word, has absolutely no ground on which to stand, no point from which to begin, until it be granted that our memory and our intellectual faculties are trustworthy informants.

We have dwelt on this consideration for two reasons: first, Mr. Mill, and those writers in general who advocate the claims of experience to be considered as the sole origin of our knowledge,

* Vol. i. p. 243.

seem to think that some additional certainty is by this view obtained for that knowledge, some additional safeguard against capricious or erroneous belief. Now, on the matter in hand, the observations just made sufficiently show how unfounded is this notion. The lower animals have as much perception of physical phenomena as we have; and yet they are able to draw from them comparatively no inferences at all, or even, which is more remarkable, are necessitated in some cases to draw inferences which are actually false. Why is it that man can, on the contrary, derive from what he perceives a structure of truth so vast and stately, and rests with such implicit confidence in the conviction that it is truth? Because of his intellectual powers, and the trust he reposes in them. No other answer can possibly be given: were it ever so probable that the *matter* of all speculation comes from the senses, it would still be absolutely certain that its *form* comes, and always must come, from those powers. And when, therefore, we proceed to examine whether an accurate survey of their nature and properties afford us grounds for confidently asserting that, in certain specified cases, they supply the *matter* of thought also, we are introducing no new instrument of inquiry whatever, nor making any new assumption. We assume no more, than that our intellectual faculties may be trusted; and we can assume no less, if we are to prosecute for a single step physical, political, or moral science in Mr. Mill's own method.*

Next, the distinction with which we started will be of advantage to us in instituting this survey. Let it be observed then, that although the ordinary course of nature, as we see it, could not by possibility convey any truth to our minds except by help of the intellect, the converse by no means follows. If the present chain of cause and effect were to be suddenly suspended; if fire were to cease from burning, and water from flowing; if birds were to descend into the streets and walk about like men, and chairs and tables were to be gifted with the power of originating motion; all this astonishing change about us would not throw even a moment's discredit on the operations of our intellect. That would proceed according to its own fixed and unchanged laws; would contemplate these new phenomena, compare them with each other, investigate how far they might be reduced to classes and traced to ascertainable sequences, and proceed in defining those classes or sequences accordingly.

Now if, as Mr. Mill contends, we derive our belief in mathematical axioms from experience, the same would be the case with

* We should observe that Mr. Mill himself in one place shows some consciousness of this, (though we cannot think that he habitually bears it in mind); he speaks (vol. i. p. 180) of something which "*cannot but follow consistently with some law which we regard as a part of the constitution either of our thinking faculty or of the universe.*"

them; and here, therefore, is the point on which issue must be joined. Taking Mr. Mill's instance, is it possible that an accurate thinker should believe in the possibility of two straight lines inclosing a space, and also in the trustworthiness of his intellectual nature?

"When we have often seen and thought of two things together," says Mr. Mill, in defending his account of the case, "and have never, in any one instance, either seen or thought of them separately, there is, by the primary law of association, an increasing difficulty, which in the end becomes insuperable, of conceiving the two things apart."

But this maxim surely tells both ways; it is quite as likely that the circumstance of experience invariably *confirming* necessary truths may prevent persons from at once perceiving that they were not originally derived *from* experience, as the reverse. And this we are compelled to consider the case with Mr. Mill. Let us appeal to his own consciousness. He tells us that

"If daily habit presents to the man of the most practised intellect for a long period two facts in combination, and if he is not led during that period, either by accident or intention, to think of them apart, . . . the supposition that the two facts can be separated in nature will at last present itself to his mind with all the character of an inconceivable phenomenon." (vol. i. p. 314.)

This he thinks a sufficient account of the certainty impressed on our minds by mathematical axioms; and of this he gives many instances, such as the antipodes, on which, nevertheless, experience has wholly disproved such alleged inconceivableness, by showing that what *was* inconceivable is found to be true. Mr. Mill has himself chosen these as instances precisely parallel in kind, whether or not in degree. Let us ask him then the question plainly; does he include the inconceivableness of mathematical contradictories in the same category? does he give us to understand that the extension of experience may possibly, some time hence, also bring before our cognizance two straight lines which *shall* enclose a space? if not, why not?

When Mr. Mill tells us that "the axiom receives confirmation in almost every instant of our lives," (p. 306,) we are at a loss to understand his meaning. Surely straight lines are not so very common in nature; still less, placed in such juxtaposition with each other as to impress on us the impossibility of their enclosing a space. When a general proposition, grounded on experience, is brought home to the consciousness of a young person, such as "fire burns," "wood swims," he immediately calls to mind, with greater or less vividness, an indefinite number of instances in which he has observed this to be the case. But when he first understands the proposition that "two straight lines cannot enclose a space,"

the process of mind is surely altogether different. It comes before him as a *new* proposition, not recognized as having been (whether consciously or not) in his mind before; but as *new*, and yet certainly true. The truth of this must of course rest with each man's individual experience, and to that experience we confidently refer it. But we have perhaps not rightly apprehended Mr. Mill's meaning in the assertion, though certainly in the original passage it seems as we have stated it; at all events, he presently takes a different point as the stay of his argument.

"We cannot so much as *call up in our imagination*," he says, "two straight lines, in order to attempt to conceive them enclosing a space, without, by that very act, repeating the philosophical experiment which establishes the contrary." (p. 318.)

What! will the "calling up in our imagination" the ideas of fire and water "repeat for us the philosophical experiment" which establishes the fact of the latter quenching the former? Who so zealous and forcible as Mr. Mill in repudiating the "theory that the investigation of truth consists in contemplating and handling our ideas and conceptions of things, instead of the things themselves? a process," he adds, "by which, I will venture to affirm, not a single truth ever was arrived at *except truths of psychology*." (p. 119.) Geometrical axioms, then, are "truths of psychology!" is that a nearer approach to Mr. Mill's opinion on the subject, or to the received one? And when the author comes to state his reasons for allowing himself in this *mental* experimentation on this particular subject, he shows, still more plainly, to how great an extent the real amount of his difference from the ordinary view is verbal and nugatory. Mr. Mill does not deny "the possibility of satisfying ourselves that two straight lines cannot enclose a space, by merely thinking of straight lines without actually looking at them," (p. 310); that is, from one single mental intuition we are justified in inferring an *universal* truth; and yet we may call that truth the result of experiment, in the same sense in which we give that name to our knowledge of the properties of horses or of gas! Surely here is mainly a question of words; Mr. Mill agrees really with Mr. Whewell and the rest of the world, but veils his agreement from his own mind, in order to serve the necessities of a theory, which he is unwilling to relinquish. He speaks for instance of

"The *peculiar property* of our impressions of form, that the ideas or mental images exactly resemble their prototypes, and adequately represent them for the purposes of scientific observation." (p. 317.)

Certainly this is a peculiar property of geometrical forms; precisely that property, which we express in other words, when we say that the truths, which concern them, are *à priori* and not experi-

mental. The very illustration he adopted a little while before, to show that it is an experimental process on which mathematical axioms rest, is very valuable in showing the reverse.

"We substitute," he says, "observation of the image in our mind for observation of the reality," "just as we should be scientifically warranted in describing the shape and colour of an animal which we had never seen, from a photogenic picture made of it with a daguerreotype." (p. 310.)

The latter inference of course being valid so long as the laws of *light* remain the same, and the former so long as the laws of the *human mind* remain the same. It would be impossible for Mr. Whewell or any one who agrees with him on this subject, to lay down the distinction with greater accuracy between *a priori* and experimental proof, than is done by this illustration.

On one point, no doubt, the author differs e. g. from Kant; viz. as to the means by which we derive our *first idea* of line and angle. But when we have passed that point (and it is surely one of minor importance) what difference remains? When once we have those ideas, Mr. Mill himself acknowledges that we are free from all further dependence on the senses; that, by a mere mental process, we are able to arrive at an indefinite number of new truths; and that these truths will be absolutely certain, neither dependent for their trustworthiness on any proof of the uniformity of the laws of nature, nor liable to overthrow from the progress of experiment. Surely then here is a most marked distinction from all the other truths of which Mr. Mill treats; and we have here a confession from himself, whatever he says to the contrary, at a distance of only two or three pages, that the contradiction of mathematical truths is inconceivable, in a wholly different sense from that in which the antipodes *were* (p. 315), or laws, other than exist, of chemical composition *are* (p. 322), by some *thought* to be so.

We obtain then these three results:—1st. Mr. Mill really holds that when the ideas of mathematical forms have once entered the mind, *new judgments*, in the strictest sense of the word, or, as we have called them, "*synthetical*," may be obtained concerning these forms by the unaided operations of the intellectual faculties: 2d. In order to conceal from himself this admission, he has been betrayed into a degree of intellectual confusion most unusual with him, in comparing the alleged necessity of mathematical truths with what has been at various times imagined to be the necessity of certain *physical facts*: 3d. That on one point nevertheless (though a minor one) he does really differ from the school of Kant on the subject. As our sole object is to vindicate against Mr. Mill the existence of *a priori* sources of knowledge, an

object which has been sufficiently attained on the matter in hand by the admissions we have drawn from the author himself, and as room is of great importance to us where so much remains to be considered, we shall not enter upon a discussion of this last point; but content ourselves with submitting to Mr. Mill's consideration the question, how the human mind obtains the ideas of a cycloid, e. g. or a lemniscata. Straight lines and angles have in nature what may possibly be considered their prototypes; but where has the student ever met with such figures as these last in the physical world? or any where except in mathematical books? As to the second point above-mentioned, the author himself reminds us of "the illusions" with which "even profound thinkers have satisfied themselves, when engaged in finding a general solution for a metaphysical problem," (p. 129); and we have no wish whatever to regard this lapse of his (for so we cannot but think it) in any other light.

But it is sorry work to rest in mere negation; and we will not therefore dismiss the subject, however much in haste we may be to do so, without laying before our readers a general outline at least of Kant's doctrine on the subject: a doctrine which, we are confident, will be the more heartily embraced, the more carefully and impartially it is considered by accurate thinkers. What then are our conceptions of Space and Time? for this question must come first in order. Generalizations from experience, answer the school to which Mr. Mill belongs: we compare tables or dogs with other tables or dogs, and obtain the general idea "table" or "dog;" so we compare parallelograms, triangles, &c. with each other and obtain the general idea Space.* Now this account of the matter will be immediately condemned by the consciousness of any man living. When we speak of London, Italy, the Sun, the fixed stars, &c. as contained in Space, it is quite certain that we do not mean to say that they are contained in Space, as Paris, Madrid, Manchester, &c. are contained in the general idea "town;" but, on the contrary, as the various squares and streets are contained in *some one particular town*: Space is not a general idea but an individual object; or, as Kant would say, our contemplation of it is not a conception but an intuition.

* Since writing the above we have referred to the "Analysis of the Human Mind," and find there an account slightly different from that in the text. From such a comparison as that in the text, says the author, we obtain the general idea "extended:" from another association of ideas we obtain the idea "infinity:" uniting these we obtain the idea of "infinitely extended." Then exactly the same relation that the idea of "sweetness" or "blackness," has to the idea of "sweet" or "black," the same, says Mr. Mill, has the idea of "Space" to the idea of "infinitely extended."—(vol. ii. 95—98.) The author of that work is the father of the present writer, and the work is frequently referred to in that which we are reviewing; more than once, however, such reference is made in order to express dissent from its views.

Now a *sensible* intuition it certainly is not ; what remains then but an *à priori* intuition ? and, moreover, the attribute of infinity is inseparably connected with that intuition ; so that the notion of Space, and of its infinity, is as essential a part of our intellectual faculties themselves, as is the process whereby we derive the conclusion of a syllogism from its premisses.* That Space indeed is peopled, as it were, in a certain particular manner, this we learn from the experience of our senses ; and we may with perfect facility conceive it peopled in a manner altogether different, or not peopled at all : but non-existent we *cannot* conceive it. Exactly the same argument may be pursued in regard to Time. The former idea moreover is the necessary condition of objective, the latter of subjective, experience ; without the former we could not perceive in their mutual relation the phenomena of the external world, without the latter those of our own mind.

Here then we have the solution of a difficulty which rather presses upon Mr. Mill. "The existence," he says, "of an extensive science of mathematics requiring the highest scientific genius in those who contributed to its creation, and calling for a most continued and vigorous exertion of intellect in order to appropriate it when created, may seem hard to be accounted for on the foregoing theory," [of his own]. Hard to be accounted for it is, we think, on any theory, except that which we are engaged in illustrating. But let us regard Time and Space as an *à priori* field, wherein the intellect may freely expatiate, unfettered by any restraint save its own *à priori* laws ; we shall then regard the fundamental propositions of arithmetic or geometry as real synthetical truths, obtained by mental experimentation in this field, and the whole most wonderful and magnificent science of mathematics as one vast record of triumph upon triumph thereon achieved.

But by far our most essential difference with Mr. Mill will turn upon the information derivable from our conscience or moral perception : and on this subject, such a vast variety of topics crowd into our mind, that the difficulty is to preserve any thing like order in what we have to say, and, where we shall have to suppress so much, to choose the most important points for discussion. It will be no part then of our direct object to prove, or even to state with scientific precision, what we believe to be

* We had almost said "as essential a part of our intellectual faculties themselves as the proposition that things equal to the same are equal to each other." But Mr. Mill (vol. ii. p. 162) considers even *this* proposition as an "inductive truth ; resting only upon the *fact* that it has been perpetually found true and never false." We cannot, however, but suppose this to be a mere oversight, which it would be unfair to press against him.

the truth of the matter; though incidentally something may be done in both ways. We wish to state those propositions, on the subject in hand, which bear directly on Mr. Mill's views, and would, if they were received by him, make it necessary that he should in some respects modify, in others altogether change them. We shall be the better able also to bring out our meaning, if we begin by considering some of the popular objections brought against the "moral sense theory;" which may be re-enforced also on the present occasion, (since the present work does not directly enter on the discussion,) by some arguments urged in one or two articles written by Mr. Mill himself.

The most popular and obvious of all objections, is that which Dr. Bowring expresses when he calls his opponents "ipse-dixit-its," and which is urged by Mr. Bentham, in an extract quoted by Mr. Mill in his review of that writer. It is surprising that persons of any acuteness should lay stress on this objection, since it lies with altogether equal force against innumerable convictions, which in no way profess to be derived from the conscience. Mr. Mill says, that "Newton saw the truth of many propositions of geometry without reading the demonstrations, but not, we may be sure, without their flashing through his mind"—(vol. i. p. 7): nothing moreover is more likely than that he would have been wholly unable to analyze the process. Here then, might Dr. Bowring have said, is an ipse-dixit-ist, who is trying to impose his own unfounded dogmas on the world for certain truth. But, in fact, "we are constantly reasoning," says Mr. Mill, "from ourselves to other people, or from one person to another, without giving ourselves the trouble to erect our demonstrations into general maxims of human or external nature" (vol. i. p. 252): and he proceeds to give some interesting instances of this habit of mind. Who is there indeed that has mixed much with others, and has not stored up within him ten thousand lessons of practical wisdom, the results of his personal experience, of which he has the most intimate and well founded conviction? And supposing him to enunciate these with that degree of confidence naturally generated by such conviction, and to be challenged for his reasons, what reply will he give? Will he not appear taken by surprise and unprepared with an answer? or if he gives an answer, how absurdly inadequate will be the reasons he produces to the weight of conviction they are brought to sustain! The various circumstances in his past life, which have assisted in giving him his conclusions, and which, if recounted in order, might satisfy Mr. Mill's most rigorous canons of induction, have perished from his memory; leaving behind them no record, save the conclusions themselves. And now what possible general criterion can be discovered, whereby the distinction shall be made clear between the

confidence which results from wisdom or knowledge, and that which is the offspring of mere shallowness, inexperience, and presumption? Or why should we be unwilling to grant, as we most cordially do, that in exactly a similar manner, in numberless cases, particular judgments are regarded as the dictates of conscience, which, in real truth, are far enough from having their origin in that sacred source: which are the mere reflexion of the opinion of those about us, or which spring from pride, passion, prejudice, indolence, bigotry, ignorance, intellectual feebleness? We certainly are not likely to overlook this fact; considering what multitudes of our countrymen at this day deem themselves but following their moral instinct, when they reject, nay denounce, Catholic doctrine and practice, as being opposed to essential principles of morality and religion.

As far indeed as regards our inability to judge rightly of *our neighbour's* conduct, we regard this as a simple benefit; we can hardly fancy a greater evil than would arise, had we power to refer other men and their actions, one by one, respectively to their true place in the moral scale: an assertion which we must be content here to leave as an assertion, having no room to pursue it. But for the purposes of *our own* guidance, doubtless it is very important that we should learn to distinguish the voice of our conscience, from that of the various clamorous intruders into its home, who strive to overwhelm it; and moreover to understand rightly that voice, *when* in fact distinguished. And it will be naturally asked, to how great an extent moral earnestness will of itself perform either of these indispensable tasks.

We answer, that as far as the first is concerned, it will perform it incomparably better than can be done by any intellectual exertion. Nor should we be surprised at learning that the same monitor, whose whispers in some are so faint, speaks in others with a voice so distinct and articulate; for what more is this than to say, that the moral faculty follows the same law with all our other faculties, that it is indefinitely strengthened by exercise and enfeebled by neglect? An uneducated person will in vain attempt to distinguish the inferences of his intellect, from the accident of prejudice, or the caprice of imagination; cultivate the former, and you bring it forward into the most distinct and un mistakeable consciousness. And yet can we even compare the difference, in this respect, between the intellectual and un intellectual, with that which exists in the parallel particular between the man of the world and the Saint? The former goes on from day to day without even the thought of thwarting his present inclination, except for the prospect of some other worldly good, health, wealth, respectability; the latter every minute of every day is sacrificing his own will to considerations of right. Is there any other faculty we possess, of

which the cultivation is one thousandth part so disproportionate in different men, as this is?—and if not, we are absolutely introducing no new law whatever into the theory of our nature, when we assert that the Saint possesses within himself a guidance on moral subjects, in its own sphere next to infallible; while the man of the world has inherited this punishment from his way of life, that the “light within him” has almost become “darkness.” Let a person then begin exercising himself in the habit of governing each action of the day, as it occurs, by the rule of right; we mean his own rule as it now is, however oblique that may happen to be, but still as being right; and he will be surprised to find how quickly, and how regularly, his moral perception will grow; and how soon he will obtain that faculty, whose very existence but now he disbelieved, of recognizing the real accents of the voice within him. We say, let him begin by acting on almost any rules whatever of conduct, so it be on the ground of doing *right*; and this is always possible even in the worst. Whatever else may be erased from the conscience, this original idea, that “right” and “wrong” are words with a meaning, and that a meaning wholly unconnected with consequences and results, this, if anything, is most certainly an innate idea of the human mind. You may bury it under the excitements of gaiety, sensuality, intellectual pursuits; but eradicate it you *cannot*: and when, from accidental circumstances, the former fail in distracting your attention from the misery which they have served to overlay, it forces itself on your consciousness, and inflicts on you a foretaste of hell. A wicked man, it has been over and over again said, cannot bear *himself*. “Is not the pain of a bad conscience different from any other pain that we know? Can it be compensated by the wages of sin, whatever they be—or rather, does it not, while it lasts, remain distinctly perceptible and entire in the midst of them?... Are not these feelings a type of the horror with which Angels now look, with which we shall look hereafter, on all transgressions of the law?” *

But as to the second of the two processes lately specified, the case no doubt is different. The conscience can express no judgment, except on a case brought before it. Now in many instances a case cannot be brought before it, clothed in all its real circumstances, and viewed in reference to such of its results as should be taken into account, without the careful performance of a very delicate and complicated intellectual operation; and in the *interpretation* again of the answer received, a similar operation is often necessary. This is the real cause of religious men going wrong in action; and this moreover is the real account of the difficulty which exists, in obtaining from the moral faculty general pro-

* Newman's Sermons, vol. v. p. 168—9.

positions, which may be applied in practice. And we use the word "difficulty," be it observed, in precisely the same sense in which Mr. Mill would speak of the "difficulty" of the inductive process: not as intimating a moment's doubt (God forbid!) on the trustworthiness of our informant; nor on the substantial accuracy, both of the results themselves, and of the means by which they are attained. Still these results have lost incalculably by the carriage as it were; by the transmission from their native soil to the repulsive and ungenial climate of definition and discussion: the delicacy of their tint is impaired, and their nicer shades of distinction blended and confused.

Here then we see one misapprehension of Mr. Mill's. "If it be true," he says,* "that man has a sense given him to determine what is right and wrong, it follows that his moral judgments cannot be susceptible of any improvement." The improvement of which he speaks must necessarily, from the nature of his theory, be one which springs from a more enlightened view on the consequences of actions. Now, though many untenable and irrational propositions have no doubt been propounded on the moral sense, who in the world ever went the length of saying, that consequences were not to be *considered* in morals? that the conscience will ordinarily decide exactly on the same line of conduct, whether the intellect presents to it the happiness or misery of those around us, as likely to accrue therefrom? On many matters, consequences almost constitute the morality of an action; and on many more, are a very important feature in its real character. On moral questions, which derive light from history or from political economy, even Saints may well differ, according to the quality of their intellect, their particular experience, or the arguments and information of which they may be cognizant; how does that tend to make their agreement less perfect or less significant, when they prefer obscurity to renown, contumely to praise, celibacy to marriage?

It would be obviously interminable to attempt specifying each class of instances, in which erroneous conclusions on moral subjects are ascribed to the conscience, while they are really due to the intellect. But one of them does deserve our notice; both from its peculiar character, and also from the circumstance, that more false judgments in it than in any other have resulted from a wrong principle of decision: we allude to judgments *concerning*

* London Review, vol. i. p. 135. We should mention that in his article on Bentham (p. 502) he revokes this opinion: "It is from no such source," (as success in the utilitarian controversy,) "that we look for the great improvements, which, we believe, are destined to take place in ethical doctrine." He proceeds, however, immediately to enumerate other propositions as "the doctrine of rational persons of all schools," with which, though not acknowledging ourselves irrational, we cannot coincide any more than with his earlier views.

others. To throw ourselves by an act of imagination into another man's circumstances, to conceive ourselves as having had the same internal and external experience with himself, to see what his actions and expressions mean, one by one, to do this fully, is a simple impossibility; to do it with any tolerable approximation to truth, requires about the highest and most difficult exercise of the mind that can possibly be conceived. But what if earnest and serious persons, in judging of others, do not so much as attempt this intellectual exertion? What then are their judgments in ordinary cases, but hasty and random sentiments, deserving no consideration whatever? A strictly conscientious person has been placed, we will say, within the pale of some religious system, which he has ever found fully congenial to himself, and well calculated to strengthen and support him in his road heavenwards; he feels accordingly every sentiment of devoted and unquestioning loyalty to that system. He meets with another, bearing equally every appearance of conscientiousness, whose circumstances have been very different, and whose feeling is very different also. He does not profess that the system is infallible; and yet he proceeds to condemn the conduct of that other as undutiful and unchristian. Or again, he has learnt from childhood to revere absolute government, as a peculiar gift of God; and has made his reverence for it the centre, round which are grouped all his feelings of devotedness and self-abnegation. He hears of persons who, on the contrary, regard political liberty as a public good so great, as to deserve well the most active and continual exertions of mind in its behalf; and, though they bear with them every appearance of religious earnestness, he without scruple loads them with the imputation of a lawless and rebellious spirit. However admirable and exemplary may be those who allow themselves in such judgments, plain reason shows us that the judgments themselves in the concrete are simply and entirely valueless; nor should all our reverence for those who form them blind us to the fact, that they are to blame for venturing on them, not we for disregarding them. They have altogether misunderstood the direct and immediate object for which conscience is given us, namely, for *our own* guidance; and they have "mistaken what is a lantern unto our feet, for the sun in the heavens."

It is desirable also to state expressly, what has been implied indeed all along, that by means of continued exercise not only does the voice within us become more distinct and authoritative, but the moral perception also to an indefinite extent more refined, delicate and true. And this again is only in accordance with the laws which regulate our other faculties; just as the painter who is familiarized with colours, by long habit acquires the power of discerning numberless varieties of shade, where the

ordinary observer sees but one uniform expanse. And if it be true, as it has been said, that "the science of morals extends indefinitely into the details of thought and conduct," each individual moral judgment through the day, performed according to this ideal standard, will be the conclusion of a syllogism; whose major premiss we derive from our moral, and minor from our intellectual apprehension; the former brought to its state of perfection by consistent and unwavering obedience to the sense of right from our infancy upwards, the latter by the appropriate cultivation of its observing, reasoning, and inventive powers. That under circumstances conceived in a certain way by the moral faculty, a practical line of conduct, conceived in another certain way, is the appropriate course, this is the major premiss; that the circumstances have been rightly conceived in the former case, and the line of conduct prescribed rightly interpreted in the latter, this is the minor. And though of course it is absurd in the last degree to suppose, either that this would be a *conscious* process, or again, that the range of things indifferent would, in the case of the best and wisest man living, be contracted in a degree comparable to this, still it is important to observe, that every individual moral judgment, so far as it is trustworthy, approaches to this type. This, too, is most essential to be remembered; viz. that of the two premisses which, as we said, produce in combination such a judgment, the major only is of vital importance to our own highest good. For the good of society no doubt it is most extremely desirable that the minor premiss be rightly assumed; and any practical system would be grossly deficient, which should not supply checks against at least the more extravagant aberrations of mind, involved in its assumption. But always supposing, of course, that we have to answer for no *faulty* neglect of our intellectual powers, it is our faithful and diligent obedience to the *moral* principle, on which depends the steady growth of our moral perception, and our real position in the scale of rational beings.

One caution requires to be added. It is impossible to rate too highly the value of prayer in purifying and enlightening the conscience; still it does not supersede the necessity of this intellectual act. This should be mentioned, because it is to be feared that some, before now, have fancied themselves to follow a commandment of God addressed to their moral perception, when they have really but surrendered themselves to the impulse of a dominant feeling. Of course, considering the wonderful rapidity with which the intellectual functions perform their office, a resolution may be formed in a moment of time, and yet all the circumstances, bearing on the case, may be virtually present to the mind, steadying it, and restraining it from a capricious movement. Only it is

necessary that they should be so present, if any weight is to be claimed as due to the decision.

From what we have said, then, it appears that the foundation of all morality is that original idea of right and wrong, which cannot be eradicated from the worst minds, but at the very lowest remains dormant and unconscious indeed, yet a really existing faculty, to which appeal may be made. As to the dictates too of this faculty, we observed that the mere random opinions about right and wrong, held by such persons as in practice follow simply the bent of their inclinations, are nothing worth as an interpreter of truth. And lastly, even in the case of those who should be habitually conscientious, were the *matter* (if we may use such an expression) of any particular moral judgment, the course of external action which the conscience might appear to recommend, ever so mistaken, this would in no way interfere with the value of such action in the way of moral discipline, and of still further quickening and enlivening the conscience itself; nor would the moral powers, but the intellectual, be responsible for the mistake. But now, what if the *form* of the judgment were wrong also, what if wrong *habits and dispositions of mind* should appear to be sanctioned as right; what remedy in this case would remain to the individual, or (which is more to our present purpose) what criterion of right and wrong to the world? We consider that the discussion of this difficulty will considerably strengthen the position which we have assumed. And that we may the better fix our ideas by a single instance, we will take an example, presenting, we suppose, as much difficulty as any that can be devised. Let us imagine the case of a person, who shall have learned to think resentment of injuries a sacred duty. And, unless this is supposed to remain hardly more than a barren and inoperative opinion on his mind, we must further imagine that he is placed in circumstances, where there is some sufficient scope for converting his speculative principle into a practical one. Further, any thing like a capricious and uncertain application of the said principle is at once excluded by the conditions we have required. The subject of our experiment may take a *wrong* view of his duty; but *on* that view he will act, by hypothesis, zealously and consistently. Let us follow him then in our conception, as he receives injuries one after the other, from a principle of duty compelling himself to resent them; taking himself bitterly to task when his anger in a particular case shall not have been adequate; going over in his mind the various details of the said case, in order the more effectually to inflame his resentment; conscientiously striving against the natural tendency of mere lapse of time to efface the memory of

injuries received, just as *we* should struggle against its tendency to weaken our good resolutions. It requires no very vivid imagination to see what indescribable confusion and disorder of mind would very speedily result, and what utter distrust of the principle itself would rise up in his mind. Then he would be led to question the authority from which he received that principle; and if he saw in them no eminent signs of conscientiousness (as he most certainly would not), he would finally, and without scruple, abandon it. But what will he substitute in its room? To suffer himself to be drifted, like the world around him, by the mere current of inclination, would be a contradiction to the one great truth, which we have assumed both to be strictly innate in all, and to have been called into vigorous exercise in his case. Supposing then even the most unfavourable circumstances, supposing him to receive no advice or suggestion from without, a very few experiments would sufficiently convince him, that if principle on the subject there must be, no other principle is possible in practice except that of patience and forgiveness. And this we may take as the fundamental condition on which our knowledge of morality may rest; not in this instance only but universally, none but the true moral principle, in each particular case, can bear the weight of earnest and consistent moral action.

And in all that we have said, we have neither taken into account the further help towards a right judgment which would be afforded by acting on fixed principles in *other* particulars in daily life (in consequence of the close and peculiar connection of all virtues with each other); nor again by the readiness with which a conscientious person recognizes a good example when placed before him, and follows it. This latter element, indeed, we hope presently to consider in some detail; but we may unite the former with the subject in hand, by making our general statement the following. First, the very idea of moral action implies that our several propensions and affections shall be governed by some fixed rule or other: secondly, that this rule shall be that which we call right, is absolutely necessary, in order to the preservation of internal peace and harmony among the various instincts of our nature; nay, in order to the very possibility of continued action in regard to any one of those instincts, on the part of a moral agent resolved to act consistently; and this last is the truth which we have endeavoured to illustrate, in the special instance we selected.

Thus far we have considered the unaided effect, in guiding and controlling our daily life, of that faculty, in virtue of which we are members of another world, wholly different from that visible one whereof our senses give us experience; and are fellow citizens, as it were, with the whole company of moral agents throughout the

creation. But in point of fact the moral faculty is not left to its own unaided powers; for one of the very earliest lessons it teaches us is the perception of superior goodness, and the duty of reposing an ardent and loving trust in the dictates of that goodness. Nor has it any other function so important and so astonishing, as that whereby it apprehends in this manner facts far beyond its own experience, appropriates them inwardly, and finally proves their truth, with a certainty wholly superseding the necessity of further evidence. In receiving some of these facts, it only anticipates its own future attainments; but the greater number, as has been often enough observed, are such as it could not have discovered, though it *can* prove.* It is not at all necessary to distinguish accurately between these respective facts; because in reality external guidance has been, in some shape or other, vouchsafed under all dispensations. Under the first class, however, would of course be ranked such truths as that "justice is a quality distinct from benevolence," "forgiveness of injuries is nobler than resentment of them," "celibacy is higher than marriage," and the like; we should be inclined also (though we speak with diffidence) to include in the same category the truth, that "duty and self-interest cannot on the whole and ultimately be separated;" a truth, the belief of which would bring with it a strong expectation of some future state of retribution. The leading and fundamental doctrine in the other class, we need not say, is the existence and attributes of God; Whose "conduct" is "as certainly determined" by the moral law which our conscience witnesses, as His "judgment" is "necessarily determined" by "speculative truth."† And while we do not profess to analyze fully the process, whereby the conscience proves what it could not discover, it is perfectly obvious, that belief in this doctrine imparts, in proportion as it is realized, a vast and inconceivable expansion to the moral nature, and endues it with a large range of powers, which it could not otherwise have possessed.‡

* Mr. Mill quotes Mr. Coleridge in his article on that writer (p. 298) as including even *Christian* doctrines among those which our unaided reason (in *his* sense of the word reason) can prove, though "it could never have discovered." On this subject we shall have to speak further on in the text.

† Butler's Analogy, part ii. chap. 8.

‡ If it be true that the idea of duty is more deeply rooted in our nature even than that of God (though it is painful to make such comparisons), a serious result follows in regard to Lutheranism. Since, however, some have considered us to have uttered "calumnious misrepresentations of this doctrine," it may be well to define what we understand by it. We consider it then a first principle, that the "*unum necessarium*," the only possible preparation for the enjoyment of any real blessings, is "obedience to the rule of right at whatever sacrifice of self;" and that any professed revelation, which should not fully recognize this cardinal principle as its very foundation, could not possibly be received by any serious mind. Now, whether or not a denial of this principle be rightly called, as we call it, Lutheranism, is a question of history; on which indeed we hope at some future opportunity to enter, but cannot enlarge here. But

Another very important caution may now be given, for the purpose of using rightly the testimony of our conscience. No moral truth can be really understood, except by one who shall have believed and acted upon it. Hence it follows, that there is the widest conceivable difference in value, between the judgment of religious persons on a doctrine which they have some time embraced, or, on the other hand, on one which they know only by its appearance, or by the representation of others. On the former their judgment may well be considered as little short of infallible; on the latter, especially when the proposed view may happen to be very far in advance of their moral experience, they are liable to the greatest errors and misconceptions. It is perfectly supposable, that some men may have acted consistently and zealously on the truths of natural religion, who might recoil from Christianity when first offered to their acceptance. They might fancy the doctrine of the Atonement to be at variance with God's attribute of justice, or even with the essential and fundamental laws of conscience; or they might fear lest belief in the Incarnation might tend to divert the minds of Christians from the pure adoration of the Invisible God to a less spiritual and heavenly worship; to draw away their affections from God the Father, and fix them on God "Manifest in the flesh;" nay, even to represent the First Person in the Blessed Trinity as less full of mercy and loving kindness than the Second. We could not be surprised if the Gospel presented this appearance, to those who regarded it from certain points of view; nor could we doubt that, so long as they believed this to be its real character, they would be bound to

that this principle itself is most inadequately apprehended among very serious persons of what are called "Evangelical" sentiments, is what we are surprised at any one doubting. Let such a sentiment as the following be heard, viz. "that the Gospel is a blessing in having relaxed the strictness of obedience required by the law;" or, in other words, "that Christians are not required to take such constant pains as the Jews were to keep their will in subjection to God's will;" surely such a view of things is not universally repudiated by them with loathing and abhorrence, as an insult to the blessed Gospel; and yet it would be so repudiated, if they held genuinely the principle above stated. This then seems the state of the case; they neither hold the Christian doctrine on the subject consistently, nor yet the Lutheran doctrine consistently; their conscience to a great extent neutralizes their Lutheran profession, and their Lutheran profession, on the other hand, to a great extent deadens their conscience. When therefore we speak of Lutheranism, we speak of an abstract doctrine, which cannot, we verily believe, be held consistently even by the devils; but which is held to an alarming extent among "Evangelicals," though inconsistently. And of this abstract doctrine we now say, that the considerations in the text show it to be worse, that is, to be more fundamentally at variance with our higher and better nature, than Atheism itself. As to the "rabid violence of language" which is laid to our charge in the same quarter, we entreat our readers to believe that our words are not the result of any temporary excitement, nor are they used oratorically to excite attention; they are no more than the adequate exponents of our deep and intimate conviction on the subject.

withhold from it their assent. A nearer and truer view of the divine system would disclose to them some glimpse at least of its holy and unearthly character; but it would not be until they had been led on by such a glimpse to union with the Church, that they would feel an intimate and intelligent conviction, how wide from the truth were their original surmises. They would then learn by sure experience, not (God forbid!) that the principles proclaimed by their conscience had been erroneous, but that they had been deceived in the matter of fact, in supposing Christianity to contradict those principles.

And now it is sufficiently plain why it has been necessary for us to go into this discussion; for Mr. Mill might have at first sight good reason for complaining, that we have entered upon ground which, in his present work, he has purposely avoided. "Truths," he says, "are known to us in two ways; some are known directly and of themselves; some through the medium of other truths;" and with the latter class, he goes on to say, logic is conversant. Now we have already, in our inquiry, accumulated a variety of cases, in which truths are known through the medium of other truths, and to which, nevertheless, no part of Mr. Mill's system will in the least apply. It appears that we obtain by means of moral action, exercised on knowledge already acquired, a *bonâ fide* increase of such knowledge, real synthetical judgments on moral subjects; and even more plainly on religious subjects also. By dint of mere contemplation, the very idea "God," e. g. while we advance in moral action under the influence of that idea, is every instant growing, expanding, developing itself into new results, within our mind; that is, we are obtaining every moment, unconsciously indeed and inexpressible by words, synthetical judgments on His nature and attributes. It appears also, that conscientious and holy men present to us, as the author would say, a most invaluable assemblage of phenomena, from which, by means of observation and induction, the most important *à priori* truths may be learnt. Nothing indeed in the whole world is surely more interesting and mysterious, than the presence of an uneducated person, who is drawing towards the close of a mortified and saintly life: possessed of heavenly truths the most marvellous and precious, yet having neither the power nor even the thought of defining them to himself, or expressing them to others; though at the same time watchful observers may, even from a gesture or a word, obtain some slight and transient glimpses of what passes within. True it is indeed that the most practised intellect must fail in conveying any, beyond the most miserably feeble and inadequate, idea of those truths, which are the life of the holy soul; but the peculiarity of the present case

is the perfect simplicity and unconsciousness of the object of our observation; the circumstance of nothing being further from his very imagination, than that he is cognizant of eternal verities, incommensurably more weighty and more transporting, than the most sagacious and wonderful conclusions from experience, or the most brilliant deductions of the pure intellect.

That such considerations, together with the whole theory of which they form a part, will be dismissed from their minds by many sensible people, as a mere illusion, we are well aware. The conscience, like the great Visible Embodiment of its lessons, does not "strive, nor cry, nor lift up its voice in the streets." "The moral element is from its very nature lowly, gentle, secret; it escapes from the thoughts of men, whose mind is dissipated in the din of outward things, and in the appearance of material objects; the spiritual eyes of such men as these find not the moral element, for they are turned outward; whereas, this element or divine seed is wholly inward, in the innermost recesses of the soul, wherein it silently dwells, secure as in some citadel."* Yet let any one for a single day act, with reasonable constancy, in disregard of other inclinations, and with a single eye to duty, he will have sufficient perception of right and wrong, both to understand, and to recognize as true, the general principles we have been laying down. Nay, which is very remarkable, a person who should have no more power than would be given him by such occasional periods of moral action, in understanding and sympathizing with real goodness, and who should be very far from having attained that "*patient continuance in well-doing*" which is absolutely required for salvation; so only he have mastered this great principle, that holy and self-denying men are the real fountains from which moral truth flows to the world, and so he have access to the company of such men; such a person, if endued with great powers of observation and analysis, may define and adjust moral truth, trace it to its ultimate laws, and compare it in its different aspects, with a depth and precision beyond the reach of very many, who are incomparably his superiors in all that is strictly valuable. So much of moral appreciation is gained by so little of moral action; and so great is the advantage of intellectual powers, even on the field where they might seem most uncongenial.

Still, though all religious persons must acknowledge, from their own experience, that conscience does speak in a manner distinct and unmistakeable, and guides them irresistibly in a certain direction, it by no means follows, nor is it true, that without such powers of analysis as fall to the lot of but few, they can make themselves

* Rosmini.

aware, by an effort of the will, what and how many truths are really engraven within its closest recesses. We were insisting, some way back, on the great desirableness that *none* should attempt the bewildering task of self-inspection, (quite a distinct thing from self-examination), until mature in mind, and prepared by special discipline. But *most* men are, to the last, unqualified for the attempt, and are grossly unjust to themselves if they make it; for those thoughts, which are most really precious, shrink back, as it were, the more from observation, in proportion to the intensity of the effort made to gaze fixedly upon them. It is not till such men meet with an external exhibition answering to their inward impression, that they become fully aware, by the spontaneous eagerness with which, waiting for no further proof, they cling to such exhibition, how much that inward impression really testified. It is as a protection against the possible effects of this disqualification, that external evidences have perhaps even their principal value; and such protection is also one signal blessing, which we derive from the history of the Church, nay, from the Scriptures themselves: for in truth this inward impression is the very element, which gives such apparent argumentative cogency to reasons drawn from those three sources. The real and complete remedy, however, for this fact in our nature is to be found in that principle of *faith*, which is so deeply implanted in our mind, and so prominently enforced in the New Testament;* and which moreover is of such unspeakable importance in our moral growth: a principle, by which we consider it our bounden duty, to accept those doctrines we have learnt, and remain in that position wherein we have been placed, not so long as we can find, by looking inwards, our warrant for so doing, but until our conscience forces on our notice that we have outgrown and overpassed them.

And the same principle holds with the mass of men. Close observation will show, that those who have not even so much cognizance of these matters as one day's real conscientiousness would give them, are nevertheless in fact governed and enlightened by their conscience, to a degree they little suspect. And this we should find, each one for himself, by a very possible contingency. Let any one of us on a sudden come into immediate peril of death, or lose great part of that which has made life dear to us, and we shall find a spontaneous analysis of our various mental

* How implicit faith, so strongly recommended in nature, in Scripture, and in the Catholic system, is prevented practically from allowing men to rest in wrong creeds, is a matter which we have so frequently of late handled in various shapes, that we fear the very mention of it may provoke ennui in our readers. Since, however, Mr. Mill might happen to see this article, and not others, we are anxious he should know that the question has been fairly grappled with.

impressions take place, of which we little fancied the results. Those truths, which are really entwined round our innermost nature, will assert themselves with a distinctness and authority, forming a most marked contrast to those other of our opinions, which have been inferred by the intellect, or impressed by education and example, or grown inveterate by habit. And this analysis may indeed be performed at once in a measure, if we will for a period shut out the world, and endeavour to make present to ourselves, by a vivid effort of imagination, the hour of approaching death. But in truth this general fact is so indisputable, that a modern writer, who differs fundamentally from such views as we have here been taking, both confesses it and explains it as follows:—

“ Men, who have rejected vulgar creeds in the days of health and prosperity, manfully opposing their clear and comprehensive views to prevailing superstitions, have sometimes exhibited the melancholy spectacle of again stooping to their shackles in the hour of sickness, and at the approach of death; . . . because that *intellectual strength, which repelled absurd dogmas*, had sunk beneath the pressure of disease, or the fears of nature, and left the defenceless spirit to the predominance of early associations, &c.”*

St. Augustine, we suppose, at the approach of death, felt a tendency to relapse into Manichæanism, or St. Cyprian into heathenism; having rejected those respective creeds, when their spiritual discernment obtained “clear and comprehensive views,” through the peculiar grace, always given for that purpose, “in the days of health and prosperity.” The acknowledgment, however, of the fact in question is only the more important, from the insolent absurdity of this attempt to account for it.

It will tend to exemplify more pointedly the power of conscience, by itself, in producing firm conviction,† and also to put in a clear light Bishop Butler’s emphatic statement of the inestimable importance of Christianity, considered only as a republication of natural religion, if we be allowed to make a hypothetical supposition. Let us suppose, then, that before our Lord’s Advent, God had thought fit to choose certain men who, He foresaw, would be faithful to His grace, as a nucleus for the establishment

* Essays on the Formation and Publication of Opinions, p. 188; written by an author of whom Mr. Mill says, (London Review, vol. i. p. 341,) that “he has excelled most of his contemporaries in the difficult art of making philosophy popular.” We are not acquainted with his other works; but we might go far before we should meet so much of self-confidence and boasting, united with so much of shallowness, nay of positive drivelling, as in this volume.

† On this subject Mr. Newman’s Sermon called “Personal Influence the Means of Propagating the Truth,” (No. 4 of the University Sermons,) should be carefully studied.

of moral and religious truth. We need not imagine them gifted with any power of imparting *supernatural* grace, but only suppose that God would give those who should join them power for the substantial fulfilment of the *natural* law. It is plain at once from our principles, that as soon as they should be able to commence a visible course of action which should embody their doctrines, whoever had in fair measure (amidst whatever doubt, inconstancy of mind, and perplexity) kept the rule of right as his great safeguard, and who came within the sphere of their attraction, would, by the very necessity of his nature, cleave to them and submit to their guidance; and would moreover find a daily increasing assurance, in the witness of his conscience, that the new and higher doctrines he had adopted were real and holy truths. Furthermore, it is plain that matters altogether beyond the range of his conscience, which they should declare to be revealed to them from God, he would believe without hesitation or demur, from the confidence he had already attained in their divine mission; and that, should they even propound doctrines which at first sight startle and alarm his conscience, he would make himself very certain that his difficulty arose from no intellectual misconception, or partial apprehension of those doctrines, before he would allow himself to question them even for a moment. As converts increased, this confidence in them would indefinitely increase also; from his observing the vast diversity of natures and dispositions, who all give the same testimony with himself on the truth of their teaching, and who perhaps find a very principal rest and support, in some parts of the system which *he* has taken wholly on faith. And as we may well conceive the disciples to embrace every variety of intellectual and practical power, and as the infant Society will be in perfect harmony with itself, we shall soon have maxims of morality, applying the general principles of that science to their particular position; in each of which, reverting to our old analysis, the minor premiss will result from the intellectual accomplishments of some, the major will be, as it were, the dictate of the united moral perception of the whole body. These again, both in their moral and their intellectual phase, will receive additional proof of their soundness, from their effects in action. And soon we shall have a systematic moral and dogmatic theology begin to arise; and so things will proceed. But we need not pursue the picture, our inference is now sufficiently plain. Here is no proof from miracles, none from fulfilment of prophecy, none even from sacred writings; yet who can doubt, that so long as the teachers are saved from inculcating error in morals or religion, and so long as external violence does not interfere, this Society contains within itself every neces-

sary principle, for permanence and indefinite extension? And what is its bond of union? truth on the part of the teachers; conscience on the part of the taught. Teaching, both truthful and authoritative, this is the great rock, on which moral truth may in safety repose; this is the $\pi\epsilon\sigma\tau\omega$, which the enlightened lover of mankind will desiderate. It was for want of this, that from the period of the fall to that of our Lord's coming, so learned men tell us, each successive generation fell below its predecessor in moral knowledge; it is for the same reason, that Protestantism, during its short three centuries, has advanced with even more frightfully hasty strides, in the road to corruption and unbelief. And if any one should suppose the idea itself of unquestioning submission to authority, in its appointed sphere, unworthy of a rational agent, let him fancy the imaginary case of a son, blessed with an infallible father. With what a confiding and enthusiastic tenderness would he regard him! Yet how hopelessly would he attempt to recall to his mind all those incidents, in every hour of every day for so many years, which, taken together, justify such confidence; how confused would he be if questioned on the subject; how enraged, or rather astounded, shocked and bewildered, if seriously advised, nay, clamorously called upon, to investigate his father's claims on his deference, with an impartial and critical eye!

There is one fact in the constitution of our mind, which we are especially bound to notice when reviewing a work on logic; a fact, which has been implied in almost all we have said (as it has been, we think, in the Church's principles from the first), and which has nevertheless been, to the best of our knowledge, universally denied, even as a fact, by modern metaphysicians. We allude to the direct power of the will over the belief. That the will has great *indirect* power of the kind, is so plain, when pointed out, that, however little it has been generally considered, it cannot possibly be denied. It is hardly more than a truism to say, that external evidence on a moral subject will produce indefinitely different impressions on different minds, according to the *internal* evidence they respectively furnish of themselves; and moreover, that such internal evidence results from moral facts, which do depend directly on the will. All this, we say, is plainly undeniable; but more than this, we are convinced, is true: though the very notion of any direct power of the will over the belief seems considered by modern writers, even those endued with the greatest subtlety of mind, to result from a mere confusion of ideas.* Let us take every-day facts. The duty is often inculcated of believing an accused party innocent until he is proved guilty; of taking the

* See e. g. Kant, Logique, p. 116.

best possible view of our neighbour's conduct; and so on: but what is the meaning of such precepts, unless we have the *power* to believe him innocent or guilty, to take the best view or the worst? I meet a perfect stranger in a railway carriage; reason forms for me the following conclusion: "most men are irreligious, ergo this man is probably irreligious." Is it really true that I have not the *power* (what may be my *duty* is beside the question) to believe him a religious person, till the reverse is proved? Can I only *act* as if I thought so, and not really think so? no one, surely, who examines his consciousness, can believe this to be impossible. Again; a new doctrine is proposed to me, which seems to promise help and expansion to my moral nature, and is yet encumbered with certain apparent difficulties. The external and internal evidence being both granted, surely I am conscious to myself of a power, which enables me either to give in to these difficulties, or, on the other hand, to leap the gulf, to make a venture, to embrace and appropriate the doctrine offered to my acceptance. And the same character of mind is exhibited by adopting the latter alternative, with that which disposes a man to act with zeal and firmness on probable evidence, rather than to sit at home, indulge in speculation, bring before his notice the whole range of moral and religious doctrines, submit them to his criticism, and with impartial coolness count the apparent flaws in each. Nor can it be asserted, that this is to tamper in our mind with the evidence on one side of the question; the fact is otherwise: I do not conceal from myself, that I believe implicitly on evidence which, by itself, will not sustain implicit belief: I so believe, because I consider it God's will that I should do so; and I expect, by the very act of believing, to *get* more evidence. Nor yet am I wavering in my loyalty to truth; for the very question with me is, how truth can most certainly be attained. And there are a very large number of subjects, not ripe as yet for discussion, on which, by accepting without question the received view, I am both approximating more closely to the real truth, and cultivating more appropriately that habit of mind which will constitute me in due time a fit judge of it, than by any other course I could adopt.

Of course we do not require to be told by our opponents, that this power ranges only within certain limits; though those limits are neither narrow nor readily discerned. Nor can we enter here, as might be wished, into the whole subject thus opened to our view. We must express, however, a conviction, that if such a power did not exist, nay, were not habitually exercised, philosophers and enquirers would be the greatest enemies existing to public peace and tranquillity. Only think of the great multitude

of existing institutions, the discussion of whose claims to public support must be wholly beyond the power of the most indefatigable speculator living. Is he then to be defended in withholding from them his generous confidence? in considering himself under no *primâ facie* obligation to look up to them and support them? may he suspend all *belief* in their claims on his veneration, because he has no *evidence* of those claims? What would become of all loyalty, devotedness, patriotism, if such views were to become really general? To distinguish, indeed, the position here maintained from mere indolent conservatism, and to draw out the principles on which belief *should* proceed, is a most essential matter for consideration, and would require to be treated specially and at length in a formal treatise on the subject; but our limits will not allow us to pursue it further. Our great guide in determining the matter, as well as our great warrant for the principle itself, will be the natural unconstrained procedure of a holy person, destitute of mental cultivation: for in moral matters this, and this only, must ever be the great prerogative of the educated, that they do consciously what others do *un-consciously*. We will not however dismiss the subject without adding, that the two classes of subjects on which there seems the greatest claim for belief without adequate evidence, in addition to that just mentioned, are 1st. the attribution of the best possible motives to all other men, and 2d. a ready reception of new doctrines, which seem favourable to our moral culture.

And now, even before carrying further our sketch of the process, by which, as we believe, moral truth accrues, we may with advantage compare its results, with those by possibility attainable in Mr. Mill's method. And first we come to a class of doctrines, on which "*habemus confitentem reum.*" That there is such a thing e. g. as right and wrong; that purity, humility, veracity, are "right" in a sense wholly distinct from "productive of good consequences in the visible order of things;" such propositions as these, if true, are certainly truths which the author's method of proof cannot establish: for he *denies* them, on the very ground that they cannot be so established. Some sentiments also he admits, which, we fully think, do flow from his view of things, but which we were rather surprised at his acknowledging. For instance, unless we wholly misunderstand his meaning, he says, (vol. i. p. 172, 173), that when the schoolmen speak of "*rationality*" as belonging to the essence of man's nature, in a different sense from that in which the power of cooking could be said so to belong, "*we lose in them even that vestige of meaning grounded in the nature of the things themselves,*" which up to this point

was to be discovered in their statement. To the same head we may refer the circumstance, that whereas moral disapprobation, in the sense in which the word is ordinarily used by serious persons, cannot be accounted for on Mr. Mill's principles, so also he does not himself show any personal experience of that feeling. For instance, in his article on Mr. Sedgwick, who had expressed his fears lest, if utilitarianism were to prevail, the sinner "would no longer be abhorred," he replies that

"while men dread and abhor a wolf, or a serpent, . . . they will abhor with infinitely greater intensity a human being who, outwardly resembling themselves, is *inwardly their enemy*, and, being far more powerful than toad or asp, voluntarily cherishes the same disposition to evil."*

It is plain, without saying another word, that the writer who, being very sufficiently acute and discerning, thought this a real account of the sentiment in question, had never at that time distinctly known what that sentiment is. And the same account must be given of a passage in another of his works, where he almost goes out of his way to distinguish formally between our feelings of "moral approbation," "admiration," and "love," as "three distinct modes of viewing an action" wholly independent of each other. Christians, it need hardly be said, consider the perception of moral truth to be that, which both justifies, and ought to regulate, all our higher tastes and emotions; they feel it their duty to aim more and more at centering all their "love" and "admiration" in Him to whom their "moral" instinct first led them; and feel more and more, that nothing is worthy of admiration or love, except so far as it is morally good. By admiration we mean, what Mr. Mill means, the sentiment usually called (though we do not wish to beg the question by a name) *moral* admiration: *intellectual* admiration we can of course feel, for such men as Laplace or as Jonathan Wild. And on the second particular it may be better to explain by the way, in order to prevent misconception, that we do not of course mean that persons should attempt to *bestow* their greatest love on those whom they judge most worthy of it; to maintain this, would be to overthrow all the particular affections, all the charities, and with them all the happiness of life: we are all placed in immediate connection with some of our fellow-men rather than others; we are all so endowed as to sympathize with some of our fellow-men rather than others. But those plainly are most *worthy* of love whom God loves most; and God loves most those who are nearest to His image.

* London Review, vol. i. p. 133.

The view however, contained in this passage of Mr. Mill's, is so closely connected with much that we have said, that it may be worth while to delay for a minute in our course, and quote an extract:—

“ Every human action has three aspects: its *moral* aspect, or that of its *right* and *wrong*; its *æsthetic* aspect, or that of its *beauty*; its *sympathetic* aspect, or that of its *loveableness*. The first addresses itself to our reason and conscience; the second to our imagination; the third to our human fellow-feeling. According to the first, we approve or disapprove; according to the second, we admire or despise; according to the third, we love, pity, or dislike. The morality of an action depends upon its foreseeable consequences; its beauty, and its loveableness, or the reverse, depend upon the qualities which it is evidence of.”*

To appreciate rightly this sentiment, let it be observed that under every conceivable complication of circumstances, there is, did we only know it, one particular course of conduct which is the right course; again under every conceivable representation of circumstances to the conscience, there is one particular direction of the will, which is the right direction. The first of these may accidentally be adopted, though the will performs its function erroneously; or the conscience and will may have a true and healthy action, while, from intellectual error, the actual thing done is wrong. The former is the *πράγμα*, the latter the *πραξις*; the former the outward *act*, the latter the inward *action*. Now is it not most extraordinary that it is the first of these, as distinguished from the second, which Mr. Mill denominates the moral aspect, and which, as he thinks, addresses itself to our *conscience*?† The second, as far as he recognises it at all (which is not much), he includes under the “*æsthetic*” aspect; but it is the *external* standard of which Mr. Mill says, a few lines on, that it ought to be “paramount though not alone,” and that it is the “the most important mode of looking at actions and characters.” We have no wish at all to underrate the importance of the right outward act, as we hope to have sufficiently shown in all that we have said: it is doubtless of very great importance, that that particular phenomenon should take place in the visible course of things, which God wills. But when we are called upon to think this more essential than that inward action, by means of which the conscience is cultivated and enlightened, and moral or religious truths rightly apprehended, than we only see the more certainly that,

* London and Westminster Review, vol. iv. p. 503.

† The use of this word in such a connection throws a painful light on a sentiment of Mr. Mill's a few pages back, which at first sight appears considerably in advance of his earlier opinions. He speaks, namely, (p. 486,) of “the existence of conscience as a thing distinct from affection for God or man, and from self-interest.”

when Mr. Mill professes to analyze the moral sense, he is dealing with a fact of which he has himself no clear experience; and that, when he speaks of moral disapprobation, he is really speaking of something differing in kind, not in degree, from that feeling of which serious men are conscious.

But in order to enter at all thoroughly into the amount and kind of pain, which the serious Christian spontaneously feels, at such a sentiment as we have quoted above, the insult to morality which he feels to be involved in it, we must take into account a principle, which is deeply felt and believed by all religious persons, whether or not they have ever brought it distinctly before their consciousness. Those mysterious feelings which are called forth by the sense of beauty, (e. g. of music or of natural scenery,) and again by marks of affection from others, (e. g. from parental tenderness,) are media, by which heavenly and supernatural truths are, in various measures, shadowed forth to the believer's mind; by which are impressed faint, confused, and unconnected ideas of that unknown Beauty and Love, whose intuition in its full and harmonious completeness is reserved for a future existence. And that faculty of the mind which, as being the fit correlative of heavenly communications, receives these ideas, is that very moral faculty, with which Mr. Mill's theory denies them all relationship or participation. So far from its being true that morality is in its essence wholly separate from these feelings, it is only by means of the careful performance of moral duty, that we are enabled to derive from them those very impressions, which it is their primary mission to convey.

But more than this; their effect even on those less carefully disciplined minds, which are sensible of their influence, cannot, we are persuaded, be rightly analyzed, without taking into account, as a very essential part of the combination of causes producing the concrete result, that innate moral sense, which, as we have more than once already said, enters as an important element in a vast number of judgments, even in the minds of those who most resolutely discredit its existence. Who that has ever experienced those indescribable emotions of awe, reverence, delight, or love, of which we have been speaking, could bear to be told that those emotions are simply and wholly an illusion, so far as they point to any thing beyond and beside themselves? Yet it must be so, if all knowledge be really from experience. "The stream cannot rise higher than its source." That notion is not more extravagant, against which Mr. Mill is so commendably zealous, that by the handling of ideas or manipulation of names we can derive real knowledge about *things*, than that other, to which he is unconsciously still in subjection, that ever so careful and exten-

sive an observation and induction of phenomena, and ever so complex an association of the ideas thence derived, can give us information on that world which is "beyond the veil."

To pursue this consideration in other instances besides that just mentioned. That certain pleasures are more permanent, intense, satisfying, than certain others; that they disqualify us less for intellectual speculation, or sympathetic feeling, or the business of life; all this may doubtless be gathered from experience. But when we apply such epithets as "high," "noble," "elevating," "worthy of rational creatures," and the like to those rather than these, we are using mere unmeaning sounds, deceiving ourselves by words without ideas, unless we have that faculty, which Mr. Mill denies us, of direct communion with the spiritual world; by means of which all of us, consciously or not, and with degrees of accuracy infinitely varying, introduce the balance of eternal and immutable morality, to test, value, and compare withal, as to their real essence, the appearances of this sensible world. Here it is that we are glad to find Mr. Mill inconsistent; for in the maintenance of so sad an error, inconsistency is the less displeasing appearance of the two. It appeared that, from his views, no such feeling as that which we call moral disapprobation, could have a rightful existence; and we found alas! accordingly, that of that feeling Mr. Mill seemed never to have received a vivid, nay a distinct, impression. It appears now moreover, that on the same theory no intrinsic superiority can be acknowledged in temperance over gluttony, in courage over sloth and cowardice, in generosity over selfishness, meanness, and sordid penuriousness: nay, that the very words "intrinsic superiority" have no meaning. But here we are able to observe with pleasure, that the author is, as we should say, unfaithful to his own philosophy; for certainly there are no writings with which we are acquainted, where such natural superiority of virtue over vice is more cordially acknowledged and taken for granted. He in plain terms recognizes "man as a being capable of pursuing spiritual perfection as an end; of desiring, for its own sake, the conformity of his own character to his standard of excellence, without hope of good or fear of evil from other source than his own inward consciousness."* And all that remains for us on this head to desire, is that he would most seriously consider, on what principle he can recognize this, which shall, on the one hand, allow such consciousness to be something altogether distinct from, and indefinitely higher than, the perception of enjoyment, (for otherwise its pursuit would be undisguised self-interest,) and shall on the other

* London and Westminster Review, vol. iv. p. 485.

hand, escape an implied confession of the whole doctrine, which we are engaged in defending. Nay, further, granting as we fully do, that a peaceful conscience is the greatest enjoyment in the world, whether the very foundation on which such enjoyment rests be not a persuasion of the objective existence of moral truth; and whether, accordingly, a person, in whose eyes morality should only be precious as tending to that enjoyment, would not from that very circumstance be debarred from all power of attaining it.

And now we are able to see in part the reason, why the study of such a work, e. g. as the elder Mr. Mill's "Analysis of the Human Mind," especially of that part which touches on morals, may well cause such distress and revulsion of feeling, even in those who most fully admit the great thoughtfulness and subtlety of mind which it displays. The distress which is felt in the perusal of similar treatises, has led some into a notion, fraught with omnigenous mischief, that the elaborate definition and analysis of moral or spiritual truth is necessarily a depressing task, perhaps almost an unlawful one; and that a sort of sentimental declamatory sing-song is admissible in a work professedly scientific, on ethics, Christian or other. Whereas a really accurate philosophy, on the most high and transcendent doctrines, is not a depressing but a most transporting study; as those can tell, who have read even a little of such writers as St. Athanasius or St. Thomas. No! the general dissatisfaction we have mentioned proves, beyond the power of mistake, that, in such works as that above specified, the philosophy is *not* accurate, that those spiritual facts, of which the mind is conscious, are *not* truly acknowledged and taken into account.* And the mental pain arises, in part from an acute perception of contrariety of view on the most deeply important of all subjects; and in part from a tendency in the human mind, distinctly attributable to our natural corruption and which must be struggled against by moral action, a tendency to waver in trusting the most certain dictates of our highest nature, when we are brought into the presence of those who disbelieve them. Hence follows a momentary doubt, which is on the mind while engaged on such productions, (a doubt sufficiently real to throw a gloom over the mind, though not sufficiently distinct to be brought into consciousness,) whether, after all, those truths may not be really disputable, deprived of which "we should be of all men most miserable."

We should mention his belief in God, as an opinion on which

* The present author, with his usual conspicuous and most exemplary candour, has himself exposed some part of the fallacy which exists in the elder Mr. Mill's reasonings. (Vol. ii. pp. 503, 504.)

and St. Augustine & Thomas the Aquinas

Mr. Mill is untrue to his own principles; were it not that wherever he alludes to religious truths, in all that we know of his writings, he does so in a tone and language which rather appear to flow from an unwillingness to shock existing "prejudices," than from his own conviction: and in the present work (vol. ii. pp. 611, 612) he tells us plainly, that when to a right method on physical science shall have been added a right method on the "philosophy of history," "*the circle of human knowledge will be complete.*" So that, most shocking though it be to say so, we doubt whether we should do justice to his intellectual consistency, if we charged him with religious belief in any shape.

Proceeding now with our sketch of the process by which moral and religious truth has been acquired, we come to the first promulgation of the Gospel; and from that period downwards, we find that which was before but hypothesis and capability, converted into actual achievements, into living and energizing results. For eighteen centuries and upwards has the Church faithfully witnessed the great truths of morality and natural religion in their full circle, whether against Manichæan heretics in earlier, or Lutheran in later times. She has drawn out the scientific statement of God's nature;* she has proclaimed and enforced the various holy tempers required by the natural law, explored the theory of their mutual relations and dependency, discussed the mode of their application to the outward state of things at each succeeding period of her progress, and provided her children with invaluable help and support for their attainment; all this she has done, with a confidence, completeness, depth and illumination of view, so absolutely beyond comparison with any feeble efforts in that direction which may have existed in heathen times,† as most fully to justify Bishop Butler's remark, lately cited, that it would be difficult for us adequately to appreciate the blessings we have obtained through the Gospel, even without reference made to the information and gifts peculiar to itself.

And that information again itself is imparted in a manner altogether analogous to the natural system; as might indeed have

* See e. g. Petavius, "De Deo Deique Proprietatibus."

† It would require of course great acquaintance with heathen literature, to pronounce a confident opinion on their real progress in philosophy. Mr. Newman has not hesitated to say that the "great endowments of mind" which existed in those times, "had been expended on vanities, or dissipated in doubt and speculation." (Univ. Sermons, p. 314). And a very different writer has drawn attention to the barrenness of heathen philosophy, with the view of gaining support from that consideration, to the wretched sentiments he is endeavouring to uphold. We allude to Mr. Macaulay's article on Lord Bacon.

been expected from the same Author. It is addressed directly, not to the intellect, but to the conscience; and the sure proof of its truth is obtained by moral action: * only, whereas the information is supernatural, so also a most wonderful gift is poured into the Christian's heart, whereby the moral action is supernatural also. Such have ever been considered the doctrines of the Blessed Trinity, Incarnation, Mission of the Holy Ghost, and whatever other truths are consequent upon these; forming the subject-matter of that majestic system of dogmatic theology, which has been the great intellectual triumph of the Church. Nor will our sketch be complete, without some slight account, though it must be very brief, of the logical process on which that system depends for proof of its truth. We are not speaking of proof addressed to individuals; the reasons, which justify and require their faith in the Church's teaching, will be perceived, by referring a few pages back to the account of our imaginary Society; the only difference being, that under the Gospel, from the nature of the gift imparted, the disciple's faith becomes more intense and more vigorous. Then as to the Church herself: in what sense, and on what principles, Christians of any particular period may be considered to throw, as it were, into one stock their united spiritual wisdom and experience, and again to inherit that of their predecessors in the faith, insomuch that in a very true sense the Church militant may be regarded as one organized being, whose life extends from the Day of Pentecost to the Day of Judgment, and whose spiritual knowledge and experience grows unceasingly throughout that period; to discuss this question would be to examine mental laws, whose existence is perfectly notorious in non-religious matters also, (and so may be fairly assumed here,)[†] while their full investigation would lead us through a series of observations and analogies, in itself sufficient to fill volumes.

Viewing then the Church collective starting after the apostles' death on her aggressive course, we find her, as might have been

* "Whosoever heareth these sayings of mine and doeth them I will liken him unto a wise man, which built his house upon a rock; and the rain descended, and the floods came, and the winds blew and beat upon that house; and it fell not, for it was founded upon a rock."

† So Mr. Mill: "There is a consensus (among the social phenomena) similar to that existing among the various organs and functions of the physical frame of man and the more perfect animals, and constituting one of the many analogies which have rendered universal such expressions as the 'body politic,' and 'body natural.'" (So, "body mystical.") Vol. ii. p. 567. We shall not be supposed to overlook the fact, how far more intimate and more mysterious is this "consensus" in the Church; but its laws and principles are the very same.

expected, fully possessed of, and energizing in, those doctrines, which are the cardinal points of faith; e. g. the Trinity, the Incarnation, the Eucharistic Presence. Her intimate assurance of the truth of these, results from spiritual action; and also, we may add, from her perception of their wonderful accordance with the dicta of the Sacred Volume, as viewed by holy eyes in all its depth, fulness and expanse. And her *idea* of them, the *impression* they form on her mind, is infinitely indeed below the original truths themselves, yet is it "the nearest approach to them which our present state allows;" as being received by the *moral* faculty: that faculty, which is more truly heavenly in its origin, and in its nature more akin to heaven, than any other part of man's constitution. On these subjects, then, the task which remains for her is, to bring before her own notice one particular after another of her complex and mysterious consciousness, to regard it steadily and distinctly, to project it, as it were, from the moral on the intellectual faculty, to express in accurate language the result of such projection, and to follow out the result so obtained into those intellectual consequences which necessarily flow from it. Or to use the words we adopted in the earlier part of our article; the science of these doctrines (and it is a science which has been in fact growing, we may say, almost to the present day, nay, which is still pregnant with an indefinite number of unexplored inferences) will consist entirely of analytical and deducible propositions. Not that these truths are of a nature different from all other religious and moral truths, so as not to grow upon the spiritual eye by contemplation, nor to germinate in the spiritual mind by appropriation; but because *in point of fact* they were impressed, in their whole fulness, on the mind of the infant Church, and, like Minerva, were born full grown and complete in all their parts. In the mind of individual Christians, indeed, they do so expand and germinate; and we may as well add, for clearness, that propositions which are but analytical of the collective Church's experience, are synthetical to believers one by one; and that these dogmatic statements are in many cases inestimably advantageous, in directing the disciple's mind to a right apprehension of the revealed Objects. In the mean time to speak at length of the gifts, moral and intellectual, called forth by these investigations, or of their inherent dignity, greatness, fruitfulness, would be hardly even becoming while the words remain on record of one, who cannot at least be accused, as others might be, of theorizing on the mere creature of his imagination, or of praising works which he has not studied.*

* See the 14th of Mr. Newman's University Sermons.

Still, though the foundations of the faith were fully realized from the first, other principles there were no doubt, and very far from unimportant ones, which were deposited, as it were, in germ within the bosom of the Church; that her internal action might gradually nurture them, or external circumstances hasten their appearance on the surface. And on these subjects the Church herself does form synthetical judgments, by dint of moral action and meditation. In other doctrines, again, the spiritual experience, which she accumulates from age to age, forms a most important part of the premisses to be taken into account;* here then also part of the premisses are synthetical. And it should be pointed out distinctly, that when this theory of "development" is maintained, it is not necessary in order to account for it to allege, as the cause of such maintenance, the necessities of some immediate object, or undue sympathy with some external system. If developments had not existed in Christianity, it *would have been necessary to suppose that God worked a continued miracle*, to separate off Christian from all other religious and moral truth. It is of the very nature of moral belief, that the same principles shall appear in each successive age, in a new aspect, or a more advanced growth, or more harmonious proportions.

Such were the principles, on which European philosophy mainly advanced, for so many centuries: and it does appear to us, we confess, that taking physical science from Mr. Mill's representation of it; putting altogether out of the question the incomparable superiority of the ancient over the modern system in the importance and high nature of the truths attained, and again the certainty of the method by which they were attained; considering the two philosophies only in regard to the interest of their processes, and fecundity of their results; it does appear to us sur-

* The following admirable observations of Mr. Mill will illustrate the indispensable importance of ever keeping alive within the Church the vivid memory of her earlier and mediæval period, through the din of present action and controversy; while they also point out the great advantage of her maintaining formulæ and expressions which have come down to her, though the present generation of believers may be far from entering into their real force. (Vol. ii. p. 237.) "There is a perpetual oscillation in spiritual truths, and of spiritual doctrines of any significance, even when not truths. Their meaning is almost always in a process either of being lost or of being recovered; a remark upon which all history is a comment. Whoever has attended to the history of the more serious convictions of mankind, of the opinions by which the general conduct of their lives is, or as they conceive ought to be, regulated, is aware that while recognizing verbally the very same doctrines, they attach to them at different periods a greater or less quantity, and even a different kind of meaning. The words in their original acceptation connoted, and the propositions expressed, a complication of outward facts and inward feelings, to different portions of which the general mind is more particularly alive in different generations of mankind. To common minds, only that portion of the meaning is in each generation suggested, of which that generation possesses the counterpart in its own habitual experience. But the words and propositions lie ready, to suggest to any mind, duly prepared, the remainder of the meaning."

prising, how in the mind of a reasonable being even a comparison can exist between the two. And we say this, with no desire to underrate the great convenience and advantages of skilful surgeons and accomplished seamen, of gas lights, railroads and steam, nay nor of Mr. Macaulay's favourite "garden-chair and boiled chicken."* That the Baconian philosophy, when it shall be applied to history and politics, has a noble field of speculation before it, and will abound in interesting discoveries, we are not denying; still such application is, according to Mr. Mill himself, altogether in its infancy; and is hardly therefore yet to be taken into account. On this matter, however, we willingly abide by the saying "*de gustibus non disputandum*." But that the scholastic theology shall have been so commonly regarded among us, not merely as less interesting than physics, but as a sort of elaborate baby house, as a mere relic of antiquarian barbarism, this is one of those instances of matchless ignorance, folly and impudence, abounding in the present day, the very existence of which future ages will be tempted to discredit.

"The schoolmen of the middle ages," says Mr. Macaulay, "show so much acuteness and force of mind in arguing from *their wretched data*, that a modern reader is perpetually at a loss to comprehend how such minds come by such data."† If this brilliant, but shallow and irreligious, writer‡ had given himself even a little trouble to understand the subject, he might perhaps have obtained some light, on the subject of his perplexity. But on what principle he takes for granted that the "*data*" are "*wretched*," while confessing himself to have no idea how they were obtained, is not so readily seen.

One more point deserves remark, before we finally quit this subject. These doctrines of the Trinity and Incarnation, though they grew not in stature during subsequent times, yet grew in evidence: if indeed that can be said in any sense to grow in evidence, of which the Church possesses, from the first, knowledge rather than mere proof. But the right-minded Christian who, for whatever reason, sought confirmation of his faith in external facts, had obtained, by the history of fifteen centuries; by the experience

* See *Critical and Historical Essays*, vol. ii. p. 393.

† *Critical and Historical Essays*, vol. i. p. 396, article on Croker's Boswell's Johnson.

‡ We wish we could honestly say any thing in defence of the *religious* tenets expressed by Mr. Mill. But in other respects we cannot forbear from drawing attention to the marked contrast between these two writers, each discussing moral and political subjects in a Review. Mr. Macaulay eloquent, pointed and brilliant, but sacrificing every thing to the object of immediate display; insomuch that one would hardly gather from his writings, that he believed truth to have existence. Mr. Mill measured, cautious, wide-seeing, earnest, consistent, progressive, as one whose object is the search and the promotion of truth, and who feels that truth is a serious, nay a sacred thing.

of heretics and heretical bodies, how uniformly they had declined into an openly low practice and an openly apostate faith; by the testimony of holy men, who, in proportion to their holiness, testified both to the truth of those doctrines, and their deep agreement with Scripture; he had obtained external proof, not at all equal indeed with the intimate conviction attainable through moral action, but still more valid in degree, as well as far more persuasive and cogent in kind, than that on which the theory of gravitation rests for its support. And this mass of evidence was, as it were, torn from his grasp by the great movement of the sixteenth century. He was relegated by Protestants to a solitary and dreary exploration of the vast expanse of Scripture, unaided and uncheered; he was urged by the more consistent philosopher to the previous inquiry, on the genuineness and authority of that Scripture itself. Is any thing to be found, in the records of the fallen world, comparable to this in absolute infatuation? Other movements have been more openly immoral: for the gross practical corruptions, which stained the Church at that period, must in all fairness be taken into account as an extenuation; and the innovators certainly bore not on their front at all the same undeniable mark of anti-Christ, e. g. as the Arian heretics. But for unspeakable fanaticism and infatuation, if any event since the fall can be named in the same breath with it, we are ignorant or unmindful of that event.

The effects of this "enfranchising" revolution on philosophy have indeed been described by a writer of this day, who is favourable to it, with such exquisite naïveté, that we cannot forbear extracting his remarks:

"The Reformation (which was the vehement protest that authority was no longer the grounds of belief, but that reason alone could claim that title) had stirred all minds to new and vigorous action; and the philosophy of Descartes is the most striking product of the newly enfranchised reason. Dissatisfied both with the scepticism and dogmatism he saw around him; unable to find any firm ground in any of the prevalent systems; *distracted by doubts of every thing high or low, holy or trivial*; mistrusting the conclusions of his own understanding and seeing that his own senses often deceived him; he resolved to make his mind a *tabula rasa* and reconstruct his knowledge. He resolved to *examine the pretensions of every conclusion, and to believe nothing but upon the clear evidence of his reason*. He began by universal doubt. He not only cleared his mind of all its previous stock of opinions, but *pursued his doubts to the very verge of self-annihilation*."^{*}

* Westminster Review for May, 1843, p. 383. The article is *not* written by Mr. Mill. For the result, however, of this experiment of his, Mr. Mill may profitably be consulted. "His works," he tells us, "*are a rich mine of almost every description of à priori fallacy*." (Vol. ii. p. 384.) Indeed, Mr. Mill in general is very far from com-

In truth, the search after moral and religious truth by Mr. Mill's method, is every whit as absurd (to speak only of the intellectual features of the case) as would be the allegation of a moral sense in support of Euclid's 12th axiom. We may take a very straightforward test of this. The most bigoted opponents of physical science, those who most resolutely shut their eyes to its advantages, never at least deny so much as this, that physical science now-a-days is effectually studied; that its truths are realized with a daily increasing certainty and progressiveness. Certainty and progressiveness: these are the great tests of the availableness of a philosophical process; and they prove to us that, whether or not physics ought to be studied *at all*, at least they have been studied by the appropriate *method*. Now, as certainly as this criterion justifies the study of the external world on the principle of free inquiry, so certainly it does *not* justify the study of morality and theology on the same principle; whether that principle be exercised on the surface of Scripture, or on that of nature. Hear again Mr. Macaulay:

"According to the doctrine of the protestant churches, in divinity there cannot be a progress analogous to that which is constantly taking place in pharmacy, geology, and navigation. A Christian of the fifth century with a Bible, is on a par with a Christian of the nineteenth century with a Bible; *candour and natural acuteness being, of course, supposed equal*. (!!) Divinity, properly so called, is not a progressive science—*no learning, no sagacity* (!!) affords a security against the greatest errors on subjects relating to the invisible world."

None indeed; that is the very position we are maintaining. The only securities against essential errors on such subjects, are of a kind, to which this writer makes small allusion throughout his works. After the detail into which we have gone, we need not take pains to avoid any possible misconception, which might otherwise arise from comparing theological to physical progress; but, making the necessary abatements, is it not a most significant fact, that those very characteristics, certainty and progressiveness, are seen on the very surface so peculiarly to distinguish Catholic Theology? And what is seen on a general survey is seen no less in particular instances. He who attempts to grasp religious doctrine with his intellect, grasps a shadow and lets the substance escape. Every

plimentary to the post-Reformation philosophers:—"Since the revolution which dislodged Aristotle from the schools, logicians may almost be divided into those who have looked upon reasoning as essentially an affair of ideas, and those who have looked upon it as essentially an affair of names." (vol. i. p. 128). No one can have read the work without being struck with the circumstance, how far more respectfully he mentions the scholastic philosophers than any others.

* Critical and Historical Essays, vol. iii. p. 212. "In religion," he says a little further, "we can trace no constant progress," for "we find it difficult to conceive in what way the Church of Rome is to perish." p. 215.

one knows what it is to meet with persons who discuss *e. g.* the corn laws or foreign politics, on principles, we do not mean inadequate merely, but wholly incommensurate. With what tenacity and positiveness do they maintain such opinions as their prejudices may have dictated! or else, with what suddenness and instability do they suddenly veer, from one most decidedly expressed view, to its very contradictory! all the time having no more real grasp or appreciation of the subjects before them, than of propositions expressed in an unknown language. Mr. Mill may have met with such persons in society or in print, and knows very well the indescribably ludicrous appearance presented by such mock combats, to by-standers or readers. We can assure him (not wishing to give offence, but to express clearly our meaning) that this is precisely the effect, on ordinary minds, produced by such discussions on religious matters as from time to time occur in his writings; or rather, that such *would* be the effect, were it not that on so awfully serious a circumstance, the sense of the lamentable overpowers that of the ridiculous.

It will perhaps be asked, how such a view of things is consistent with our previous recognition, that there is really such a science as "ethology," according to Mr. Mill's conception of it; nay, our claim, on behalf of Catholic ages, to considerable proficiency in its knowledge. The answer to this will still further clear our position. It is absolutely essential, if we desire any tolerable distinctness of ideas on the matter in hand, to distinguish most carefully psychology, and the sciences which flow from it, from that science of morality, which we have been lately discussing, and on which theology is built.* The latter science is exclusively conversant with eternal and immutable principles, which bind the conscience of all intelligent beings; nay, as we have already said, (with reverence be it spoken,) which are the necessary laws on which God's actions proceed. Its organ of inquiry is that moral faculty, which is directly conversant with such principles; and the only fit depository of its truths, among beings constituted like ourselves, is such a Society as the Church; a Society challenging allegiance and teaching with authority. Mr. Mill, accordingly, as denying the moral faculty, disbelieves the science which depends on it, and rejects the claims of the Church. But psychology, though the art of holy living is most intimately connected with

* The Christian Revelation has, it must not be forgotten, its *external* evidences also; which are appeals to principles derived from experience. We have already alluded to some benefits which we derive from these; there are many others also which it is not necessary here to particularize. So again the historical facts in which the Christian doctrines are, as it were, imbedded, such as the contemporary high-priests, procurator of Judæa, &c. &c. must have been obtained by the Church through testimony, or through direct revelation. To speak of conscience as corroborating *them*, would be simply unmeaning.

many of its truths, still, taken as a science, is wholly experimental, and subject to Mr. Mill's canons; as much so as botany or geology. True indeed that none but religious persons can adequately cultivate it, but that is only because none others *have* experience of the most interesting and mysterious of its phenomena; and thus it happens, that even if Protestant* or infidel metaphysicians should appear to have more successfully examined other parts of our mental constitution (which we do not however think will be found the case), still, the Catholic ascetic writers and moral theologians must be considered as, beyond competition, the leaders of human intelligence in psychological inquiries.

The *art* of morals, be it then observed, while making incidental use of an indefinite number of other sciences, employs as habitually the science of psychology, as that of morality itself. For instance. I have injured my neighbour's fame by random talking; the science of morality pronounces as the dictate of natural justice, that I am bound to make him restitution, humble myself before God for my sin, and take pains to reform my character in respect of unguarded use of the tongue. Waiving the second of these duties; to perform the first, I must have recourse to a knowledge of practical life; to perform the last, I consult the science of psychology. In the first, I receive no help from practical religious works; nor will a serious Christian be likely to understand better, rather worse, than others the best *means* of performing the object in view: in the last, on the contrary, I receive every help from such books; and religious men, from the interest and care which they, and they only, take in such matters, understand the way to such an object incomparably better than other men: but in either case the *source* of the knowledge is *experience*. From morality, in a word, the art of morals draws its *laws*, from

* We are obliged here to use the word "Protestant" in order to include such writers as Locke. Of course *religious* Protestants would have such experience as is now in question, but they seem diverted by their Lutheran profession from any methodical contemplation of the subject. Indeed, were they to acquire sufficient clearness of ideas to form a correct analysis, they would be led to see, how repugnant to the dictates of their conscience their profession has hitherto been. A sentiment is very commonly found in Mr. Maurice's writings, which puzzles us more than any other of his sentiments; viz. that "the truths, which constitute Protestantism, are those which concern man as a personal being, which assert his individual responsibility and relation to God, and provide that this responsibility and relation shall be realities, and not dreams." We should be not more surprised than pleased, if Mr. Maurice could show us in Protestant writers any knowledge or realization of these subjects, which can for very shame be compared with that displayed scientifically by St. Thomas and the moral theologians who have followed him; and practically by a hundred such works as the "Spiritual Combat," St. Francis de Sale's "Love of God," St. Alphonsus Liguori's Sermons, and "Preparation for Death," Salazar's "Sinners' Conversion reduced to Principles;" we may be allowed to add Mr. Newman's Sermons. We keep St. Ignatius' "Spiritual Exercises" for a separate place, since the wonderful insight into such subjects which that work displays, has led many to think it inspired, in a lower sense of the word.

psychology its *rules*. A pupil, who starts by taking both on faith, and acts upon them, learns from his *conscience* that the former are right, and from his *experience* that the latter are wise. He has been taught e. g. when he receives an insult, to think of Christ Crucified; in habitually doing so, he receives proof from his conscience that to endure an insult patiently is his duty, and from his experience that the thought of Christ crucified is a powerful help towards such endurance. The first, in Kant's expressive language, are *categorical*, the last *hypothetical*, imperatives.

And thus we are fairly embarked in a question, which it is impossible wholly to decline; viz. the compatibility and mutual relation of these two philosophies. That the right time indeed is as yet very distant for the methodical examination of this question, we are quite certain. Men, in general, have so long been in the habit of fixing their attention exclusively on experience, as the foundation of scientific knowledge; at all events they have so very little realized the independent and absolutely impregnable basis, on which moral and religious truths rest for their support; that to do this habitually and vividly must be the first step. If without such a preparation, they should dream of modifying the conclusions of one science by those of the other, they will make the most irrational and the most fatal concessions. This is all very certain; but the foundation itself, on which Mr. Mill places science, is simply and clearly subversive of all religion and morality; as serious men will at once see, and those who are not should be made to see: and this, being a matter of first principles not of final results, does demand from us some attempts towards its consideration and adjustment.

To prevent dangerous misconceptions, however, we must state plainly at starting, that it is wholly for the sake of experimental, not of *a priori*, science, that we are anxious for such adjustment; we must claim plainly for moral and religious truth its undoubted privilege, of being wholly independent, for its direct evidence, on any experimental or any intellectual support. "*Si fractus illabatur orbis*" of physical, "*ethological*" and "*sociological*," nay mathematical deductions,—"*impavidam ferient ruinæ*" scientiam illam: the expressions, arguments, intellectual deductions of the science might indeed suffer, but the science itself, in all essential respects, would remain scathless. Just as we formerly claimed for the intellect an absolute independence, for its own operations, of any observed regularity in the course of nature, so now we claim for the moral faculty an equally absolute independence of *it*; independence and superiority. So certain is this, that we hold it perfectly conceivable (we do not say probable) that, as part of the last fierce contest on earth between the powers

of good and evil, when "great signs and wonders" shall be shown, "insomuch that if it were possible they shall deceive the very elect," Satan may be permitted to bring a cloud over man's intellectual faculties, and represent, as certainly deduced by the reasoning powers, doctrines inconsistent with revealed truth. On such an hypothesis, it will be the evidence and the reward of men practised in piety, that their perception of holiness shall attach them even the more indissolubly to the doctrines and ordinances of the Church, from the unprecedented confederacy of strength which would be ranged in opposition. Nor is this all; nor is the moral faculty *independent* only of the intellectual; the former is the only sufficient warrant for the confidence we repose in the latter. Whether indeed that instinctive trust which all men feel in the operations of the intellect, be explainable on the mere principles of association, or whether it be not rather among that large class of judgments, in the formation of which the conscience is a most important, though unsuspected, element; this we need not discuss. But that, in order to *justify* such trust, we must resort to that element, we hold as perfectly indisputable. Most lofty indeed and splendid are the triumphs of the intellect in its own sphere; but by what possible inherent power of its own can it escape from that sphere? Most wonderful and sublime is the contemplation of that inexhaustible variety and subtlety of methods, which it evolves from itself; that vast assemblage of inferences, which it draws from a few simple principles through those methods, and unites in one consistent body by its own laws; but whither can it resort, when it wishes to prove those laws? Surely it must fret as it were impatiently, pent up within its own, infinite indeed, and yet limited, range, ignorant of truths and realities, and incompetent to discover or even think of them; until it be visited by some ray of heaven, imparting the hues of economical at least and adumbratory truth, to the dark agglomerated mass. Sceptics, not less than dreary fatalists, are refuted by one and one only thought. There be many that say, who will show us any *truth*; Lord, lift *THOU* up the light of Thy countenance upon us!

"All is dreary till we believe that we are subjects of God's governance; nothing is dreary, all inspires hope and trust, directly we understand that we are under His hand, and that whatever comes to us, is from Him as a method of discipline and guidance." "And what is true of this, is true of all the information which it has pleased God to give us." "What is it to us whether it be exact or vague, if He bids us trust it? What have we to care, whether we are or are not given to divide substance from shadow, if He is training us heavenwards by means of either?"

Mr. Mill may probably consider the argument we have pur-

sued for some way past, to be sophistical, yet he will hardly deny that the line of conduct, which it professes to justify, is practically successful. Whether or not men *ought* to be principally influenced by arguments consciously weighed, and canons of induction rigorously fulfilled, it is quite certain that they *are* not.

“The heart is commonly reached not through the reason, but through the imagination; by means of direct impressions, by the testimony of facts and events, by history, by description. Persons influence us, voices melt us, looks subdue us, deeds inflame us. Many a man will live and die upon a dogma; no one will be a martyr for a conclusion.”*

Consider the effects wrought by the mere sight of holy men; by the visible marks they bear of mortification and an unearthly life; by their manifest renunciation of those objects which reign in the hearts of the many, renown, or ease, or wealth, or personal aggrandizement; nay, only by their simple and straightforward fulfilment of those duties, which our conscience tells us are duties, and which the frailty and instability of our nature prevent us from attempting. That with such men as these in our front ranks, teaching with authority, rebuking with dignity, admonishing with spiritual wisdom, comforting the afflicted, restraining the wayward, cherishing the poor, we need care but little for any amount and any weight of mere arguments, by which it might be attempted to draw the people from us, Mr. Mill knows history and human nature too well to doubt. “O fortunatos nimium sua si bona nôrint.” Oh, that we knew, as well as our adversaries, wherein our real strength lies! Oh, that we would dismiss all petty cavils on this and the other doctrine, on this and the other supposed result! that we would prove to ourselves by experience, how real and consistent a basis of truth our conscience supplies, and throw ourselves resolutely and unreservedly on the dictates of our higher nature! And truly there are plain signs on all sides that we are beginning to move in that direction. Let us only realize to ourselves our position; let us be fully persuaded, that if we dread the aggressive progress of physical science, or other new and strange thing, our safety lies not in parley with the enemy, nor in special pleading on minute points, but in a frank and hearty defiance; not in coming down from our fortified city into the field, but in repairing the breaches and strengthening the walls of Sion.

It is from no fear then of any possible danger to Revelation, that we confess a desire to harmonize the respective principles of faith and experimental inquiry. If the state of things which has lately prevailed amongst us is to continue, one argument more

* Letters of Catholicus, p. 33.

or less matters little: protestantism will never be without its specious pretext, at each period of its descent, to lead its victims just one step further towards the abyss of unbelief. And on the other hand if, as we cannot but tremblingly hope, some revival be really at hand of Christ's Church amongst us in her native holiness, beauty and majesty, as soon would Mr. Mill's arguments keep iron apart from the magnet, as divert even one humble believer from bowing at her feet. But it appears to us plainly God's will, that the inductive philosophy shall be studied; and moreover, when it shall have been applied to those higher objects which Mr. Mill desires to include within its scope, it cannot but be a most valuable minister in the Church's service. And yet, if Mr. Mill's own principles alone could serve for its appropriate basis, it would carry with it on the surface its own condemnation. For what are those principles?

Mr. Mill considers that the "nooumenal" world is absolutely inaccessible to our faculties; but that the "phenomenal" proceeds unrelentingly and undeviatingly on laws fixed and defined from the first.

"If any particular state of the entire universe should ever recur a second time" (including, be it observed, mental phenomena of all kinds as well as physical) "*all subsequent states would return too, and history would, like a circulating decimal of many figures, periodically repeat itself.*" "The whole series of events in the history of the universe past and future, is . . . capable, in its own nature, of being constructed *a priori*, by any one whom we can suppose acquainted with the original distribution of all natural agents, and with the whole of their properties." —vol. i. p. 420.

Principles these, inconsistent with all belief in God's watchful Providence, in the efficacy of prayer for any temporal good, nay (as the author in terms confesses) of free will itself. The well-disciplined Christian learns to see the hand of God in all that befalls him, in health and sickness, in prosperity and adversity, and to trust in His gracious Providence, that he will be visited by no more of trial than he may well and salutarily endure. "It is no hand of God," says Mr. Mill, "and such trust has no foundation. Whether some overwhelming grief may or may not oppress you, depends on the concatenation of mental, social, physiological, and other phenomena, whose fixed laws enclose you on every side." We know as a fact, of a Roman Catholic nurse, who, being a most consistent and edifying person, told her employer, that in all her life she had neither taken nor administered medicine, without a prayer for its efficacy. "Delusion all," says Mr. Mill; "fit only for the earliest of the three great

stages of speculation.* The laws of physiology are not to be broken through, for every old woman who chooses to offer up a prayer." We trust to have already shown sufficiently, by our remarks at an earlier stage, that it is the old woman who is deep in the matter, and Mr. Mill who is shallow; the old woman who believes on evidence, Mr. Mill who theorises at random: and we shall have more to add presently in the same direction. But what serious man living is there, who would not rather die a death of torture, than believe that on essential truths of religion, such as this, holy and humble souls, "the little ones of Christ," shall be allowed to err, and an intellectual speculator to find the truth. "I thank Thee, O Father, Lord of Heaven and Earth, that Thou hast concealed these things from the wise and prudent, and revealed them unto babes."

We have forbore to comment on an allusion made cursorily by Mr. Mill to a divine interposition, in a passage close to that quoted above; and again in vol. ii. (p. 185); and on the inconsistency thereby introduced into his whole theory; for a reason which we lately specified.

The author's theory on necessity will also be best illustrated by a practical instance. I wake at a certain hour in the morning, and know very well that I ought to rise; I am nevertheless strongly tempted to remain in bed. Which shall I follow, duty or inclination? Shall I add to the number of those little acts of self-discipline, the performance of which, time after time, is that very thing which constitutes the strength of moral habit; or of those sins, which will appear against me at the Great Day? "You have no choice in the matter," answers Mr. Mill; "a being sufficiently sagacious, who might know the whole assemblage of phenomena, external and internal, at the previous moment, might prophesy with absolute certainty your determination." Now we have no wish at all to be unfair: we willingly concede, that such a view of things is perfectly consistent with an ardent zeal for training the will aright by means of education; for promoting in individuals habits of perseverance, justice, benevolence, of deriving enjoyment from the pleasure of others rather than from their own. That with which it is *not* consistent, is belief in the existence of *sin*: for where the will is as necessarily determined by the impulses which draw it in their respective direc-

* "Speculation" has, "on every subject of human inquiry three successive stages; in the first of which it tends to explain the phenomena by supernatural agencies, in the second by metaphysical abstractions, and in the third, or final state, confines itself to ascertaining their laws of succession and similitude." Vol. ii. p. 610. This detestable sentiment is adopted by the author from a French writer.

tions, as a mechanical point by given forces, there can be no room for the idea of responsibility, nor consequently of sin. And here again we are obliged to confess the author's consistency: such sentiments as "the proneness to evil in man," or "the corruption of human nature," or any other of the modifications which flow from the parent idea "sin," you may in vain seek throughout his writings. Those who well know that a deep and piercing conviction of sin is the only soil in which moral truth can really take root, will sufficiently appreciate the amount of disqualification for moral inquiries which this negation entails: the mere notion of a science of morals, without the idea of sin or of responsibility, will in truth be to most of our readers sufficiently startling. And indeed, with that peculiar candour which we have continually fresh occasion to eulogize, Mr. Mill fully confesses, that in fact "the free-will doctrine has fostered, especially in the younger of its supporters, a much stronger spirit of self-culture"* than the necessitarian; though he differs from us, in ascribing this result to a popular misconception of the latter scheme.

"It has never been admitted," says Mr. Mill, "by the religious philosophers who advocated the freewill doctrine, that we must feel not free, because God foreknows our actions."† It is curious here to see, how the author's ignorance of theology interferes with him in his own province. Theologians have ever drawn a *distinction* between God's foreknowledge, which we may call simple, and that fore-knowledge, which results from a deep insight into causes and a deep sagacity in calculating effects. The former is absolutely peculiar to God; but that a being endowed only with the latter, even in an absolutely perfect degree, could strictly foreknow our actions, is an allegation which we as certainly know to be false, as we know the doctrine of freewill to be true. They are mutually incompatible.

The human will then *is*, to use the common phrase, a "self-originating principle of causation;" and now let us see what this involves, and what it does not. The freedom which we claim extends to that class only of mental actions (of which we have already spoken so often, and the consideration of which, we are convinced, thinkers will more and more discover to be the very root of all moral science) in which some particular thing (thought, word, or deed) is done or forborne, from considerations of duty. Now if this be so, we have no more wish than Mr. Mill himself, to deny that the mass of men are really bound by the chain of necessity, which he winds around them; any one of them indeed may escape from it at any moment, by springing, as

* Vol. ii. p. 487.

† Vol. ii. p. 481.

it were, heavenwards; but so long as this one definite motive of *right* does not intervene, we are quite prepared to confess, as far as any supposed freedom of the will is concerned, that "given the motives which are present to an individual's mind, and given likewise the character and disposition of the individual, the manner in which he will act may be unerringly inferred."* There are explanations indeed, as we shall presently see, under which only this proposition can be admitted; but the consideration of these may as yet be waived. Every man living is free as to his will just so often, and just in proportion, as he exercises an act of self-control,† from the simple recognition of a rule of right. It will be probably true then to say, that the ordinary Christian is free for certain periods in each day, which may almost be counted. At the intervals of those periods, such moral action is left to work its appropriate effect, according to the established constitution of human nature,‡ in elevating the character. At the same time, the conscience is so far alive, as to be prepared any moment to interfere, when such established constitution shall be drawing him into serious sin; and even without such necessity, repeatedly stimulates him to fresh acts of freedom, to the more cheerful and energetic performance of some duty, to prayer at stated periods, to voluntary acts of self-denial. But the Saint, and the Christian in so far as he approaches to be a Saint, is free throughout the day; his life is one continued sacrifice of himself on the altar of duty, his conscience ever active, his will ever vigorous.

And now we shall be able to allay a misgiving of the author's, which, if we are not widely mistaken, is at the bottom of most of his anxiety on this matter. As early as the preface he adverts to the question, "whether moral and social phenomena are really exceptions to the general certainty and uniformity of the course of nature; and how far the methods" which have availed in physics "can be made instrumental to the gradual formation of a similar body of received doctrine in moral and political science." He trembles for his favourite "sociology" and "ethology," if the impertinent intrusion of free-will into a philosophical system be admitted. Yet he tells us himself that in such sciences an inquirer "can get on well enough with approximate generaliza-

* Vol. ii. p. 480.

† We say *self-control*, because we speak of *men*. Saints and Angels have, as it were, a *necessity* of freedom.

‡ Here would require to be considered the special characteristics belonging to the established constitution of *regenerate* human nature. We indicate this, but have no space to pursue it. And further, we do not forget that as the soul is possessed of *habitual* grace, so also the particular acts of freewill are encompassed by grace, both preventing and co-operating.

tions on human nature, since what is true approximately of all individuals is true absolutely of all masses.* And so long as conscientious persons form a miserably small minority in the world, the irregularities, and, as it were, eccentricities introduced by their freedom, will no more interfere with the general calculations of social science, than the mutual perturbation of the planets with the substantial truth of Kepler's laws. True indeed that the sanctity, which results in individuals from the steady and persevering practice of their freedom, does exercise over the mass of men an influence surpassing every other; but this influence may be calculated, according to the fixed laws of human nature. You will say that Saints are here and there maturing, and preparing, as it were, to exercise this singular power over the rising generation: true; and this consideration would effectually overthrow any pretence which science might make to *prediction* on the course of history. But other reasons altogether independent, as we shall presently see, would equally overthrow such pretence; and Mr. Mill, for *his* reasons again, expressly disclaims it.†

However we cannot deny, that the whole study of such subjects sinks more than a little, not in importance but in dignity, by introducing Christian considerations; by remembering that the whole class of phenomena, with which they deal, are mainly the various ramifications and results of human sin. Man, left to his own nature, acts on laws, which may be calculated and made the subject of a science: granted; yet that he was left to his own nature was a punishment for sin; the laws on which he acts are habits of sin; and that any individual does act on them exclusively is in his case an instance of sin. Psychology, we lately said, can only be adequately cultivated by religious men; the temptation is great, but it should not be irresistible, to leave history and "sociology" to those who seem their fitter exponents, men of a very opposite character. For the former science, we may observe, is not only not vitiated by belief in free-will, but indebted to it for its most recondite and interesting theorems. Free-will is possible only in one direction; and the discovery of the effects resulting on the other facts of the mind, by its exercise in all its various degrees, is fully practicable by means of observation and experiment.

And now our readers will naturally inquire, what is Mr. Mill's *proof* of this doctrine of necessity? and much surprised they will be by our answer, which is nevertheless a very true one;—

* Vol. ii. p. 152, 153, see also p. 494, 495.

† Vol. ii. p. 535. Mr. Mill is not always quite consistent in his statements as to the degree in which prediction will be possible in such matters, and, of course, even where he is least sanguine on the subject, he is a great deal too sanguine.

—none whatever, unless the misgiving above alluded to can be called proof. He has attempted to show that men's consciousness is not adverse to his view; in which, so long as a sense of sin is part of that consciousness, he must be considered to have signally failed. But for positive proof we look in vain, (vol. ii. p. 479—489,) except that in one place he calls his "proposition" "a mere statement in words of what every one is internally convinced of," which he will now see to be a mistake of fact.

Free-will then must be made an exception to a proposition, which needs no other exception, viz. that all the phenomena, external and internal, of the sensible world, proceed ordinarily on fixed laws. Ordinarily; for in every age of the Church, there are those particular interruptions of these laws, which make up one class of miracles; and, perhaps, externally to the Church also, a miraculous power has been from time to time permitted to instruments of Satan. The very notion however of such miracles implies, that those laws, to which they are exceptions, shall be so constant and universal, that the evidence of their *being* laws shall be accessible, nay obtrusive, on all. These fixed laws are indeed the very means of training us for heaven. Were the external world otherwise ordered, experience, forethought, prudence, diligence, nay beneficence, would be empty names; were the internal, neither growth in virtue nor cultivation of moral principle would be possible: the most vivid imagination would indeed vainly attempt to conceive the indescribable confusion, aimlessness, unmeaningness, which would result in our whole way of life. But more than this; these laws are to us the exhibition, type, and pledge of those Eternal Laws, whereby God's providence harmoniously and consistently works throughout His creation; Laws, the real nature of which is absolutely inaccessible to our limited knowledge; but which Saints and Angels, each according to his measure, behold with admiring love throughout their entire range. The regular course of nature then is even a sacred subject of contemplation; and it has ever been a matter of pious belief, that Angels are entrusted with the office of carrying on that course, in its undeviating and regular round.*

* A contemporary writer, endowed with the true poetical spirit, has spoken very beautifully of the "empyrean legions,"

"whose unwearied love
May wield the host of Heaven, until man's heart,
Used to behold rebellion in himself
And creatures like himself, hath disbelieved
That ought but matter dead is seen to roll
In that harmonious order."

Nature, a Parable, p. 4.

Moreover such laws are not on the surface merely, if such an expression may be used, but extend to an indefinite extent beneath it; as is plain from the continual generalizations and discoveries made by physical or psychological inquirers. Belief in this is indeed the very soul of those sciences. What then is the point of difference between the religious and the atheistical philosopher? This: The latter considers that we might be able to trace up all around and within us, had we sufficient faculties, to certain laws impressed on the phenomenal world from the first, and acting with relentless accuracy; from whose combination in their various proportions, all of which we have consciousness or sensation has its origin. But the Church has ever held, that were the whole course of nature laid bare to our view, with all its causes and constituent elements, when we had arrived in our search at the ultimate phenomenal agencies, (whether in the mental or physical order,) we should not see them acting on a blind unbending law, but the very contrary: we should see the finger of God moving them in turn, at the time, and in the degree, which may make them fit instruments for furthering His designs. And thus we perceive in some measure the vast array of mechanism which He has provided for His own use, wherewith to carry out, in all its minute details, that watchful and particular providence, which we learn from our conscience that He momentarily exercises. All things past, present, and future being present to Him as it were, in one glance, He impresses on mind and matter from the first those laws which, He foresees, will best co-operate with His purposes; and then His providence marches on with silent and steady tread, *through* those laws, *to* those purposes. These purposes, as has been often said, are individual as well as general, extending to the very humblest detail of every-day life; and one very principal cause which influences them, as we well know, are the prayers, and other intercessory sacraments and works, of faithful Christians. Those holy and humble souls, of whom we lately spoke, see far more real and essential truth, when they discern God's hand in all that befalls them, than do those who are most acute and sagacious in referring it to natural causes, if they stop there; and our friend the Roman Catholic nurse may have had her prayers answered, over and over again, as to the very object for which she prayed, not by the suspension of physiological laws, but by God's providence stimulating them, and acting through their means.

It has often been pointed out, that many of the miracles in Scripture and Ecclesiastical History, are of this character; such e. g. as the sudden calm after a sudden storm in the Sea of Galilee. These may be probably no more than instances of that

providence of God, which is proceeding everywhere and at every moment; their miraculous character consisting in the presence of the Divine Teacher, who lays bare the act of that providence, and summons it moreover at His bidding. In like manner, it is an extremely important truth, that Christian grace acts through, not to the exclusion of, human nature; as we have lately had occasion to point out at greater length. By self-discipline are formed habits of virtue, in heathens as in Christians; to issue in results and developments, is a property belonging to the doctrines of natural, as of revealed, religion; nor, again, can any one read e. g. the history of the Council of Constantinople or of Ephesus, without observing that if these Councils have really availed to fix irrevocably the doctrine of the Church, it was by means of no miraculous suspension of the laws of fallen human nature. And we are fully satisfied, that in proportion as genuine "sociology" makes progress, it will be found that such an institution as the Christian Church is peculiarly calculated, according to the ordinary laws of society, for the temporal as well as spiritual advancement of mankind.*

There is one only class of cases which can give a moment's difficulty. That some few out of the fixed laws of the universe act simply on an uniform principle, without providential interference, has been clear to all from the first; such as the alternation of day and night, and of the seasons; nay, that there should be some of this character might have been expected, from

* Of this we have already one curious instance. An eminent writer, now deceased, who grounded his speculative principles in a special manner on historical grounds, (but whose views, we may add, on history present a contrast to Mr. Mill's almost grotesque,) felt apparently as his greatest difficulty in receiving Catholic doctrine, that there would be always, on Church principles, two rival powers contesting the sovereignty of a state. Now an admirable article in the Edinburgh Review on M. de Tocqueville's "Democracy in America," (an article bearing the plainest internal evidence of Mr. Mill's authorship,) speaks of some such unceasing antagonism, as not only not an evil, but as our only possible protection against "an era of stationariness and decline." "The European family of nations," he says, "is the only one which has ever shown any capability of spontaneous improvement, beyond a certain low level. Let us beware of supposing that we owe this peculiarity to any necessity of nature, and not rather to combination of circumstances which have existed nowhere else, and may not exist for ever among ourselves." The writer presently discusses plans for the construction of "a great social support, for opinions and sentiments different from those of the mass." (October, 1840, p. 44, 45). And in one of his acknowledged writings (the article on Bentham) Mr. Mill says, even more expressly, "all countries which have long continued progressive, or been durably great, have been so because there has been an organized opposition to the ruling power." (p. 499.) We very much wish the author would turn his attention to M. de Maistre's book, "Du Pape dans son Rapport avec la Civilisation et le bonheur des Peuples," (Du Pape, liv. 3me,) especially the fourth chapter; for we are quite aware that M. de Maistre, notwithstanding his brilliant abilities, is a one-sided writer, and does not always go to the bottom of things; so that we should highly value the opinion of a competent judge on the arguments in that book, considered in the light merely of a historical investigation.

the infinite diversity of God's operations. Let us be allowed for our purpose, and that we may make our remarks clearer, to call these "fatal" laws. And without discussing the question how far ordinary persons are allowed to pray for a miracle, it is very plain, at all events, that to pray for a long day in December, is a very different thing e. g. from praying for rain or for fine weather. But is it not conceivable that the progress of physical science may bring to light such hidden laws, even of these events, that they may be capable of prediction as surely as the length of the day? So far as this is possible, those events even now take place by "fatal" laws, though we do not know them: and *if* this be possible, will not our prayers for those objects be damped and deadened by the apprehension that such prayer may be a mockery? such is the difficulty. Now we cannot take upon ourselves to deny the abstract possibility of some future revolution of physical science, which may discover that certain other phenomena proceed by "fatal" laws, as well as those of which we know this to be the attribute: there is nothing contrary to Revelation in supposing it, though the assumption seems extremely gratuitous. But if it be meant that the present aspect of physical science may well prepare us for such a notion, such speculators have the high authority of Mr. Mill directly in their teeth. In the vast field of new physical truths which the last two centuries have brought to light, we believe we are correct in saying that the only one discovery of a "fatal" character is the periodical return of comets. There is nothing else which physical philosophers can predict now, which all the world did not habitually predict in the days of Homer. And Mr. Mill tells us, (vol. ii. p. 5,) that the "ideal limit of the explanation of natural phenomena" is one "towards which we are constantly tending, *without the prospect of ever completely attaining it.*" Even Mr. Mill, with his determinedly "fatal" views, and very sanguine temperament on experimental subjects, is not sufficiently sanguine to see in the present state of physics even any *possibility* of discovering a "fatal" order in general phenomena. Again, of the particular event to which we just now referred, the succession of rain and sunshine, he speaks more in particular, (p. 490,) and concludes his inquiry by pronouncing that "the science is still very imperfect," and, "*were it perfect*, might probably of little avail in practice," so that we might "predict the state of the weather at any future time." His own "fatal" views then are a mere hypothesis; almost necessary indeed to one, who will not adopt Theism as the centre of thought, but unsupported by the faintest scintilla even of probable evidence.

We have said that it is impossible to set definite bounds to the
 NO. LXVIII.—OCT. 1843.

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conceivable results of some future revolution in science ; still so much as this will be confidently held by *all* Christians, viz. that the attribute of "fatality" will not be found *generally* inherent in phenomena. On two subjects, viz. the organization of the human mind and body, they know *as a point of faith* that they have not this attribute: the latter, because we all most intimately and habitually feel that the "issues of life and death" are, in a peculiar sense, in God's hands, depending very far more on moral than on physical laws ; the former, because, as we have already shown, *fixed* laws do not even universally prevail in it, much less "fatal;" and because also the overruling of those who are bent on evil to one evil rather than another, by means of the ordinary laws of the mind, is universally received as one of the most especial agencies, by which God carries out His providential purposes. Now these very two sciences, it is a most remarkable fact, psychology and physiology, are mentioned by Mr. Mill, as those which, very much more than any other, baffle the experimental inquirer in his attempt to penetrate the origin and causes of things. (vol. i. pp. 444, 537.)

And now the way is clear to our proposed inquiry. It is the dictate of our conscience, confirmed by revelation, to pray for those temporal blessings which, by God's dispensation, appear to us contingent; e. g. for rain. On the other hand—in the first place, there is no reason whatever to suppose, the very contrary, the "fatal" laws do prevail, among other phenomena than those in which they are at present acknowledged. In the second place, even were such "fatal" laws ultimately discoverable in this one phenomenon, how inconceivably bold to imagine that He, Who holds the hearts of men in His hand, might not suspend those laws, while unknown, in answer to prayer, without interfering with those observations, which He has predestined as leading to their ultimate discovery.* In the third place, granting that there is a possibility, indefinitely small, that neither of these suppositions has place, what results? Only, that prayer for such an event is offered up, under precisely the same conditions under which all prayer for temporal blessings *has* been offered, from the commencement of Christianity to this present day. Religious writers have always enforced the duty of implying in our mind a condition, when we pray for any worldly good ; for this reason, that we might misconceive the nature of that good, or that to us it might not be a real benefit, or (which this case would be) that *it might interfere with God's providential appointments in other directions*. And it has always been added that the prayer, even

* In the particular instance we have chosen for illustration, we know from Scripture that it is given or withholden in answer to prayer. See James, v., 17, 18.

if not granted in the sense in which it was put up, still remains not without fruit: that as to have omitted it would perhaps have been a sin, so the prayer itself avails in some other way to influence the course of God's providence here below, as well as to the sanctification and highest good of the suppliant himself.

For preferring what we have been compelled to call the atheistic theory to that which we have attempted to set forth, Mr. Mill, as we have already said, gives no reason whatever. He is not one of those philosophers who maintain (most strangely as appears to us), that we have evidence *à priori* of a "fatal" law of causation; his argument against such a view (vol. ii. 109, 110) seems to us very conclusive. He bases his own proof on Bacon's "*inductio per enumerationem simplicem ubi non reperitur instantia contradictoria*;" which, he very truly says, "is by no means the illicit logical process in some cases which it is in most." Still any one who will take the pains of reading the short chapter (p. 107—120) in defence of his position, will find that it leaves *absolutely untouched* the question between him and ourselves. This is very instructive. Those who are most bent against *à priori* presumptions, cannot avoid them. Mr. Mill is compelled by the instinct of his nature to seek for protection under some consistent, and as it were ultimate, view of things: that view which is graciously provided he will not accept; and at the expense of all his logic and all his caution, he flies, with what we may most truly call an enthusiastic and superstitious credulity, into the doctrine of necessity. And the result is this; that he promulgates a statement, which could not be received by the Christian even on demonstrative evidence (for the trustworthiness itself of his intellectual powers is a matter of less certain conviction, than are the truths which that statement opposes) and in proof of this harrowing theory adduces—simply nothing at all.

It would be a deeply interesting task to follow Mr. Mill's chapters on the social science, point by point, and trace what changes or modifications would be required by the Christian view of things. We cannot attempt even the most general outline of such a picture; and must content ourselves with observing that, referring to the quotations we made towards the beginning of the article, we have very much more confidence in the feasibility of a "social statics," than a "dynamics." The *differential* of a trajectory may be scientifically treated, though a force be continually acting, which follows a law wholly beyond our powers of calculation. The following account of the capabilities of social science is not, we think, interfered with by any thing we have said.

"The science of society would have attained a very high point of perfection, if it enabled us in any given condition of social affairs, in the

condition, for instance, of Europe or of any European country at the present time, to understand by what causes it had, in any and every particular, been made what it was ; whether it was tending to any, and to what, changes ; what effects each feature of its existing state was likely to produce in the future ; and by what means any of those effects might be prevented, modified, or accelerated, or a different class of effects superinduced."—(Vol. ii. p. 535.)

In truth all good men are occupied, whether in a wide or a narrow sphere, in observing the various tendencies around them, and introducing natural agencies, in order to check such as are dangerous, and promote those of an opposite character. Nor is such an one at all wavering in his allegiance to God, if he endeavour to obtain a scientific rather than empirical view of such tendencies and such agencies ; rather it may be his positive duty to make the attempt. But while he uses also a certain forethought for the probable and foreseeable exigencies of the morrow, he shrinks from forming more than the most humble and diffident conjecture, as to what new appearances that morrow may bring forth. Such contingencies are in God's hand, and are for ever issuing in fresh and varying forms from the unbounded resources of His power. Except so far as "fatal" laws may have place, we cannot even distantly approximate to *prediction*, through the processes of experimental science. This fact the Christian must look well in the face and thoroughly master, or he will continually be going wrong in the business of practical life, whether in its more retired or its public posts. This thought doubtless deprives political and historical science of much interest ; but can that be remedied ? If earth be our place of banishment, and heaven our home, can the Christian's chief interest be allowably placed, in studying the records of political changes or the growth of worldly civilization ?

And mark another evil, which has resulted from the concentration of interest on such sciences. There is an acknowledged tendency of the human intellect, to delight in the contemplation of that which is consistent and progressive in its course ; to be impatient of that which gains and loses ground alternately, which ebbs and flows, which oscillates, as if unmeaningly, to and fro. This is one great charm in the history of mathematical, and again of physical science ; and for those who feel that such a study is far too alien in kind from the summum bonum to deserve the highest place in their regard, a similar gratification is afforded in the history of theology, where they will find in like manner, as has been already said, a track of harmonious evolution and development. But with the history of human affairs it is widely different. If each period has its own gains to boast, so

has it also its own losses to lament; the later ages of the Church are not, as such, purer than the earlier; nay it would appear that "when the Son of Man comes" He will hardly find "faith" existing "on the earth." But in truth "every century is like every other; and to those who live in it seems worse than all times before it:" the peculiar note of disgrace attaching to our own times and country being, that to religious men among us (speaking generally) it does *not* so seem. And the cause of this fact is as obvious as the fact itself; being the perverse sinfulness of man's nature. Social science ranks below even physical in this important particular, as we have already said, that the laws on which it rests are the mere exhibitions of that sinfulness. But philosophers of Mr. Mill's school, acknowledging neither the cause nor effect, are led to devise for themselves the idea of some moral and political Utopia; they look forward with enthusiasm to some, perhaps not distant, period, when an education based on sound and enlightened "ethology" shall introduce a golden age into the world, which poets have dreamed of, but Baconian philosophers can alone realize. Considering indeed how strong a bias of this kind necessarily exists in the mind of the present author, he is most surprisingly candid in stating the "pro" and "con" of social progress; still when he comes to sum up, his mind seems more than a little warped; and his conclusion in the present work, though cautiously expressed, that "the general tendency is and will continue to be, saving occasional exceptions, one of improvement," seems a good deal more than the premisses in his other writings warrant.

And now we have at length brought to an end our protracted course of remarks; remarks on which we have the rather entered, because it seems that such matters must very soon come before the Church in a practical shape. It seems characteristic of the present period (and is probably a result of the same habit of mind which has, in our own memory, given new life to historical studies), that schools of thought, which have advanced perhaps for centuries on their own respective principles, are becoming, as it were, for the first time conscious of each other's existence. Again, the Protestant and infidel philosophies have had full scope to exhibit themselves; and the miserable issue to which they are tending is displayed, with more or less distinctness, to numbers who have been nurtured in them, or have been seduced by their show of wisdom and depth. Thinkers then are becoming, from distinct causes, at once less narrow-minded and also searchers for a new position. Thus we hear in Germany of intellectual Protestants resorting once more to the works of the schoolmen; nay even Mr. Mill professes himself to have derived much benefit from their philo-

sophical system. The eyes of men seem turned, in a certain inexplicable manner, towards the Catholic Church, as though expecting some unknown good to come forth from her bosom, to help us in our present extremity of both social and speculative perplexity. Now it would be most untrue to say that the Church has been stationary, during these three centuries; to mention none other of her intellectual achievements, she has strengthened the cord which binds together the various periods of her history, by the revival of patristic studies (a revival which, so learned men tell us, is attributable to nothing so much as to Petavius's great works); and moreover the heresies of Baius,* Jansenius, and Quesnel have been the occasion of imparting increased light and precision, nay partly giving a new direction, to the all-important doctrines of grace.† Still these are not the special accomplishments, which seem required by the exigencies of her present position. If she is to fulfil her appointed office; if she is to be a haven, to which those may flee for refuge, who are wearied and exhausted with doubt and speculation; if she is to be a light, whither those may resort for guidance, who have lost their way in the maze of human philosophies;—she must be prepared with a view, on the relative position due to the respective tenets of those philosophies; she must be enabled to satisfy inquirers, that the doctrines they have therein learnt, and of whose partial truth at least they have obtained a certain conviction, the methods of inquiry, the principles of evidence, need not, so far as they are true, be neglected, (rather may be the more effectually cherished,) when philosophers shall receive her authority. Such is the lesson, with which we may humbly hope that God will inspire His Church in His appointed time; and were an attempted contribution to this end absolutely worthless, as at best it must be most insignificant, merely to have drawn attention to it is, we are convinced, an important service.

Another reason which has induced us to seize the opportunity of noticing the present work, is the interest which we cannot help feeling in Mr. Mill himself. At a time when so much of charlatanerie, so much of reckless and audacious sophistry, is written and finds acceptance on the most serious subjects of thought, to meet with an inquirer, who bears every mark of a single-minded and earnest pursuit of truth, cheers and relieves the spirits. We trust we have not been wanting in strong and

* Baius *retracted* his condemned propositions.

† Our own more orthodox divines have, for various reasons, taken but little part in the continental development of doctrine; being indeed busily engaged at home in defending the very foundations of the faith. Still Bishop Bull e. g. seems considered by learned men to have made permanent additions to the theological fabric.

pointed rebukes of his miserable moral and religious deficiencies;* still we cannot but recur to that "manifest purity and intensity of devotion to truth" which is commemorated by a contemporary eulogizer of the present work, that "susceptibility to every breath of reason" which has so "refreshing and delightful an effect on the reader." Nay more; we cannot avoid building hopes on Mr. Mill's faithfulness to his own principles. His literary researches may lead him to study church history; can he fail of perceiving there, as a plain and prominent fact, phenomena of such a nature, that the very attempt to account for them on the principles of which he is cognizant, is extravagant beyond expression? He may chance to open such a work as Butler's "Lives of the Saints;" and when he has duly authenticated the main facts there recorded, can he fail to see in each one of those individuals, indefinitely varying from each other in natural disposition, in period, rank, education, sex, and age, a certain inward character surprisingly similar to that of all the rest; a character evidencing a certain complete fulfilment of the natural law; which, by his own methods of induction, he will be wholly unable to trace to any sufficient cause, unless he allows that conscience speaks articulately, and that the Christian character is its legitimate exponent? We do not of course forget that a Higher than human Agency must be at work, in such an effect as that to which we allude; we are speaking of these as occasions, rather than causes, of a change of mind. And for our own parts we take our leave of Mr. Mill with the assurance, that we never think of him without indulging the hope, that one who treats on so great a variety of subjects with such conspicuous power, knowledge, far-sightedness, earnestness, and single-mindedness, will not always remain so ignorant, and so careless of acquiring knowledge, as his writings show him to have hitherto been, on those things which alone really "belong unto his peace."

* We cannot however conclude our notice of these, without severely condemning his utterly gratuitous introduction of a most objectionable anecdote. We trust he will be advised to omit it, should his work reach another edition.

ART. IV.—*A Visit to the East; comprising Germany and the Danube, Constantinople, Asia Minor, Egypt, and Idumea.* By the Rev. Henry Formby, M. A. London: Burns. 1843.

THIS unpretending volume forms part of "The Englishman's Library," and is the most original work that has yet appeared in that series. It is original even to eccentricity, and to the very verge of paradox and over-statement; we will not say to the verge of error, though we confess that the writer sometimes leaves us breathless a little in the rear. This originality makes the volume come rather oddly in that otherwise very well marshalled and uniformed procession; but Mr. Formby evidently is not a man to keep time or walk in file. In vain does the serjeant call on him to fall in; vain are fife and drum, vain the obvious beauty of fifty men all dressed exactly alike, doing exactly the same thing: the empty show delights him not, and a music in his own mind prevents the intrusion of more social harmonies. He reminds us of an expedient adopted by that most brilliant of our contemporaries,—need we say *Punch*,—to supply a place in a procession of which it was kindly suggesting a programme for the opening of a city market,—a chimney sweep, walking in the contrary direction. It is true, that at the moment of his appearance, Mr. Formby is No. 25, in the Englishman's Library; but whether that relation to the series is more than instantaneous, time alone can show. It is indeed no easy matter to drill authors. Mind, if it exist, will have its way, and break out sooner or later in its own line. A notable instance occurs to our memory. Two men, at that time fellow collegians, and in daily communication, were invited, almost on the same day, to contribute parts to a Theological Library. Their subjects were given them; one, the history of the four first general Councils; the other, the history of the Inquisition. We had the opportunity of witnessing, day by day, the difficulties, the amplifications, the simplifications of their respective subjects. One grew into the "History of the Arians," and what else after that we need not say; the other into "Observations on Heresy and Orthodoxy." It is unnecessary to add, that neither appeared in the Library. Now Mr. Formby's book is not quite calculated to set the world on fire, but we cannot help thinking it is not the book one expects in "a series of cheap publications, adapted for popular reading; suited for presents, class-books, lending libraries, &c. &c." Conceive the upper class in a ladies boarding school, or a farmer's daughter, or your little god-child, just ten years old, stumbling upon such views as that the Mahometan religion is a very good one as things go; that the misfortunes

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